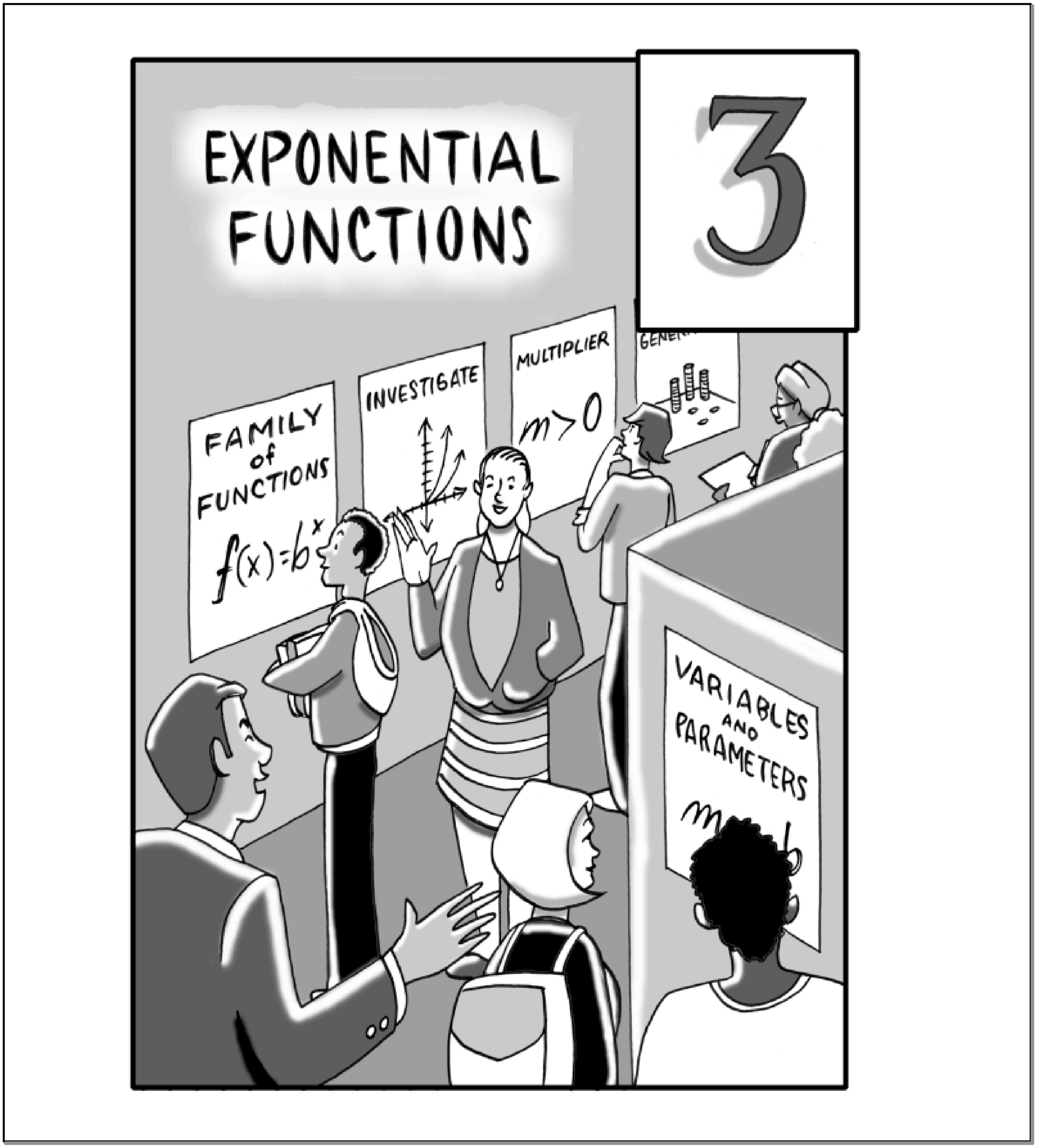
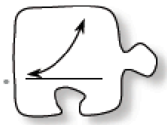

EXPONENTIAL FUNCTIONS
3
FAMILY of FUNCTIONS
f(x)=b^x
INVESTIGATE
MULTIPLIER
m>0
VARIABLES AND PARAMETERS

Chapter 3 Teacher Guide

Section	Lesson	Days	Lesson Title	Materials	Homework
3.1	3.1.1	2	Investigating $y = b^x$	• "Function Investigation" questions from Chapter 1 • Lesson 3.1.1 Res. Pg. (optional) • Computer and projector • Dynamic tool: *Exponential Functions:* $y = b^x$	3-7 to 3-12 and 3-13 to 3-21
	3.1.2	1	Multiple Representations	• Poster paper and markers (or transparency and overhead pens)	3-26 to 3-33
	3.1.3	1	More Applications of Exponential Growth	• Exponential web poster from Lesson 3.1.2 • Transparencies and pens	3-39 to 3-47
	3.1.4	1	Exponential Decay	• Rolls of pennies • Paper cups • Lesson 3.1.4 Res. Pg.	3-53 to 3-61
	3.1.5	1	Graph to Rule	• Lesson 1.1.2A Res. Pg. ("Team Roles") on a transparency (optional)	3-64 to 3-71
	3.1.6	1	Completing the Web	• Lesson 1.1.2A Res. Pg. ("Team Roles") on a transparency (optional)	3-78 to 3-86
3.2	3.2.1	1	Curve Fitting and Fractional Exponents	None	3-95 to 3-104
	3.2.2	1	More Curve Fitting	None	3-109 to 3-116
	3.2.3	1	Solving a System of Exponential Functions Graphically	None	3-121 to 3-130
Chapter Closure			Varied Format Options		

Total: **10 days plus optional time for Chapter Closure**

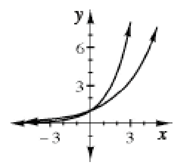
3.1.1 What do exponential graphs look like?



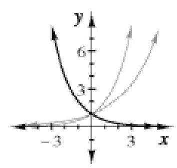
Investigating $y = b^x$

In this lesson you will **investigate** the characteristics of $y = b^x$. As a team, you will generate data, form questions about your data, and answer each of these questions using multiple representations. Your team will show what you have learned on a stand-alone poster.

3-1. BEGINNING TO INVESTIGATE EXPONENTIALS



In Chapter 2, you graphed several exponential functions. Some graphs, like those that modeled the rabbit populations in problem 2-4, were *increasing* exponential functions and looked similar to the two exponential functions graphed at right.



Other graphs, such as the rebound-height graphs from the bouncy-ball activity (problem 2-23), represented *decreasing* exponential functions and looked similar to the third curve, shown in bold at right.

You already know that equations of the form $y = mx + b$ represent lines, and you know what effect changing the parameters m and b has on the graph. Today you will begin to learn more about exponential functions. In their simplest form, the equations of exponential functions look like $y = b^x$.

Find three equations in $y = b^x$ form that have graphs appearing to match the three graphs shown above. Confirm your results using your graphing calculator.

3-2. INVESTIGATING $y = b^x$, Part One

What other types of graphs exist for equation of the form $y = b^x$?

Your task: With your team, **investigate** the family of functions of the form $y = b^x$. Decide as a team what different values of b to try so that you find as many different looking graphs as possible. Use the questions listed in the "Discussion Points" section below to help get you started.

Discussion Points

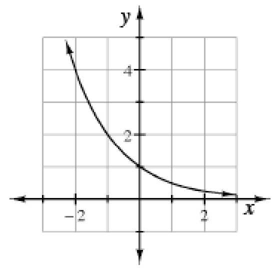
What special values of b should we consider?

Are there any other values of b we should try?

How many different types of graphs can we find?

How do we know we have found all possible graphs?

3-3. The graph of the function $D(x) = (\frac{1}{2})^x$ is shown at right.



a. Describe what happens to y as x gets bigger and bigger. For example, what is $D(20)$? $D(100)$? $D(1000)$? D(a very, very larger number)?

b. Does the graph of $D(x)$ have an x-intercept? Explain how you know.

c. When x is very large, the graph of $D(x) = (\frac{1}{2})^x$ approaches the x-axis. That is, as x gets bigger and bigger and gets farther to the right along the curve, the closer the curve gets to the x-axis. In this situation, the x-axis is called an **asymptote** of $D(x) = (\frac{1}{2})^x$. You can read more about asymptotes in the Math Notes box at the end of this lesson.

Does $f(x) = (\frac{1}{2})^x$ have a vertical asymptote? In other words, is there a vertical line that the graph above approaches? Why or why not?

d. Which cases for $y = b^x$ that you found in problem 3-2 have asymptotes? Review your graphs and record any asymptotes you find.

3-4. INVESTIGATING $y = b^x$, Part Two

Now that you, with your class, have found all of the possible graphs for $y = b^x$, your teacher will assign your team one of the types of graphs to **investigate** further. Use the "Discussion Points" section below in addition to the function-investigation questions your class generated in Chapter 1 to guide your **investigation** of this graph. Look for ways to **justify** your summary statements using more than one representation (equation, table, graph).

As a team, organize your graphs and summary statements into a stand-alone poster that clearly communicates what you learned about your set of graphs. Be sure to include all of your observations along with examples to demonstrate them. Anyone should be able to answer the questions below after examining your poster. Use colors, arrows, labels, and other tools to help explain your ideas.

Discussion Points

For our value(s) of *b*, what does the graph look like?
Why does the graph look this way?

What values of *b* generate this type of graph?

What are the special qualities of this graph?

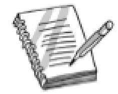
3-5. Exponential functions have some interesting characteristics. Consider functions of the form $y = b^x$ as you discuss the questions below.

a. Exponential functions such as $y = b^x$ are defined only for $b > 0$. Why do you think this is? That is, why would you not want to use negative values of b?

b. Can you consider $y = 1^x$ or $y = 0^x$ to be exponential functions? Why or why not? How are they different from other exponential function?

3-6. LEARNING LOG

Look over your work from this lesson. What questions did you ask yourself as you were making observations and statements? How does changing the value of b affect a graph? What questions do you still have after this **investigation**? Write a Learning Log entry describing what mathematical ideas you developed during this lesson. Title this entry "Investigating $y = b^x$" and label it with today's date.

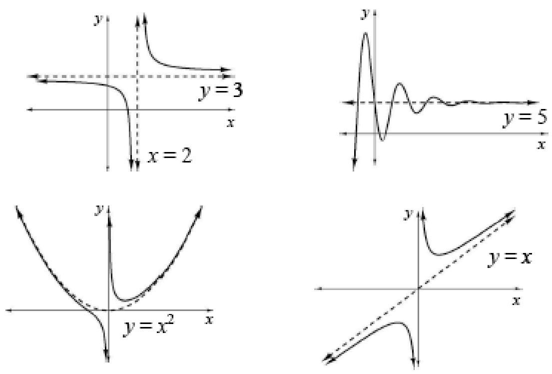


MATH NOTES

METHODS AND MEANINGS

Graphs with Asymptotes

A mathematically clear and complete definition of an asymptote requires some ideas from Calculus, but some examples of **graphs with asymptotes** might help you recognize them when they occur. In the following examples, the dotted lines are the asymptotes, and their equations are given. In the two lower graphs, the *y*-axis, $x = 0$, is also an asymptote.



As you can see in the examples above, asymptotes can be diagonal lines or even curves. However, in this course, asymptotes will almost always be horizontal or vertical lines. The graph of a function $y = f(x)$ has a **horizontal asymptote** if, as you trace along the graph out to the left or right (that is, as you choose *x*-coordinates farther and farther away from zero, either toward infinity or toward negative infinity), the distance between the graph of $f(x)$ and the asymptote gets closer to zero.

A graph has a **vertical asymptote** if, as you choose *x*-coordinates closer and closer to a certain value, from either the left or right (or both), the *y*-coordinate gets farther away from zero, either toward infinity or toward negative infinity.

Review & Preview

3-7. Solve each equation below for x.

a. $2^3 = 2^x$ b. $x^3 = 5^3$ c. $3^4 = 3^{2x}$ d. $2^7 = 2^{(2x+1)}$

3-8. A grocery store is offering a sale on bread and soup. Khalil buys four cans of soup and three loaves of bread for \$11.67. Ronda buys eight cans of soup and one loaf of bread for \$12.89.

a. Write equations for both Khalil's and Ronda's purchases.

b. Solve the system to find the price of one can of soup and the price of one loaf of bread.

3-9. If two expressions are equivalent, they can form an equation that is considered to be **always true**. For example, since $3(x-5)$ is equivalent to $3x-15$, then the equation $3(x-5) = 3x-15$ is always true, or true for any value of x.

If two expressions are equal only for certain values of the variable, they can form an equation that is considered to be **sometimes true**. For example, $x+2$ is equal to $3x-8$ only when $x=5$, so the equation $x+2 = 3x-8$ is said to be sometimes true.

If two expressions are not equal for any value of the variable, they can form an equation that is considered to be **never true**. For example, $x-5$ is not equal to $x+1$ for *any* value of x, so the equation $x-5 = x+1$ is said to be never true.

Is the equation $(x+3)^2 = x^2+9$ always, sometimes or never true? **Justify** your reasoning completely.

3-10. Solve $2x^2 - 3x - 7 = 0$ for x. Give your solutions in both radical form and as decimal approximations.

3-11. Consider the sequence that begins 40, 20, 10, 5, …

a. Based on the information given, can this sequence be arithmetic? Can it be geometric? Why?

b. Assume this is a geometric sequence. On graph paper, plot the sequence on a graph up to $n = 6$.

c. Will the values of the sequence ever become zero or negative? Explain.

3-12. If a ball is dropped from 160 cm and rebounds to 120 cm on the first bounce, how high will the ball be:

a. On the 2^{nd} bounce?

b. On the 5^{th} bounce?

c. On the n^{th} bounce?

3-13. Simplify each of the following expressions.

a. $(3x^2yz^4)^2$ b. $\frac{2mn^5}{6m^2n}$

c. $(pq^2)(p^3q^5)$ d. $(\frac{r^2s}{rs^3t})^3$

3-14. *Without using a calculator*, perform each operation indicated in the expressions below.

a. $\frac{3}{4}-\frac{2}{5}$ b. $\frac{3}{y}-\frac{5}{4}$ c. $(\frac{3m}{n})\cdot(\frac{m}{6n})$ d. $(\frac{5x^2}{y})\cdot(\frac{10}{x})$

3-15. Sketch the shape of the graph of the function $y=b^x$ given each of the following values of b.

a. b is a number larger than 1.

b. b is a number between 0 and 1.

c. b is equal to 1.

3-16. Find rules for each of the following sequences.

a. 108, 120, 132, … b. $\frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \ldots$

c. 3741, 3702, 3663, … d. 117, 23.4, 4.68, …

3-17. Calculate the x-intercepts for the graph of each function below.

a. $y=(x-2)(x+1)$ b. $y=2x^2+16x+30$

3-18. Write the multiplier for each increase or decrease described below.

a. a 25% increase

b. a decrease of 18%

c. an increase of 39%

d. a decrease of 94%

3-19. Simplify each of the following expressions.

a. $3x^2yz^3\cdot 2xyz^4$ b. $\frac{21m^5p^4}{3mp^3}$

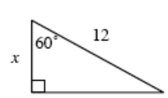
c. $(3rs^2t^3)^5$ d. $\frac{4a^2}{b}\cdot ab^4$

3-20. Solve the system of equations at right for m and b.

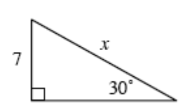
$$15=5m+b$$
$$7=3m+b$$

3-21. Find the length of the side labeled x in each triangle below.

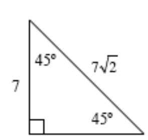
a.



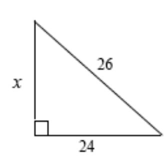
b.



c.



d.



Lesson 3.1.1 Resource Page

Student's Name	Team Roles
	Facilitator: Make sure the problem is read aloud. If conversation stops, or if your teammates think you have completed the **investigation**, reread the discussion questions for the team. *"Are there any other types of values for b that we could try?"*
	Recorder/Reporter: Listen for observations and make sure they are clearly stated and supported with reasons. Record these observations as a start to the team's summary statements. "*Wait, will you state that again? How do you see that? Why do you think that happens?*"
	Task Manager: Support open communication. Make sure all voices are heard. *"What values of b are you trying? What does it look like as a graph?"*
	Resource Manager: Organize the technology. Make sure teammates agree on how to enter equations and use the technology to see the graphs. If your whole team has a question, call the teacher over. *"How did you type that equation? Are we ready with a question for the teacher?"*

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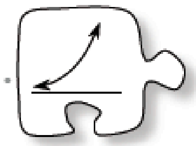
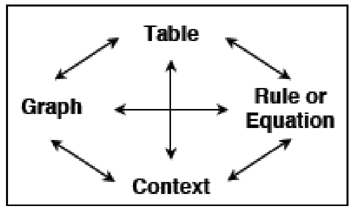
3.1.2 What is the connection?



Multiple Representations

So far, you have used multiple representations (such as a table, graph, rule, or context) to solve problems, **investigate** functions, and **justify** conclusions. But how can the connections between the different representations help you understand more about the family of exponential functions? Today you will **investigate** this question as you develop a deeper understanding of exponential functions.



3-22. What can a graph tell you about a rule? Can you predict the value in a table if you know about the particular situation it represents? Use what you have learned about exponential functions to answer the questions below.

Bounce Number	Height (cm)
0	
1	
2	84.5
3	67.6
4	54.1

a. Arnold dropped a ball during the bouncy-ball activity and recorded its height in a table. Part of his table is shown at right. What was the rebound ratio of his ball? At what height did he drop the ball? Write a rule that represents his data. Explain your rule.

b. A major technology company, ExpoGrow, is growing incredibly fast. The latest prospectus (a report on the company) said that so far, the number of employees, y, could be found with the equation $y = 3(4)^x$, where x represents the number of years since the company was founded. How many people founded the company? How can the growth of this company be described?

c. A computer virus is affecting the technology center in such a way that each day, a certain portion of virus-free computers is infected. The number of virus-free computers is recorded in the table at right. How many computers are in the technology center? What portion of virus-free computers is infected each day? How many computers will remain virus-free at the end of the third day? **Justify** your answer.

Day	Uninfected Computers
0	27
1	18
2	12
3	8

d. As part of a major scandal, it was discovered that several statements in the prospectus for ExpoGrow were false. If the company actually had five founders and doubles in size each year, what rule should it have printed in its report?

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3-23. Most of the exponential equations you have used in this chapter have been in the form $y = ab^x$.

a. What does *a* represent in this equation? What does *b* represent?

b. How can you identify *a* by looking at a graph? How can you find it using a table? In a context or situation? Give an example for each representation.

c. How can you determine *b* in each representation? Use arrows or colors to add your ideas about *b* to the examples you created in part (b).

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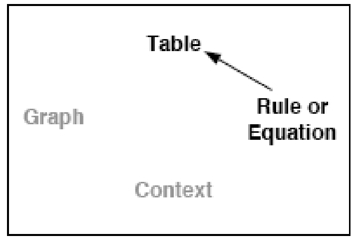
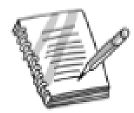
3-24. MULTIPLE-REPRESENTATIONS WEB

What connections are you sure you can use in an exponential-functions web? For example, if you have an exponential rule, such as $y = 20(3)^x$, can you complete a table? If so, draw an arrow from the rule and point at the table, as shown at right.



Copy the web into your Learning Log (but do not draw the arrows yet). Discuss with your team the connections you have used so far in this chapter. Use arrows to show which representations you can connect already. Which connections have you not used yet but you are confident that you could? Which connections do you still need to explore? Title this entry "Multiple-Representations Web for Exponential Functions" and label it with today's date. Be ready to share your findings with the rest of the class.



3-25. RULE → GRAPH

How can you sketch the graph of an exponential function directly from its rule without making a table first? Discuss this with your team. Then make a reasonable sketch of the graph of $y = 7(2)^x$ on your paper.

Review & Preview

3-26. Each table below represents an exponential function of the form $y = ab^x$. Copy and complete each table on your paper and find the corresponding rule.

a.

x	y
0	1.8
1	5.76
2	18.432
3	
4	

b.

x	y
0	5
1	
2	245
3	
4	

3-27. Brianna is working on her homework. Her assignment is to come up with four representations for an exponential function of her choosing. She decides it is easiest to start by writing an equation, so she chooses $y = 1200(\frac{1}{2})^x$. Help Brianna create the other three components of the web.

3-28. Sketch the graphs of $y = x^2$, $y = 2x^2$, and $y = \frac{1}{2}x^2$ on the same set of axes. Describe the similarities and differences among the graphs.

3-29. Factor each expression completely.

a. $x^2 - 49$ b. $4x^2 - 1$ c. $x^2y^2 - 81z^2$ d. $2x^3 - 8x$

3-30. Derek and Donovan were trying to solve the equation $4^4 = 16^x$. Derek had an idea.

"I know," he said. *"Isn't* 16 *equal to* 4^2 *?"*

"Yeah, so what?" said Donovan.

"That means that we can rewrite the equation to look like $4^4 = (4^2)^x$. *This is much easier to solve!"* replied Derek.

"Yes," said Donovan. *"That makes sense. Isn't there another way, too? Since* 4 *is the same as* 2^2 *and* 16 *is the same as* 2^4, *can't we rewrite it as* $(2^2)^4 = (2^4)^x$ *?"*

a. What do you think of Derek's and Donovan's methods? Will they both work?

b. Use both methods to solve $4^4 = 16^x$.

c. Now solve $3^5 = 9^{2x}$.

3-31. Solve each equation below for x. For parts (c) and (d), use the ideas from problem 3-30 so that you do not have to use Guess and Check.

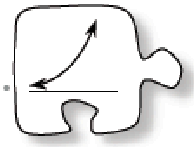
a. $2^{(x+3)} = 2^{2x}$ b. $3^{(2x+1)} = 3^3$

c. $9^x = 3^{40}$ d. $8^{70} = 2^x$

3-32. The probability of landing on blue using a three-colored spinner is $\frac{2}{5}$. What is the probability of landing on blue on both of the next two spins?

3-33. Mary has a piece of wood 12 feet long. She needs to cut off a section to use in a project. **Investigate** the function defined by inputs that are the amount of wood she cuts off and outputs that are the amount of wood remaining on the original piece of wood.

3.1.3 How does it grow?



More Applications of Exponential Growth

You may have heard the expression, "Money does not grow on trees." However, money does, in a sense, grow in a savings account. In today's lesson you will apply your understanding of exponential functions to solve problems involving money and interest. As you work, use the questions below to help focus your team's discussions.

How does it grow?

How is the rate written as a percent? As a decimal?

How is it the same (or different)?

3-34. SAVING FOR COLLEGE

Suppose you have $1000 to invest and know of two investment options. You can invest in bonds (which pay 8% *simple* interest) or put your money in a credit-union account (which pays 8% *compound* interest). Will the option you choose make a difference in the amount of money you earn? Examine these two contexts below.

Bonds with Simple Interest:

a. If you invest in bonds, your $1000 would grow as shown in the table at right. How does money grow with simple interest?

b. By what percent would your balance have increased at the end of the 4th year? Show how you know.

VALUE OF BONDS

Number of Years	Amount of Money (in dollars)
0	1000.00 (initial value)
1	1080.00
2	1160.00
3	1240.00
4	

Accounts with Compound Interest:

c. Instead, if you invest your $1000 in the credit union at 8% compound interest that is compounded once a year, its value would grow as shown in the table at right. Why is there $1166.40 in your account at the end of the second year? Explain how the money grows with compound interest.

VALUE OF CREDIT-UNION ACCOUNT

Number of Years	Amount of Money (in dollars)
0	1000.00 (initial value)
1	1080.00
2	1166.40
3	1259.71
4	

d. What will be the balance of the credit-union account at the end of the 4th year? By what percent would this account balance increase in four years? Show how you know.

e. Which type of account—a bond with simple interest or a credit-union account with compound interest—grows most quickly?

f. How much would your original $1000 investment be worth at the end of 20 years in this credit union? Show how you got your answer.

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3-35. Examine these two types of investments through other representations below.

a. The sequence below represents the value of an investment earning *simple* interest at the beginning of each year.

   1000, 1080, 1160, 1240, …

   Is this sequence arithmetic, geometric, or neither? What calculation is done to each term to get the next term?

b. Write a sequence for the value of the investment when $1000 is invested in an account with 8% annually *compounded* interest. Is this sequence arithmetic, geometric, or neither? What calculation is done to each term to get the next term?

c. Write an equation for each type of interest (simple and compound), where y represents the value of the investment after x years.

d. For each type of investment (simple and compound), draw a graph showing the value of $1000 over the first 8 years. Should these graphs be discrete (points only) or continuous (connected)? Explain.

e. What interest rate would the bonds with simple interest need to earn so that you would earn the same amount in both accounts after 6 years? After 20 years? Show how you know.

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3-36. A third option for investing money is a money-market account, which offers 8% annual interest *compounded quarterly* (four times per year). This means that the 8% is divided into four parts over the year, so the bank pays 2% every three months.

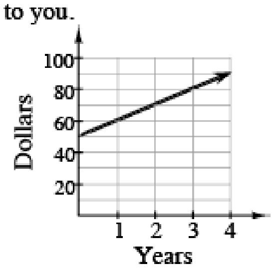
a. Represent the value (at the end of every three months) of the $1000 investment in this money-market account with a sequence. List at least six terms.

b. Write an equation that determines the value of the investment, y, in the money-market account after x quarters.

c. What will be the value of your $1000 investment at the end of four years? How does this compare with your other investment options?

3-37. You have $500 to invest and have several options available to you.

a. Your banker shows you the graph at right to explain what you can earn if you invest with him. Does this graph represent simple or compound interest? How can you tell? What is the interest rate? Write an equation to represent how much money you would have as time passes. Let x represent time in years.



b. Jerry says, *"I've got my money in a great account that compounds interest* ***monthly****. The equation* $y = 388(1.008)^m$ *represents how much money I have at the end of any month."* What is Jerry's annual interest rate? Write an equation to represent *your* total money if you invest your $500 in an account with the same rate of return. Let m represent the number of months the money has been invested.

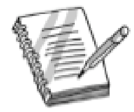
c. An investment advisor shows you the table of earnings you see at right. Write an equation for the table, letting q represent the number of quarters the money has been invested. What is the annual interest rate?

Quarter	$
0	500
1	515
2	530.45
3	546.36

d. Compare the earning potential of the options described in parts (a), (b), and (c) above. Which account is the best investment if you plan to leave your money in for only one year? Which would grow the most after ten years? Show and explain your work.

3-38. BACK TO THE WEB

Examine your web from problem 3-22. Which connections did you use in this lesson? Did you use any new connections during this lesson that you did not already have represented on your web with arrows? If so, update your web. Which connections do you still need to find?



October 24, 2010

Review & Preview

3-39. Each table below represents an exponential function in $y = ab^x$ form. Copy and complete each table on your paper and find a corresponding rule.

a.

x	y
−1	3
0	
1	75
2	
3	

b.

x	y
0	
1	
2	96.64
3	77.312
4	

3-40. Tickets for a concert have been in incredibly high demand, and as the date for the concert draws closer, the price of tickets increases exponentially. The cost of a pair of concert tickets was \$150 yesterday, and today it is \$162. As you complete parts (a) through (c) below, assume that each day's percent increase from the day before is the same.

a. What is the daily percent rate of increase? What is the multiplier?

b. What will be the cost of a pair of concert tickets one week from now?

c. What was the cost of a pair of tickets two weeks ago?

3-41. Dusty won \$125,000 on the *Who Wants to be a Zillionaire?* game show. He decides to place the money into an account that earns 6.25% interest compounded annually and plans not to use any of it until he retires.

a. Write an expression that represents how much money Dusty will have in t years.

b. How much money will be in the account when he retires in 23 years?

3-42. In 1999, Charlie received the family heirloom marble collection consisting of 1239 marbles. Charlie's great-grandfather had started the original marble collection in 1905. Each year, Charlie's great-grandfather had added the same number of marbles to his collection. When he passed them on to his son, he insisted that each future generation add the same number of marbles per year to the collection. When Charlie's father received the collection in 1966, there were 810 marbles.

b. How many marbles are added to the collection each year?

c. Use the information you found in part (b) to figure out how many marbles were in the original collection when Charlie's great-grandfather started it.

d. **Generalize** this situation by writing a function describing the growth of the marble collection for each year (n) since Charlie's great-grandfather started it.

e. How old will the marble collection be when Charlie (or one of his children) has more than 2000 marbles? In what year will this occur?

3-43. Solve the following systems of equations.

a. $3x - 2y = 14$
$-2x + 2y = -10$

b. $y = 5x + 3$
$-2x - 4y = 10$

c. Which system above is most efficiently solved by using the Substitution Method? Explain.

d. Which system above is most efficiently solved by using the Elimination Method? Explain.

3-44. If $f(x) = \frac{6}{x-1}$, find the value of x that will make $f(x) = 5$.

3-45. Factor $2x^2 + 3x - 2$ and use an area model to demonstrate the equivalence of your expression to the original.

3-46. Examine each sequence below. State whether it is arithmetic, geometric, or neither. For the sequences that are arithmetic, find the formula for $t(n)$. For the sequences that are geometric, find the sequence generator for $t(n)$.

a. 1, 4, 7, 10, 13, …

b. 0, 5, 12, 21, 32, …

c. 2, 4, 8, 16, 32, …

d. 5, 12, 19, 26, …

e. $x, x+1, x+2, x+3, \ldots$

f. 3, 12, 48, 192, …

3-47. If you flip a fair coin, what is the probability that it comes up "heads"? "Tails"?

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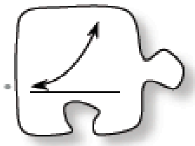
3.1.4 What if it does not grow?



Exponential Decay

To learn more about exponents, today you will study a new context that can be represented with an equation of the form $y = ab^x$.

3-48. THE PENNY LAB

What about situations that do not grow? In this activity, you will explore a situation that behaves exponentially, but results get smaller. This is an example of **exponential decay**.

Your task: Follow the directions below to model exponential decay using pennies.

Trial #0: Start with 100 pennies.

Trial #1: Dump the pennies out on your team's workspace. Remove any pennies that have "tails" side up. Record the number of pennies that *remain* in a table where the input is the trial number and the output is the number of "heads."

Trial #2: Gather the "heads-up" pennies, shake them up, and dump them on your workspace again. Remove any pennies that have the "tails" side up and count the number of pennies that remain.

Trial #*x*: Continue this process until the last penny is removed. Be sure to record all of your results in your table and then answer the questions below.

a. Is it possible that a team conducting this experiment might never remove their last penny? Explain.

b. Would the results of this experiment have been significantly different if you had removed the "heads" pennies each time?

c. If you had started with 200 pennies, how would this have affected the results?

3-49. Decide what your dependent and independent variables are for "The Penny Lab" data, clearly label them, and graph your data on your own graph paper. Then graph your data carefully on a team Lesson 3.1.4 Resource Page transparency obtained from your teacher.

a. Stack your team's transparency with of those from other teams on an overhead projector so that the axes are aligned. Then examine and describe the resulting scatter plot. Where does the graph cross the y-axis? Does the graph have any asymptotes? Should the graph be continuous or discrete?

b. Write an equation for an exponential function that approximates the data.

c. What output does your function give for $x = 0$? What could this mean in relation to the situation?

d. Could there be an output value for $x = -1$? If so, what might it mean?

3-50. HALF-LIFE

Carbon-14 dating is used to approximate the age of ancient discoveries and to learn more about things like dinosaur fossils. Scientists have studied the rate of decay of carbon-14 and have learned that no matter how much of this element they start with, only half of it will remain after about 5730 years (which is called its **half-life**).

All living things on this planet contain the same proportion of this carbon-14 relative to overall carbon in their bodies. Knowing how much carbon-14 to expect, scientists can then measure how much carbon-14 is left in ancient items to figure out how much time has passed since the object was living.

a. If a living object is supposed to contain 100 grams of carbon-14, how much would be expected to remain after one half-life (5730 years)? After two half-lives (11,460 years)?

b. Draw a graph showing the expected amount, y, of carbon-14 (in grams) remaining after x half-lives.

c. Write an equation for a function that represents the amount of carbon-14 that will remain after x half-lives.

d. What output does your function give for $x = 0$? Does this make sense? **Justify** why or why not.

e. What output would the function give for $x = -1$?

3-51. In addition to helping you learn about exponential decay, half-life can also provide insight into some special exponent properties.

a. For example, in part (d) of problem 3-50, you determined that $100(\frac{1}{2})^0 = 100$. So what must $(\frac{1}{2})^0$ equal? What do you think 3^0 or $(-5)^0$ equals? How do your graphs from Lesson 3.1.1 help you predict this? Use your calculator to check your predictions. Then write a conjecture about the value of x^0 (when $x \neq 0$).

b. What if $x = -1$? According to your graph, how much carbon-14 should there be when $x = -1$? Use this information to make sense of the value of $(\frac{1}{2})^{-1}$. Confirm your conclusion with your calculator.

c. Now find the value of your equation when $x = -2$. Use this information to make sense of the value of $(\frac{1}{2})^{-2}$. Then, as a team, write a conjecture about the value of x^{-2} when $x \neq 0$. Test your conjecture by predicting the value of 3^{-2} and $(\frac{2}{3})^{-2}$. Be sure to test your predictions with your calculator.

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3-52. Use the graphing calculator to compare the graphs of $y = (\frac{1}{2})^x$ and $y = 2^{-x}$.

a. What do you notice? How does a negative exponent affect the base number?

b. Use this idea to rewrite each of the following expressions in a different form. If you and your team members disagree, check your results with the calculator.

i. $(\frac{1}{5})^{-1}$	*ii*. 100^{-1}	*iii*. $(\frac{5}{8})^{-1}$	*iv*. $(\frac{1}{3})^{-2}$
v. $(\frac{2}{3})^{-3}$	*vi*. 6^{-3}	*vii*. $(\frac{3}{2})^{-1}$	*viii*. 2^{-5}

METHODS AND MEANINGS

MATH NOTES

Basic Laws of Exponents

The following laws of exponents follow logically from the basic definition $x^n = \underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ times}}$, where n is a postive integer or counting number.

If $x > 0$ and $y > 0$, the following equations are always true.

$$x^m \cdot x^n = x^{m+n} \qquad (xy)^k = x^k y^k \qquad \frac{x^m}{x^n} = x^{m-n}$$

$$\left(\frac{x}{y}\right)^k = \frac{x^k}{y^k} \qquad (x^m)^n = x^{mn}$$

Review & Preview

3-53. Assume that a DVD loses 60% of its value every year it is in a video store. Suppose the initial value of the DVD was $80.

a. What multiplier would you use to calculate the video's new values?

b. What is the value of the DVD after one year? After four years?

c. Write a function $V(t) = ?$ to represent the value in t years.

d. When does the video have no value?

e. Sketch a graph of this function. Be sure to scale and label the axes.

3-54. Use the basic definition of exponents to show an example that demonstrates each of the laws of exponents listed in the Math Notes box in this lesson. An example demonstrating $x^m \cdot x^n = x^{m+n}$ is shown at right.

$$\begin{aligned} x^3 \cdot x^2 &= (x \cdot x \cdot x)(x \cdot x) \\ &= x \cdot x \cdot x \cdot x \cdot x \\ &= x^5 \\ &= x^{3+2} \end{aligned}$$

3-55. This problem is a checkpoint for simplifying expressions with positive integral exponents. It will be referred to as Checkpoint 4.

Simplify each expression.

a. $(2x^2y)^4$

b. $\frac{-3x^2y^3}{(-6)^2}$

c. $\frac{(2x^2y)^4}{3xy^5}$

d. $5(5xy)^2(x^3y)$

e. Check your answers to parts (a) through (d) by referring to the Checkpoint 4 materials located at the back of your book.

If you needed help simplifying these expressions correctly, then you need more practice. Review the Checkpoint 4 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to simplify expressions like these quickly and easily.

3-56. Consider the pattern at right.

$$\frac{1}{2^3} = \frac{1}{8} \qquad \frac{1}{2^2} = \frac{1}{4} \qquad \frac{1}{2^1} = \frac{1}{2} \qquad \frac{1}{2^0} = 1$$

a. Continue the pattern to find $\frac{1}{2^{-1}}$, $\frac{1}{2^{-2}}$, $\frac{1}{2^{-3}}$, and $\frac{1}{2^{-4}}$.

b. What is the value of $\frac{1}{2^{-n}}$?

c. Write a conjecture about how to rewrite $\frac{1}{a^{-n}}$ without a negative exponent.

3-57. Solve each equation below for x. Refer back to problem 3-30 for examples, if necessary.

a. $2^{(x+3)} = 64$

b. $8^x = 4^6$

c. $9^x = \frac{1}{27}$

3-58. Dr. Sanchez asked her class to simplify the expression $x + 0.6x$, but some students disagree on how to simplify it. Terry says that $x + 0.6x = 1.6x$, but Jo says that $x + 0.6x = 0.7x$. Who is correct? **Justify** your conclusion.

3-59. Use your exponent patterns to rewrite each of the expressions below. For example, if the original expression has a negative exponent, then rewrite the expression so that it has no negative exponents, and vice versa. Also, if you can simplify the expression, go ahead and do so. Note: In part (b), assume that $m \neq 0$.

a. k^{-5}

b. m^0

c. $x^{-2} \cdot x^5$

d. $\frac{1}{p^2}$

e. $\frac{y^{-2}}{y^{-3}}$

f. $(x^{-2})^3$

g. $(a^2b)^{-1}$

h. $\frac{1}{x^{-1}}$

3-60. Factor each expression below.

a. $x^2 + 8x$

b. $6x^2 + 48x$

c. $2x^2 + 14x - 16$

d. $2x^3 - 128x$

3-61. Multiply and simplify each expression below.

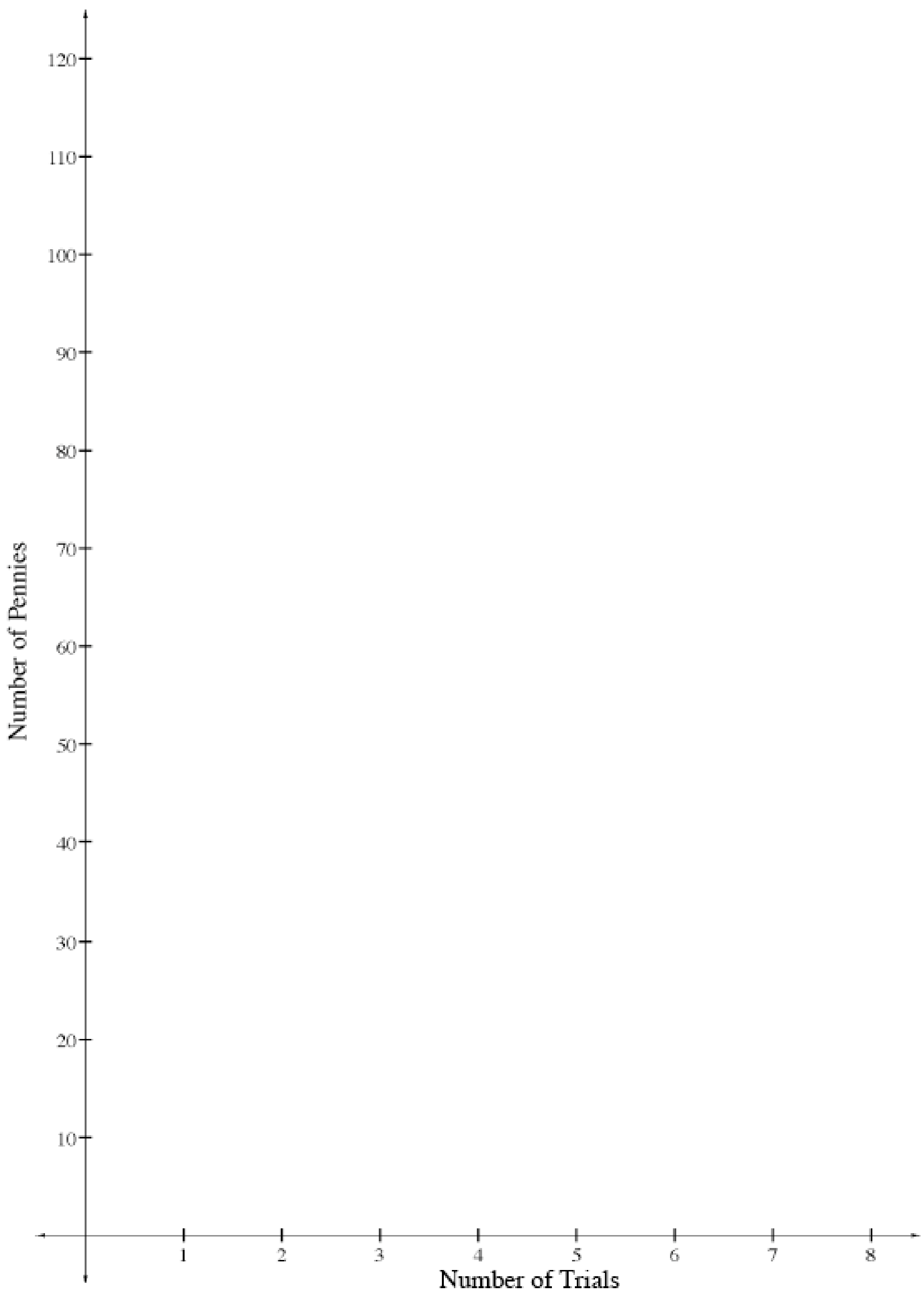
a. $(x-3)^2$

b. $(2m+1)^2$

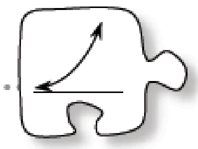
c. $x(x-3)(x+1)$

d. $(2y-1)(y^2+7)$

Lesson 3.1.4 Resource Page

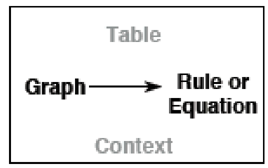


3.1.5 What are the connections?



Graph → Rule

In Lesson 3.1.2, you started an exponential multiple-representations web. Today your team will develop methods for finding a rule from a graph. As you find ways to write rules based on a graph, you will build deeper understanding of exponential functions.



What information do we have?

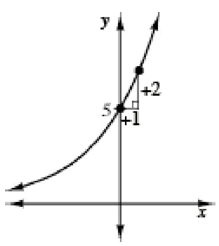
Can we use other representations to help us think about our rule?

How can we be sure that our rule works?

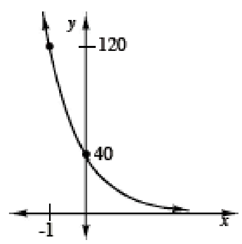
3-62. GRAPH → RULE

Use the clues provided in each graph below to find a possible corresponding rule in $y = ab^x$ form. Assume that if the graph has an asymptote, it is located on the x-axis.

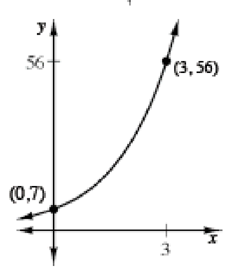
a.



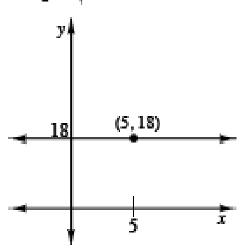
b.



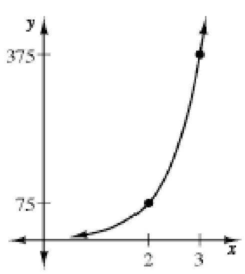
c.



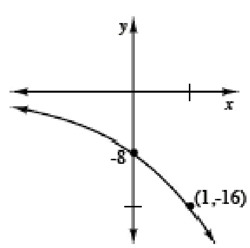
d.



e.

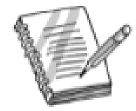


f.



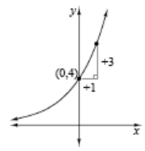
3-63. LEARNING LOG

Create a Learning Log entry in which you describe methods for creating an exponential rule given a graph. Be sure to include examples to illustrate your reasoning. Title this entry "Graph $\rightarrow$ Rule for Exponential Functions" and label it with today's date.



Review & Preview

3-64. Use the clues in the graph at right to find a possible corresponding rule in $y = ab^x$ form. Assume the graph has an asymptote at the x-axis.



3-65. Kristin's grandparents started a savings account for her when she was born. They invested $500 in an account that pays 8% interest compounded annually.

a. Write an equation to express the amount of money in the account on Kristin's x^{th} birthday.

b. How much money is in the account on Kristin's 16^{th} birthday?

c. What are the domain and range of the equation that you wrote in part (a)?

3-66. Jack and Jill were working on simplifying the expression at right, but they were having some trouble. Then Jill had an idea. $\frac{3x^2y^{-3}}{x^{-1}y^2}$

"Can't we separate the parts?" she said. *"That way, it might be easier to tell what we can simplify."* She rewrote the expression as shown at right. $3 \cdot x^2 \cdot \frac{1}{x^{-1}} \cdot y^{-3} \cdot \frac{1}{y^2}$

"Okay," said Jack. *"Now we can rewrite each of the parts with negative exponents and simplify."*

a. Help Jack and Jill finish simplifying their expression.

b. Use their idea to rewrite and simplify $\frac{m^2pq^{-1}}{4m^{-2}pq3}$.

3-67. Consider the sequence $2, 8, 3y+5, \ldots$

a. Find the value of y if the sequence is arithmetic.

b. Find the value of y if the sequence is geometric.

3-68. The solution to the equation $x^3 = 64$ is called the **cube root** of 64. The idea is similar to the idea of a square root, except that the value must be cubed (multiplied by itself three times) to become 64. One way to write the cube root of 64 is using the notation $\sqrt[3]{64}$. Use this information to evaluate each of the following expressions.

a. $\sqrt[3]{64}$ b. $\sqrt[4]{16}$ c. $\sqrt[3]{-8}$ d. $\sqrt[3]{125}$

3-69. Determine which of the following equations are true for all values (always true). For those that are not, decide whether they are true for certain values (sometimes true) or not true for any values (never true). **Justify** your decisions clearly.

a. $(x-5)^2 = x^2 + 25$

b. $(2x-1)(x+4) = 2x^2 + 7x - 4$

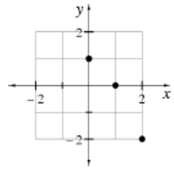
c. $\frac{2x^2y^3}{y^2} = 2x^2y$

d. $(3x-2)(2x+1) = 6x^2 - x - 5$

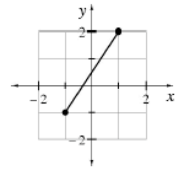
3-70. Graph $y = x^2 + 3$ and $y = (x+3)^2$. What are the similarities and differences between the graphs? How do these graphs compare to the graph of $y = x^2$?

3-71. Find the domain and range for each of the relations graphed below.

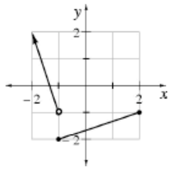
a.



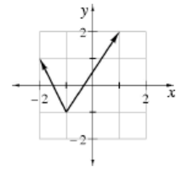
b.



c.



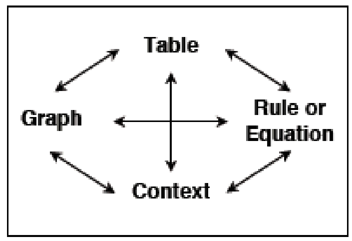
d.



3.1.6 What's the connection?

Completing the Web

Review the exponential multiple-representations web that you created in Lesson 3.1.2. Are there any connections you have made since Lesson 3.1.2 that you need to add to your web? What connections between representations do you still need to explore?

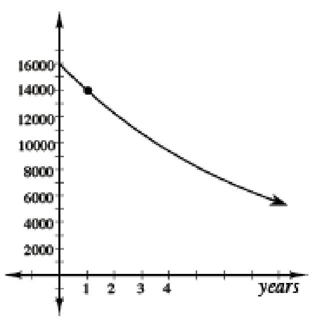


In today's lesson, design your own teamwork based on the connections that are incomplete in your web. Plan to begin your teamwork today with the problems that appear to your team to be the most challenging. The goal is for your team to complete the web by the end of this lesson.

3-72. WRITING A CONTEXT

Each representation below represents a different set of data. For each part, brainstorm a context that could fit the data. Provide enough information in your "problem" description so that someone else could generate the graph, table, or rule for the data. Be creative! Your team's context may be selected for a future assessment!

a.



b. $B(t) = 180(1.22)^t$

c.

Year	Amount	
1980	226 million	x 1.02
1981	________	x 1.02
1982	________	
.		
.		
.		
1990	________	
____	________	
	1,000,000,000	

3-73. CONTEXT → GRAPH

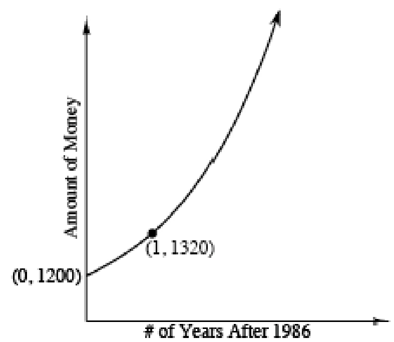
A virus has invaded Leticia's favorite mountain fishing lake. Currently there are an estimated 1800 trout in the lake, and the Fish and Game Department has determined that the rate of fish deaths will be one-third of the population per week if left untreated.

a. Sketch a graph showing how many fish are left in Leticia's favorite lake over several weeks.

b. Theoretically, will the trout ever completely disappear from the lake? Use the graph to **justify** your answer.

3-74. GRAPH → EQUATION

Suppose the annual fees for attending a public university were \$1200 in 1986 and the annual cost increase is shown in the graph at right. Note that x represents the number of years after 1986.



a. Write an equation to describe this situation.

b. Use this model to predict the cost of attending a public university in the first year you would be eligible to enroll.

c. What was the cost in 1980, assuming the 10% increase still applied?

d. By 1993 the annual cost was actually \$3276. How accurate was the model? What actually happened?

3-75. EQUATION → GRAPH

For each equation below, make a reasonable sketch of the graph without making a table first. Discuss your **strategy** with your team before you begin.

a. $y = 5(3)^x$ b. $y = 10(\frac{1}{2})^x$ c. $y = \frac{1}{10}(5)^x$

3-76. CONTEXT → ?

Use each of the situations below to complete missing pieces of the web or to practice moving from a context directly to a specific representation. For each part, decide which representation you will generate from the context description based on where your team needs to work.

a. A 100-gram sample of a radioactive isotope decays at a rate of 6% every week. How big will the sample be one year from now?

b. The math club is fast becoming one of the most popular clubs on campus because of the fabulous activities it sponsors annually for Pi Day on March 14. Each year, the club's enrollment increases by 30%. If the club has 45 members this year, how many members should it expect to enroll 5 years from now?

c. Barbara made a bad investment. Rather than earning interest, her money is decreasing in value by 11% each week! After just one week, she is down to just $142.40. How much money did she start with? If she does not withdraw her money, how long will it be before she has less than half of what she originally invested?

d. Larry loves music. He bought $285 worth of MP3 files on his credit card, and now he cannot afford to pay off his debt. If the credit-card company charges him 18% annual interest compounded monthly, how does Larry's debt grow as time passes? How much would he owe at the end of the year if he had a "no payments for 12 months" feature for his credit card?

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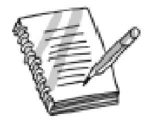
3-77. LEARNING LOG

Consider all of the things you have learned so far in this chapter. If you were creating a presentation for families for Back to School Night and wanted to teach them the main ideas about exponential functions, what would they be? Write a Learning Log entry describing the main ideas and why they are important. Title this entry "Important Ideas about Exponential Functions" and include today's date.



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Review & Preview

3-78. According to the U.S. Census Bureau, the population of the United States has been growing at an average rate of approximately 2% per year. The census is taken every 10 years, and the population in 1980 was estimated at 226 million people.

a. How many people would the Census Bureau have expected to count in the 1990 census?

b. How many people should the Census Bureau have expected to count in the 2000 census?

c. How does your answer compare with the actual 2000 census population data of 281.4 million? What does this mean?

d. If the rate of population growth in the U.S. had continued at about 2%, in about what year would the population in the United States reach and surpass one billion?

3-79. Solve each equation below.

a. $2^{(x-1)} = 64$ b. $4.7 = x^{1/3}$

c. $8^{(x+3)} = 16^x$ d. $9^3 = 27^{(2x-1)}$

e. $x^6 = 29$ f. $25^x = 125$

3-80. Solve each equation below for x.

a. $5^x = 5^{-3}$ b. $6^x = 216$

c. $7^x = \frac{1}{49}$ d. $10^x = 0.001$

3-81. Find values of a and b that make each system of equations true (i.e., solve each system). Be sure to show your work or explain your thinking clearly.

a. $6 = a \cdot b^0$
$24 = a \cdot b^2$

b. $32 = a \cdot b^2$
$128 = a \cdot b^3$

3-82. Determine whether each of the following sequences is arithmetic, geometric, or neither. Then find a rule for the sequence, if possible.

a. 12.2, 13, 13.8, … b. 90, 81, 72.9, …

c. 1, 1, 1, … d. 2, 4, 16, …

3-83. A particular sequence can be represented by $t(n) = 2(3)^n$.

a. What are $t(0)$, $t(1)$, $t(2)$, and $t(3)$?

b. Graph this sequence. What is the domain?

c. On the same set of axes, graph the function $f(x) = 2(3)^x$.

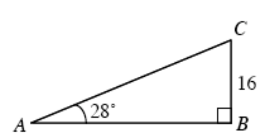
d. How are the two graphs similar? How are they different?

3-84. Solve the system of equations at right algebraically.

$2x - 3y = 12$
$y + x = -9$

3-85. An integer between 10 and 20 is selected at random. What is the probability that 2 is a factor of that integer?

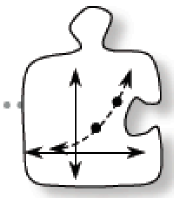
3-86. Place the triangle at right on a set of x and y axes so that B is located at the origin.



a. Find the coordinates of A and C.

b. Find the area and perimeter of triangle ABC.

3.2.1 How can I find the equation?



Curve Fitting and Fractional Exponents

In this section, you will use your knowledge of linear equations to help develop algebraic **strategies** for finding linear and exponential functions. You will also learn more about working with roots and exponents.

3-87. How can finding linear equations help you form **strategies** for finding exponential equations? Parts (a) and (b) below will help you answer that question.

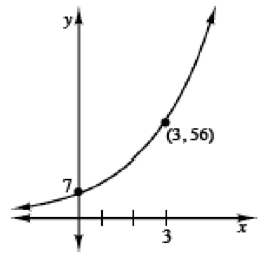
a. In the equation $y = mx + b$, which letters represent variables? Which represent parameters? Briefly describe the difference between a variable and a parameter.

b. Find the equation of the line with slope 3 that passes through the point (5, 19). Take careful note of the method that you use.

3-88. Can you use similar ideas to find an exponential equation?

a. In the equation $y = ab^x$, which letters represent variables? Which represent parameters?

b. While trying to find the equation for the graph in part (c) of problem 3-62 (shown again at right), Errol stated, *"I think 'a' must be 7 because the y-intercept is at (0, 7)."* Do you agree? **Justify** your answer.



c. *"But we still don't know what 'b' is,"* Errol noticed. His teammate, Sandy, had an idea. *"I think that* $56 = 7(b)^3$ *."* How did she get this equation? Is it valid? Explain.

d. If you have not done so already, solve $56 = 7(b)^3$ for b. Explain how you solved this equation.

e. Use a and b to write the equation for this graph. Does it agree with the equation you found in part (c) of problem 3-62?

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3-89. Use Errol's and Sandy's method from problem 3-88 to find the equation of an exponential function with an asymptote at $y = 0$ that passes through the points (0, 5) and (3, 320).

3-90. NEW NOTATION FOR ROOTS

The solution of the equation $b^3 = 8$ is called the **cube root** of 8. The idea of a cube root is similar to the idea of a square root, except that the cube root of 8 must be cubed (multiplied by itself three times) to become 8. One way to write the cube root of 8 is using the notation $\sqrt[3]{8}$.

a. How can this notation be used to write the solution of $t^3 = 11$?

b. Addison wondered how to find $\sqrt[3]{17}$ with her graphing calculator. *"If we could write $\sqrt[3]{17}$ in an exponent form instead of with a root symbol, we could use the graphing calculator to find it."* She showed her team this equation: $17^x = \sqrt[3]{17}$. Do you have any guesses about what the exponent might be? Discuss this with your team.

c. *"But we also know that $(\sqrt[3]{17})^3 = 17$, and we want to write $\sqrt[3]{17}$ with an exponent instead, like 17^x. So why don't we combine these and write $(17^x)^3 = 17$?"* Addison asked. What do you think?

d. Addison continued, "*Oh, so $(17^x)^3 = 17^{3x}$?*" Is she correct? Is it true that $(17^x)^3 = 17^{3x}$? Be ready to share your reasoning with the class.

e. Addison wrote: $17^{3x} = 17^1$. Complete Addison's work to find the value of x in this equation. What does this tell you about another way to write $\sqrt[3]{17}$?

f. Use similar logic to find exponential expressions for $\sqrt{5}$ and $\sqrt[5]{11}$. Show your reasoning. Then use your graphing calculator to find their decimal equivalents, rounded to the nearest 0.01.

3-91. REWRITING EXPRESSIONS

The property $(k^m)^n = k^{mn}$ can help you rewrite expressions with roots and fractional exponents, as it helped you in part (e) of problem 3-90.

For example, since $16^{3/2} = (16^3)^{1/2}$, $16^{3/2}$ can be rewritten as $\sqrt{16^3}$. However, since $16^{3/2} = (16^{1/2})^3$, $16^{3/2}$ can also be rewritten as $(\sqrt{16})^3$ or 4^3.

With your team, find ways to rewrite the expressions below *two different ways*. Be ready to **justify** your answers.

a. $10^{2/3}$ b. $(\sqrt[3]{9})^4$ c. $\sqrt[5]{x^3}$

d. $(\sqrt{2})^5$ e. $5^{7/2}$ f. $y^{3/3}$

3-92. Fractional powers can give surprising results when used with negative bases. Answer the following questions using what you now know about how to rewrite fractional powers and your mental math skills. Avoid using your calculator.

a. Show or explain why $(-27)^{1/2}$ has no real solution but $(-27)^{1/3} = -3$.

b. Given that $(-27)^{1/3} = -3$, is $(-27)^{2/3}$ positive or negative, or does it have no real solution? What about $(-27)^{1/4}$? And $(-27)^{1/5}$? **Justify** your answers.

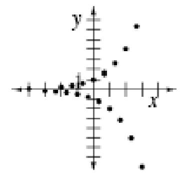
c. Mischa was working with her team on the idea of negative bases, but she got confused. Consider her thinking below.

i. *"Wait,"* she said. *"Isn't it true that* $(-100)^{1/2}$ *has no real solution?"* What does Mischa mean? Is she right?

ii. *"But,"* she continued, *"I can figure out that* $(-100)^{2/4} = 10$ *."* Check her calculation. Is she correct?

iii. *"That doesn't make any sense, since* $\frac{2}{4}$ *can be reduced to* $\frac{1}{2}$*!"* What do you think?

3-93. Recall that when you **investigated** $y = b^x$ in problem 3-2, the graphing calculator produced graphs like the one at right for negative values of b when you used the "zoom decimal" option. Use what you have learned about the meaning of fractional exponents to explain why $y = (-2)^x$ is impossible to graph accurately.



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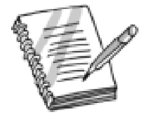
3-94. You have now worked with exponents that have been zero, exponents that have been negative numbers, and exponents that have been fractions. In your Learning Log, explain everything you know about these kinds of exponents. Show equivalent ways to write expressions with zero, negative, and fractional exponents. What does each kind of exponent mean? Explain using both words and examples. Title this entry "Zero, Negative, and Fractional Exponents" and label it with today's date.



Review & Preview

3-95. Find a possible exponential function in $y = a \cdot b^x$ form that represents each situation described below.

a. Has an initial value of 2 and passes through the point (3, 128).

b. Passes through the points (0, 4) and (2, 1).

3-96. Solve the following systems of equations. In other words, find values of *a* and *b* that make each system true. Be sure to show your work or explain your thinking clearly.

a. $3 = a \cdot b^0$
$75 = a \cdot b^2$

b. $18 = a \cdot b^2$
$54 = a \cdot b^3$

3-97. Evaluate each expression below.

a. $\sqrt[3]{-64}$ b. $\sqrt[5]{32}$ c. $\sqrt[3]{27}$ d. $\sqrt[4]{10000}$

3-98. Rewrite $16^{3/4}$ in as many different ways as you can.

3-99. Show two steps to simplify each of the following expressions, and then calculate the value of each expression.

a. $64^{2/3}$ b. $25^{5/2}$ c. $81^{7/4}$

3-100. Solve each of the following equations for *x*.

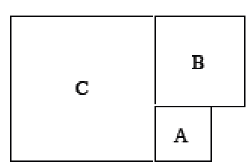
a. $2^{1.4} = 2^{2x}$ b. $8^x = 4$ c. $3^{5x} = 9^2$

3-101. For each of the problems below, find the initial value.

a. Five years from now, a bond that appreciates at 4% per year will be worth $146.

b. Seventeen years from now, Ms. Speedi's car, which is depreciating at 20% per year, will be worth $500.

3-102. In the diagram at right, the area of square A is 121 square units and the perimeter of square B is 80 units. Find the area of square C.



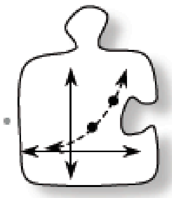
3-103. Solve each system of equations below.

a. $2x + y = -7y$
$y = x + 10$

b. $3x = -5y$
$6x - 7y = 17$

3-104. Find the equation of the line passing through the points (7, 16) and (2, −4). Then state the slope and *x*- and *y*-intercepts. Explain how you found them.

3.2.2 How can I find the equation?



More Curve Fitting

In this lesson, you will continue your work from Lesson 3.2.1 as you develop a new method to find linear and exponential equations given two points.

3-105. Mitchell was working on his Algebra 2 homework, when suddenly he had an idea about finding linear equations. He was trying to find the equation of the line that passes through the points (5, 15) and (3, 7). *"Look!"* he exclaimed. *"We know that the line can be written in the form* $y = mx + b$*, and we also know that the points (5, 15) and (3, 7) have to make the equation true. So we can substitute in these two points to create a system of equations. When we solve that, we'll know the values of m and b, and we'll have our equation!"*

a. What is Mitchell talking about? Use his method to find the equation of the line through the points (5, 15) and (3, 7).

b. Will Mitchell's method work to find the equation of a line through any two points? **Justify** your answer.

3-106. Use Mitchell's method from problem 3-105 to find the equation of the line that passes through the points (2, 3) and (5, –6).

3-107. Can Mitchell's method from problem 3-105 be used to find the *exponential* function that passes through the points $(2,16)$ and $(6,256)$? Consider this as you answer the questions below.

a. What is the general form for an exponential function that has an asymptote at $y=0$?

b. Use the two points that you know to create a system of equations.

c. Solve your system of equations to find the values of a and b. What is the equation? Be prepared to share your method with the class.

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3-108. Find an exponential function that passes through each pair of points.

a. $(-1, -2)$ and $(3, -162)$

b. $(2, 1.75)$ and $(-2, 28)$

METHODS AND MEANINGS

MATH NOTES

Summary of Exponents

For all x not equal to zero:

$x^0 = 1$ Examples: $2^0 = 1$, $(-3)^0 = 1$, $(\frac{1}{4})^0 = 1$

For positive values of x:

$x^{-n} = \frac{1}{x^n}$ Examples: $x^{-3} = \frac{1}{x^3}$, $y^{-4} = \frac{1}{y^4}$, $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

$\frac{1}{x^{-n}} = x^n$ Examples: $\frac{1}{x^{-5}} = x^5$, $\frac{1}{x^{-2}} = x^2$, $\frac{1}{3^{-2}} = 3^2 = 9$

$x^{a/b} = (x^a)^{1/b} = \sqrt[b]{x^a}$ or $x^{a/b} = (x^{1/b})^a = (\sqrt[b]{x})^a$

Examples: $5^{1/2} = \sqrt{5}$, $3^{2/3} = \sqrt[3]{3^2} = \sqrt[3]{9}$,

$16^{3/4} = (16^{1/4})^3 = (\sqrt[4]{16})^3 = 2^3 = 8$

Review & Preview

3-109. Find an exponential function that passes through each pair of points.

a. (1, 7.5) and (3, 16.875)

b. (–1, 1.25) and (3, 0.032)

3-110. Solve the following equations for x, if possible. Some you can solve exactly, others approximately. If a solution is not possible, explain how you know.

a. $1^x = 5$

b. $\sqrt{27^x} = 81$

c. $2^x = 9$

d. $25^{(x+1)} = 125^x$

e. $8^x = 2^5 \cdot 4^4$

3-111. This problem is a checkpoint for factoring quadratic expressions. It will be referred to as Checkpoint 5.

Factor each expression below.

a. $4x^2 - 1$

b. $4x^2 + 4x + 1$

c. $2y^2 + 5y + 2$

d. $3m^2 - 5m - 2$

e. Check your answers to parts (a) through (d) by referring to the Checkpoint 5 materials located at the back of your book.

If you needed help factoring these expressions correctly, then you need more practice. Review the Checkpoint 5 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to factor expressions like these quickly and easily.

3-112. To what power do you have to raise:

a. 3 to get 27?

b 2 to get 32?

c. 5 to get 625?

d. 64 to get 8?

e. 81 to get 3?

f. 64 to get 2?

g. (x^2) to get x^1?

h. (x^3) to get x^{12}?

i. x to get x^a?

3-113. Simplify each expression below.

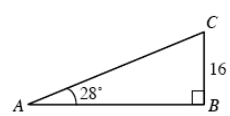
a. $\frac{x}{3} - \frac{x+1}{5}$

b. $\frac{5}{x} + \frac{3}{x^2}$

c. $(\frac{7x^2}{m}) \cdot (\frac{m}{x^2})$

d. $\frac{1}{3} \div m^2$

3-114. Find the length of $\overline{AC}$ in the diagram at right.



3-115. If $f(x) = 3(2)^x$, find the value of the expressions in parts (a) through (c) below. Then complete parts (d) through (f).

a. $f(-1)$

b. $f(0)$

c. $f(1)$

d. What value of x gives $f(x) = 12$?

e. Where does the graph of this function cross the x-axis? The y-axis?

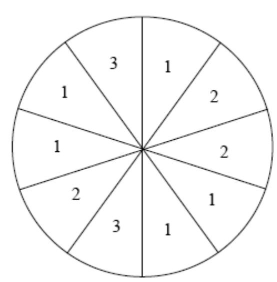
f. If $g(x) = \frac{1}{3x}$, find $f(x) \cdot g(x)$.

3-116. On the spinner at right, each "slice" is the same size. What is the probability that when you spin you will get:

a. The number 1?

b. The number 2?

c. The number 3?



3.2.3 How can I use exponential functions?

Solving a System of Exponential Functions Graphically

In this lesson, you will apply your skills with exponential functions to a system of equations as you explore the value of cars in an **investigation** called "Fast Cars."

3-117. FAST CARS

The moment you drive a new car off the dealer's lot, the car is worth less than what you paid for it. This phenomenon is called *depreciation*, which means you will sell the car for less than the price that you paid for it. Some cars depreciate more than others (that is, they depreciate at different rates), but most cars depreciate over time. On the other hand, some older cars actually increase in value. This is called *appreciation*. Let's suppose that in 2008, Jeralyn had a choice between buying a 2006 Fonda Concord EX for $21,000, which depreciates at 6% per year; a 2006 Padillac Escalate for $33,000, which depreciates at 22.5% per year; or a 1967 Fyord Rustang for $15,000 that is appreciating at 10% per year.

Your task: Investigate the value of each of the three cars over time.

- Generate multiple representations of the value of each car over time.
- For each of the new cars, determine how much value they lost (in dollars) from the time they were new in 2006 until 2008.
- Decide which car Jeralyn should buy and defend your choice in as many ways as you can.

Discussion Points

What is the multiplier?

How can we represent this situation in a table? a graph? an equation?

What should we consider when deciding which car to buy?

Further Guidance

3-118. **Investigate** the changing values of each of the cars by addressing the questions below.

a. What is the multiplier for the Concord? For the Escalate? For the Rustang?

b. Make a table like the one below and calculate the value for each car for each year shown.

Year	Concord	Escalate	Rustang
0	$21,000	$33,000	$15,000
1	$19,740	$25,575	
2			
3			
4			
5			
...			
10			
...			
n			

c. On your own graph paper, graph the data for all three cars on the same set of axes. Are the graphs linear? How are they similar? How are they different? You may want to use a different color for each car.

d. Write a function to represent the value of each car.

e. What were the values of the Concord and the Escalate when they were new? How much value (in dollars) did each car lose from 2006 to 2008?

f. Using the graph, which of the three cars is worth the most after one year? After 3 years? After 10 years? In how many years will the values of the Concord and Escalate be the same?

g. Pick one of the three cars and explain why Jeralyn should buy it. Has this problem changed your view of buying cars?

Further Guidance section ends here.

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3-119. As you saw in "The Penny Lab," half-life applies to situations other than radioactive decay. In fact, the idea can be applied to anything that is depreciating or decaying exponentially.

a. From the values in problem 3-117, Fast Cars, estimate the half-life of the value of the Concord and the Escalate.

b. According to the mathematical model (not necessarily corresponding to reality), when will each car have no value?

c. Why do you think some cars depreciate so much more quickly than others?

3-120. In 2004, a brand new SUV cost $26,000 to drive off the lot. In 2007 that same SUV was valued at $18,000. Write an exponential equation to represent this information. Then find the rate of depreciation for the SUV.

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Review & Preview

3-121. **Investigate** the function $y = 2 \cdot 4^x$. What would be the effect of replacing the 2 with a 5? What would be the effect of adding +1 to the end of the equation, making it $y = 2 \cdot 4^x + 1$? Include sketches as a part of your **investigation**.

3-122. Find the equation of an exponential function that passes through the points (2, 48) and (5, 750).

3-123. Billy Rich and Michael Million are two *very* wealthy, elderly men. Since neither of them have any heirs, they decide to give away all but $1 of their fortune before they die. Billy Rich has $1,340,000 and is giving $\frac{1}{3}$ of his remaining money away each year. Michael Million has $980,000 and is giving away $\frac{1}{4}$ of his money away each year. Who will get down to their final dollar first: Billy or Michael? How many years will it take each of them to give away their fortune? **Justify** your answer.

3-124. Factor each expression below.

a. $x^2 - x - 72$　　b. $9x^2 - 100$

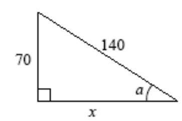
c. $2x^2 - 8$　　d. $3x^2 - 11x - 4$

3-125. Decide whether each sequence below is arithmetic, geometric, or neither. Then find equations to represent each sequence, if possible.

a. 10.3, 11.5, 12.7, …　　b. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

c. 1, 4, 9, …　　d. 1.1, 1.21, 1.331, …

3-126. Find x and $m\angle A$ in the triangle at right.



3-127. Decide which of the following pairs of expressions are equivalent. For those that are not equivalent, determine if there are any values of the variables that would make them equal (in other words, determine if they are *sometimes* equal). **Justify** each of your decisions thoroughly.

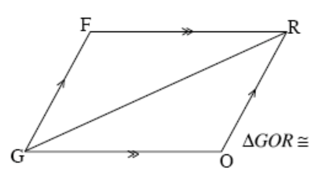
a. $(3x^2y)^3$ and $3x^6y^3$　　b. $(3x^2y)^3$ and $27x^6y^3$

c. $(3x^2y)^3$ and $27x^5y^4$　　d. $(3x^2y)^3$ and $27x^5y^3$

3-128. When data is arranged from least to greatest, the middle number (or the number between the two middle numbers, in the case of an even number of data pieces) is the median. The median of the lower half of data determines the boundary of the **first** and **second quartiles**. The median of the upper half of data determines the boundary of the **third** and **fourth quartiles**. The scores for 12 golfers were as follows: 68, 73, 80, 95, 86, 68, 74, 72, 90, 85, 70, and 82. Find the scores in the third quartile.

3-129. Solve $2x = \sqrt{8x+12}$ for x by first rewriting the equation as an equivalent equation without the square-root sign. After solving the new equation, answer these questions: Do both answers make the original equation true? Was the equation that led to the two solutions equivalent to the original equation? Why or why not?

3-130. Examine the diagram below. Complete the congruence statement and **justify** your answer.



$\Delta GOR \cong \Delta$ ____

CL 3-131. Find an exponential function in $y = ab^x$ form that satisfies each of the following sets of conditions.

a. Has a y-intercept of $(0, 2)$ and a multiplier of 0.8.

b. Passes through the points $(0, 3.5)$ and $(2, 31.5)$.

c. Passes through the points $(2, 20)$ and $(7, 640)$.

CL 3-132. Sam wants to create an arithmetic sequence and a geometric sequence, both of which have $t(1) = 8$ and $t(7) = 512$. Is this possible? If it is, help Sam create his sequences. If not, **justify** why not.

CL 3-133. Write each expression below as an equivalent expression without negative exponents.

a. 3^{-2} b. m^{-4} c. $(\frac{1}{2})^{-3}$ d. $(\frac{3}{5x})^{-1}$

CL 3-134. Write each expression below in radical form and compute the value without using a calculator.

a. $8^{1/3}$ b. $16^{3/4}$ c. $125^{-4/3}$

CL 3-135. Rewrite the following expressions using fractional exponents.

a. $(\sqrt{3x})^3$ b. $\sqrt[x]{81}$ c. $(\sqrt[3]{17})^x$

CL 3-136. Solve each equation.

a. $8^{x+3} = 16^x$ b. $y^{1/3} = 9$ c. $x^6 = 35$

CL 3-137. Find the annual multiplier and the percent decrease if a share of CPM stock was worth \$60 in 2000 and only worth \$45 in 2005.

CL 3-138. On January 17, 2005, the average price of a gallon of gasoline in the United States was \$1.84 per gallon. Nineteen months later, on July 17, 2006, the average price per gallon had risen to \$2.99, an average monthly growth of about 2.6%. At this rate, what will be the average price of a gallon of gas the summer after you graduate from high school?

CL 3-139. Simplify each expression.

a. $\dfrac{-8x^6y^2}{-4xy}$ b. $x^2y^3 \cdot x^3y^5z$

c. $(3x^2)^2 \div (6x^4)$ d. $\dfrac{(3x^2y)^2}{2z^2y^3} \cdot \dfrac{4x(y^2z^3)^3}{3x^2(z^2)^2}$

CL 3-140. Below are several situations that can be described using exponential functions. They represent a small sampling of the situations where quantities grow or decay by a constant percentage over equal periods of time. For each situation (a) through (d):

- Find an appropriate unit of time (such as days, weeks, years).
- Find the multiplier that should be used.
- Identify the initial value.
- Write an exponential equation in the form $f(x) = ab^x$ that represents the growth or decay.

a. A house purchased for \$120,000 has an annual appreciation of 6%.

b. The number of bacteria present in a colony is 180 at noon, and it increases at a rate of 22% per hour.

c. The value of a car with an initial purchase price of \$12,250 depreciates by 11% per year.

d. An investment of \$1000 earns 6% annual interest, compounded monthly.

CL 3-141. Write an equation for the line that passes through the points $(-5, 4)$ and $(3, -2)$.

CL 3-142. Add the fractions in part (a) and show two ways to solve the equation in part (b).

a. $\frac{x}{3} + \frac{2}{5}$ b. $\frac{x}{3} + \frac{2}{5} = x + 1$

CL 3-143. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in previous math classes? Use the table to make a list of topics you need help on, and a list of topics you need to practice more.

Chapter 3 Closure Resource Page: Multiple Representations of Exponential Functions GO

Create multiple representations of an exponential function. Write and **justify** summary statements using all of the representations. Use colors, arrows, and other tools to show connections between the representations.

Situation

Graph

Summary statements:

Table

Equation