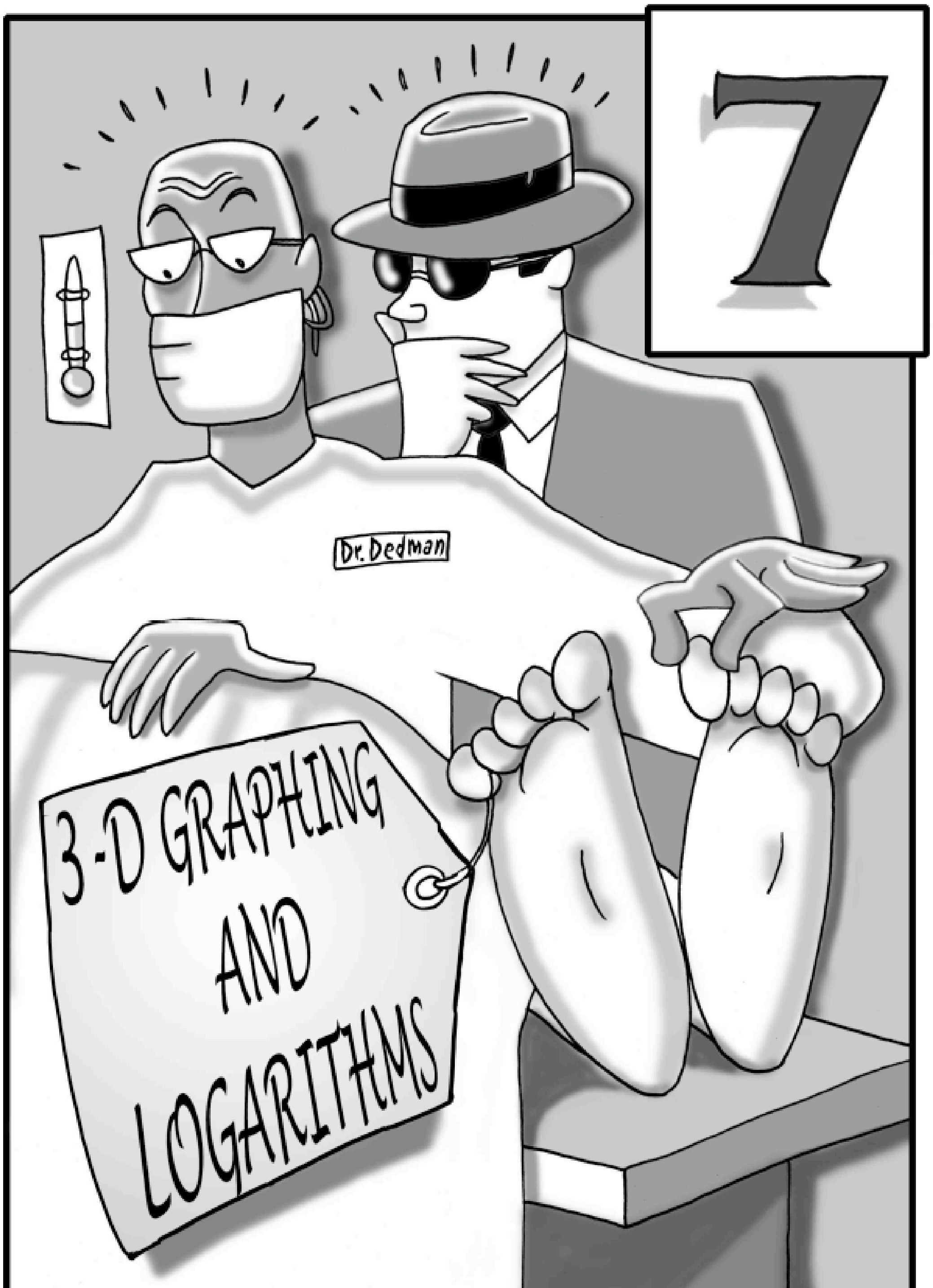


7
Dr. Dedman
3-D GRAPHING
AND
LOGARITHMS

Chapter 7 Teacher Guide

Section	Lesson	Days	Lesson Objective	Materials	Homework
7.1	7.1.1	1	Creating a Three-Dimensional Model	• Lesson 7.1.1A or B Res. Pg. • Lesson 7.1.1C Res. Pg. • Scissors, tape, small place markers such as dimes • Centimeter or multilink cubes	7-8 to 7-15
	7.1.2	1	Graphing Equations in Three Dimensions	• Lesson 7.1.1C Res. Pg. • Lesson 7.1.2 Res. Pg. • Computer and projector • Dynamic tools: *3D Point Plotter* and *Graphing Linear Equations in Three Variables*	7-21 to 7-28
	7.1.3	1	Systems of Three-Variable Equations	• Lesson 7.1.1C Res. Pg. • Colored pencils • First octant 3-D model • Computer and projector • Dynamic tool: *Intersection of 3 Planes*	7-34 to 7-42
	7.1.4	1	Solving Systems of Three Equations with Three Unknowns	• Lesson 7.1.1C Res. Pg. • Colored markers • First octant 3-D model • Computer and projector • Dynamic tool: *Intersection of 3 Planes*	7-50 to 7-59
	7.1.5	2	Using Systems of Three Equations for Curve Fitting	• Index Cards (optional)	7-71 to 7-78 and 7-79 to 7-86
7.2	7.2.1	1	Using Logarithms to Solve Exponential Equations	None	7-94 to 7-102
	7.2.2	1	Investigating the Properties of Logarithms	• Lesson 7.2.2 Res. Pg.	7-111 to 7-122
	7.2.3	1	Writing Equations of Exponential Functions	None	7-127 to 7-136
	7.2.4	1	An Application of Logarithms	• Poster paper and markers (opt.) • Lesson 7.2.4 Res. Pg. (opt.)	7-138 to 7-147
7.3 (optional)	7.3.1	1	Introduction to Matrices	None	7-155 to 7-164
	7.3.2	1	Matrix Multiplication	None	7-171 to 7-178
	7.3.3	1	Matrix Multiplication with a Graphing Calculator	• Lesson 7.3.3A or B Res. Pg.	7-185 to 7-193
	7.3.4	1	Writing Systems as Matrix Equations	None	7-200 to 7-208
	7.3.5	1	Using Matrices to Solve Systems of Equations	• Lesson 7.3.3A or B Res. Pg.	7-218 to 7-226
Chapter Closure		Varied Format Options			

Total: **10 or 15 days plus optional closure time**

7.1.1 How can I plot points in three dimensions?

Creating a Three-Dimensional Model

In geometry, you worked with objects that existed in different dimensions. You considered lines and line segments, which have only one dimension: length. You also looked at flat shapes like circles, rectangles, and trapezoids that have two dimensions: length and width. Prisms, cones, and most objects that we encounter in the world have volume, and therefore have three dimensions: length, width and height.

When you worked with graphs in Algebra 1, you represented points, the number line, and curves on a **two-dimensional** (flat) surface called the *xy*-plane. So far, you have only been able to represent relationships with at most two unknowns, usually the variables x and y. However, many problems have more than two unknowns. Today, you and your team will build a model that will help you graph in three dimensions. As you work on this lesson, consider the following questions with your team:

How can we plot a point in three dimensions?

How can we write the coordinates of a point in three dimensions?

How can we show three dimensions on flat paper?

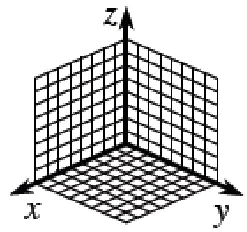
7-1. The following questions ask you to consider when it is appropriate to graph a situation in one, two, and/or three dimensions. It may be helpful to think about your experience representing numbers and relationships on a number line or an *xy*-plane, and how that could be adapted to work in three dimensions. Discuss each question with your team before writing your response.

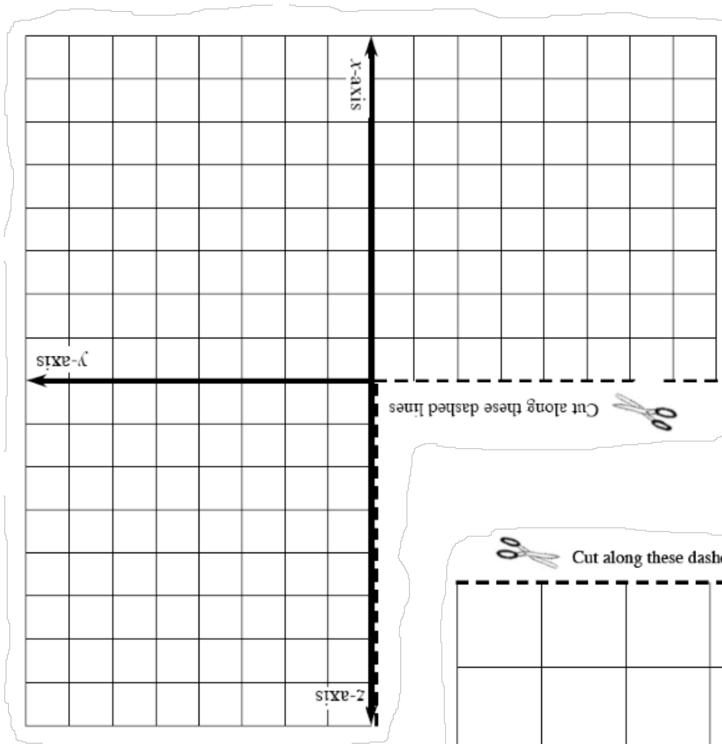
a. How can you represent the solution to $x = 5$ graphically? Can you think of more than one way?

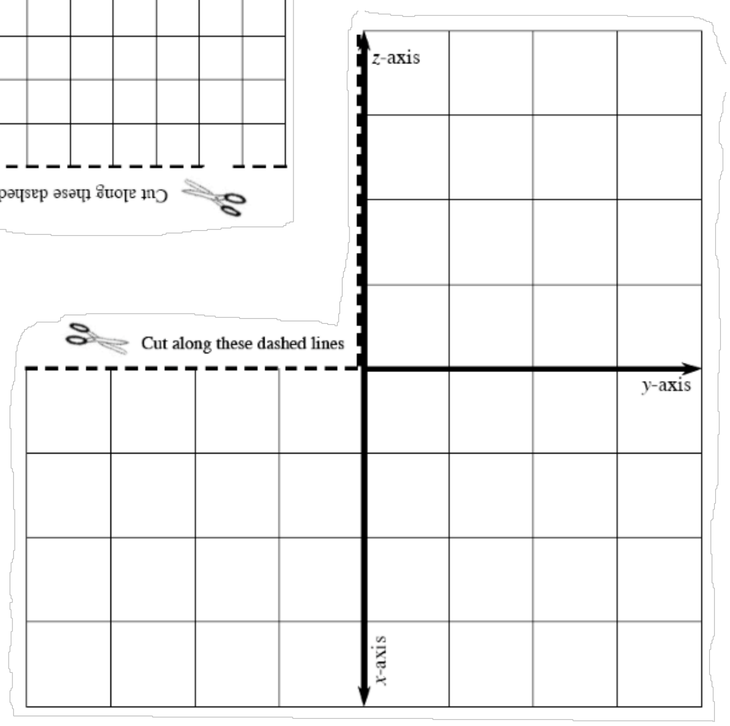
b. How can you represent the solutions to $x + 2y = 5$ graphically?

c. How could you represent the solutions to $x + 2y + z = 5$? What would the solutions look like? Discuss these questions with your team and write down any ideas that you have.

7-2. To graph solutions to equations with three variables, you need to use a three-dimensional coordinate system. Obtain a Lesson 7.1.1A or 7.1.1B Resource Page from your teacher. Use scissors to cut out the region indicated on the page. Then fold along each of the axes and use tape to attach the dashed edge to the z-axis. Be sure that the grid ends up on the *inside* of your model (rather than the outside). The result should look similar to the diagram at right.







7-3. Place a dime (or other marker) on the bottom surface of your model at the point where $x = 4$ and $y = 2$. Now lift your dime straight up so that it you are holding it 3 units above the bottom of the model.

a. With your team, find a way to write the coordinates of this point.

b. In your model, find the point where $x = 3$, $y = 4$, and $z = 2$. Use your team's method to write the coordinates for this point.

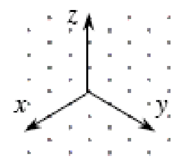
c. The model you have created is only a portion of the entire coordinate system used to represent three dimensions mathematically. How many of these models would you have to put together to create a model that represents the entire three-dimensional coordinate system? Think about the regions you would need to graph points like $(5, -2, -7)$ or $(-1, -2, -4)$.

7-4. Use cubes to build each shape described below inside your three-dimensional model. Make sure that one corner of each shape you build lies at the **origin** (at the point (0, 0, 0)).

a. Build a $2 \times 2 \times 2$ cube. Use coordinates to name the vertex that is farthest from the origin.

b. Build a rectangular prism that is 2 units in length along the *x*-axis, 1 unit in length along the *y*-axis, and 3 units in length along the *z*-axis. Use coordinates to name the vertex that is farthest from the origin.

c. Draw and label a three-dimensional coordinate system on isometric dot paper like the one shown at right. Now add the prism from part (b) to the drawing. On your dot paper, label the coordinates of *all* of the vertices.



7-5. Build a rectangular prism that will have vertices in your model at (1, 0, 0), (0, 0, 4), and (0, 3, 0).

a. Find the coordinates of the other five vertices.

b. Move the rectangular prism so that three vertices are at (–1, 0, 0), (0, 0, 4), and (0, 3, 0). Now where are the other vertices?

c. Is it possible to build another rectangular prism that has the same coordinates for the vertex farthest from the origin as the prism in part (b)? Be sure to **justify** your conclusion.

7-6. On isometric dot paper, draw a three-dimensional coordinate system and plot the following points: $(0, 1, -1)$, $(1, 2, 0)$, and $(2, 3, 1)$.

a. What do you notice about the three points?

b. With your team, find a **strategy** to make each point clearly different from the others. Be prepared to share your **strategy** with the class.

c. Identify the coordinates of two points that appear to be the same as $(-2, 0, 0)$.

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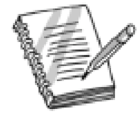
7-7. In your Learning Log, show and explain how to graph points in three dimensions. Include clear pictures to illustrate your method. Title this entry "Plotting Points in *xyz*-Space" and label it with today's date.



Review & Preview

7-8. Make a table like the one below. Choose points in each of the locations listed at the top of the table and write in the coordinates of the points you have chosen.

	Points on the x-axis	Points on the y-axis	Points on the z-axis	Points not on the x-, y-, or z-axes
1st point	(, ,)	(, ,)	(, ,)	(, ,)
2nd point	(, ,)	(, ,)	(, ,)	(, ,)
3rd point	(, ,)	(, ,)	(, ,)	(, ,)
4th point	(, ,)	(, ,)	(, ,)	(, ,)

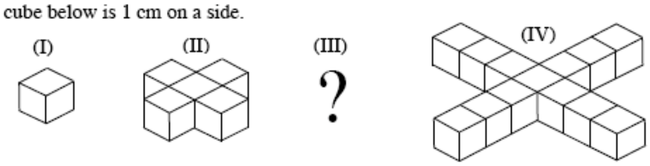
a. What do you notice about the coordinates of the points on the x-axis?

b. Make a conjecture about the coordinates of points that lie on any of the coordinate axes.

7-9. Solve the system of equations at right.

$$3x + 8 = 2$$
$$7x + 3y = 1$$

7-10. Each cube below is 1 cm on a side.



a. Based on the pattern, find the volume of Figure III.

b. If the pattern continues, write an expression to represent the volume of figure N. What kind of sequence is this?

7-11. Solve each exponential equation for x.

a. $10^x = 16$ b. $10^x = 41$ c. $3^x = 729$ d. $10^x = 101$

7-12. Rewrite each expression below as an equivalent expression without negative exponents.

a. 5^{-2} b. xy^{-2} c. $(xy)^{-2}$ d. $a^3b^4a^{-4}b^6$

7-13. Graph the system of $y \geq x^2$ and $y \geq (x-4)^2 + 2$ and shade their overlapping region. How is the graph of $y \geq (x-4)^2 + 2$ positioned in relation to the graph of $y \geq x^2$?

7-14. Given the two points (–2, 0) and (0, 1), complete parts (a) through (c) below.

a. Find the slope of the line that passes through these two points.

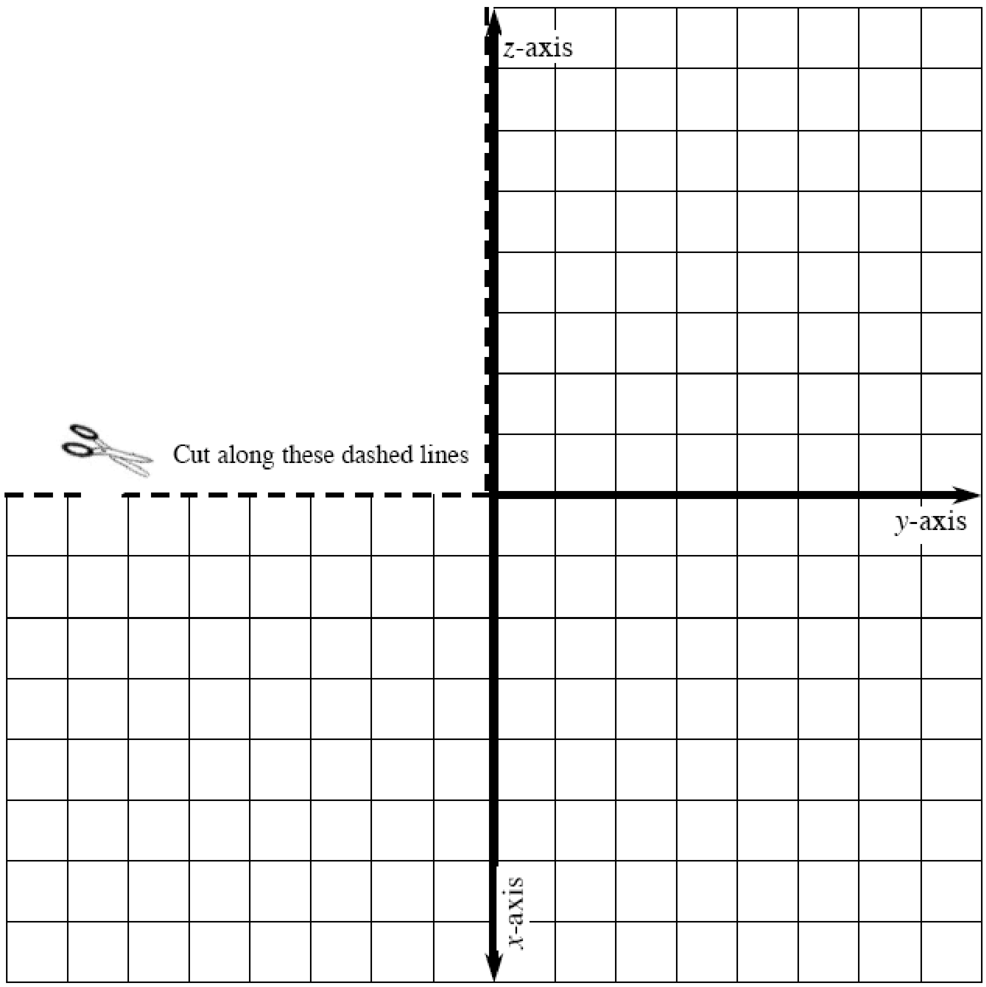
b. Find the slope of the line perpendicular to the line that passes through these two points.

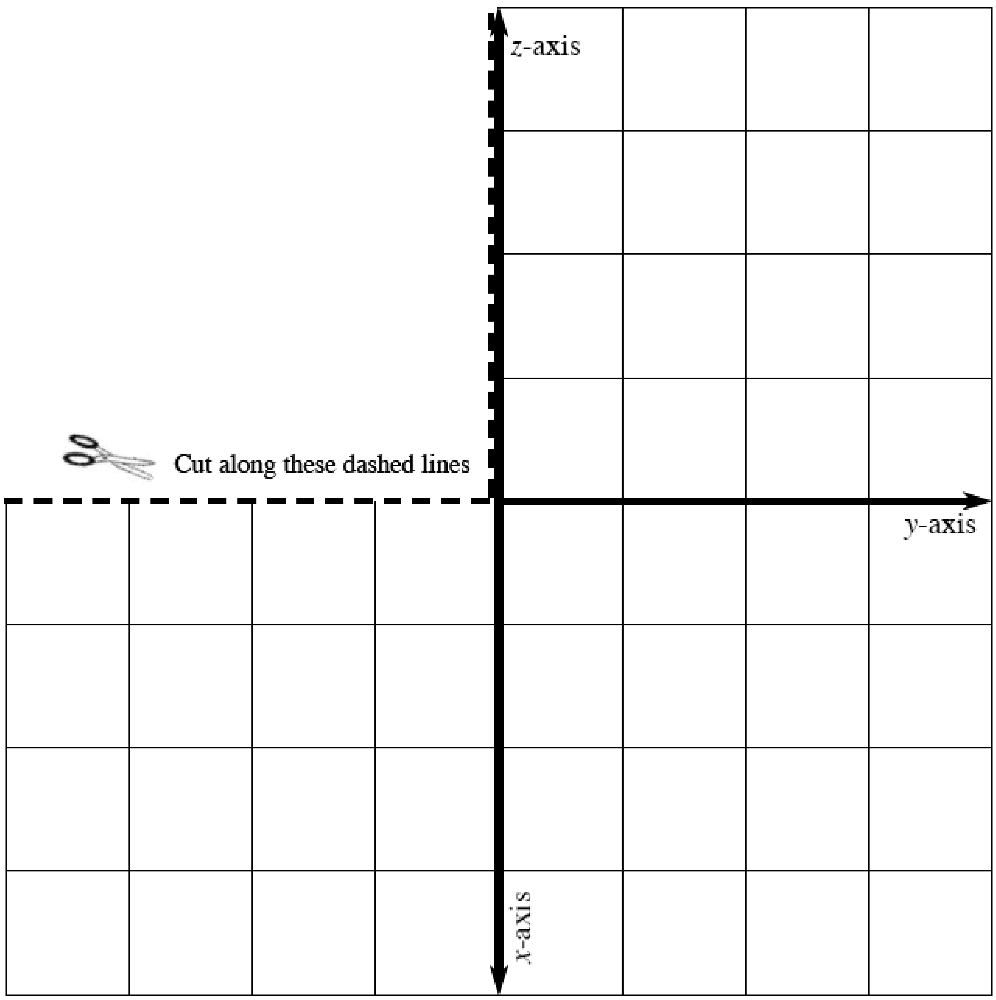
c. Describe the relationship between the slopes of perpendicular lines.

7-15. The cost of food has been increasing by 4% per year for many years. To find the cost of an item 15 years ago, Heather said, *"Take the current price and divide it by* $(1.04)^{15}$*."*

Her friend Elissa said, *"No, you should take the current price and multiply it by* $(0.96)^{15}$*!"*

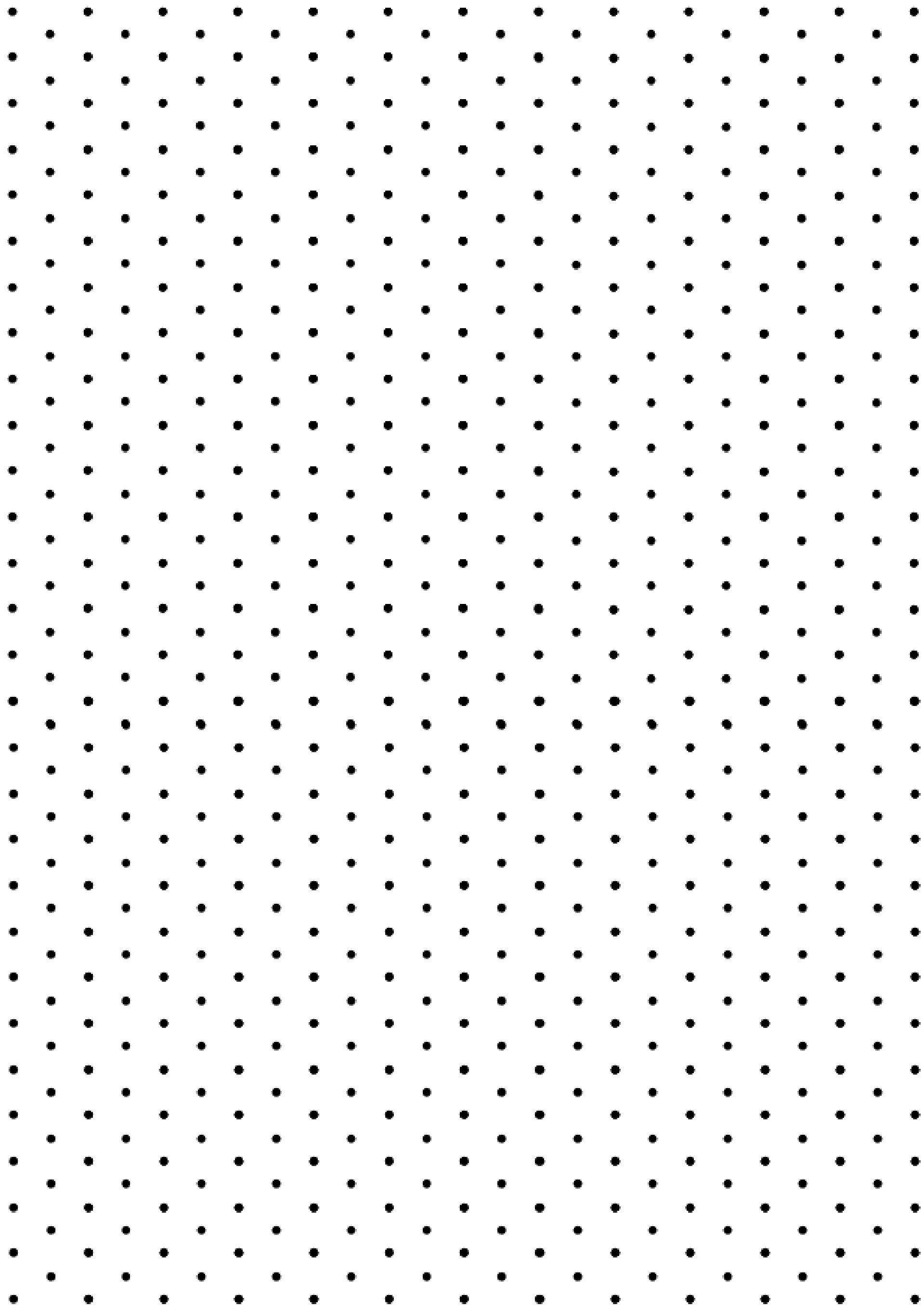
Explain who is correct and why.


z-axis
Cut along these dashed lines
y-axis
x-axis



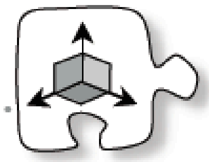
Cut along these dashed lines
z-axis
y-axis
x-axis

Lesson 7.1.1C Resource Page: Isometric Dot Paper



March 03, 2011

7.1.2 How can I graph a rule in three dimensions?



Graphing Equations in Three Dimensions

In the past, you have used the two-dimensional Cartesian coordinate system (x- and y-axes) to graph equations involving two variables. In the previous lesson, you used a three-dimensional coordinate system to plot points. Today you will use the three-dimensional coordinate system to graph rules that have three variables. As you are working through the lesson, use the following questions to help focus your discussion:

How can we use what we know about graphing in two dimensions
to help us graph in three dimensions?

What does a solution to a three-variable equation represent?

7-16. Consider the 3-D equation $5x + 8y + 10z = 40$.

a. Discuss with your team what you think the shape of the graph would be. Explain how you decided.

b. Is the point (4, 5, –2) a solution to the equation $5x + 8y + 10z = 40$? **Justify** your answer.

c. Your team will be given a list of points to test in the equation. Plot each point that makes the equation true on the three-dimensional grapher your teacher has set up.

d. Now examine the solutions displayed on the grapher. With your team, discuss the questions below. Be ready to share your discoveries with the class.

(*i*) Are there any points that you suspect are solutions, but do not have a point showing on the graph?

(*ii*) How many solutions do you think there are?

(*iii*) Are there any points showing that you think are not solutions? Explain.

(*iv*) What shape is formed by all of the solutions? That is, what is the shape of the graph of $5x + 8y + 10z = 40$?

7-17. How can you graph an equation like $12x + 4y + 5z = 60$ in three dimensions? To come up with a **strategy** to graph a three-variable equation, look at the **strategies** you can use to graph a two-variable equation in two dimensions. For example, consider $5x + 8y = 40$.

a. What is the shape of the graph of $5x + 8y = 40$? How can you tell?

b. With your team, brainstorm all of the **strategies** you could use to graph $5x + 8y = 40$. Which **strategy** do you prefer? Why?

7-18. Now you will work with your team to graph $12x+4y+5z=60$.

a. What do you think it will look like?

b. Which of the **strategies** you used to graph a two-variable equation in problem 7-17 can be used to graph this three-variable equation? Work with your team to find a **strategy** and then graph $12x+4y+5z=60$ on your isometric dot paper. Be prepared to share your **strategy** with the class.

7-19. Use your new **strategy** to graph each of the following equations in three dimensions.

a. $13x + 4y + 5z = 260$

b. $12x - 9y + 108 = 0$

7-20. Consider the graph of $x = 4$ for each of the following problems.

a. Graph the solution to $x = 4$ in one dimension (on a number line).

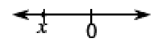
b. Graph the solutions to $x = 4$ in two dimensions (on the *xy*-plane).

c. Graph the solutions to $x = 4$ in three dimensions (in the *xyz*-space).

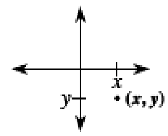
MATH NOTES

METHODS AND MEANINGS

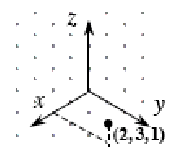
Locating Points in Three Dimensions



When locating a point on a *number line*, a single number, x, is used.



The location of a point in a *plane* is given by two numbers, (x, y), called an ordered pair.



To locate a point in *space*, three numbers, (x, y, z), are used, which are called an **ordered triple**. The point (2, 3, 1) is shown at right. The dotted lines help clarify which coordinate was graphed.

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Review & Preview

7-21. For each of the following equations, find every point where its *three-dimensional* graph intersects one of the coordinate axes. That is, find the x-, y- and z-intercepts. Express your answer in (x, y, z) form.

a. $6y + 15z = 60$

b. $3x + 4y + 2z = 24$

c. $(x+3)^2 + z^2 = 25$

d. $z = 6$

7-22. Answer each of the following questions. Illustrate your answers with a sketch.

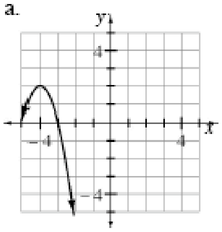
a. What do you think the intersection of two planes looks like?

b. What do you think it means for two planes to be parallel?

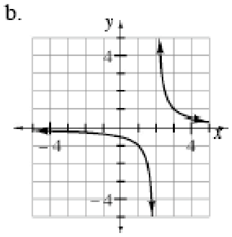
c. What do you think it means for a line and a plane to be parallel?

7-23. Find an equation that will generate each graph.

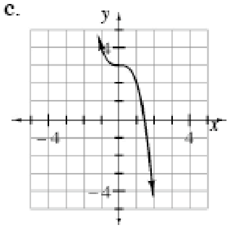
a.



b.



c.



7-24. Is $y = \frac{1}{x}$ the parent of $y = \frac{1}{x^2+7}$? Explain your reasoning.

7-25. Solve each equation below for x.

a. $2x + x = b$

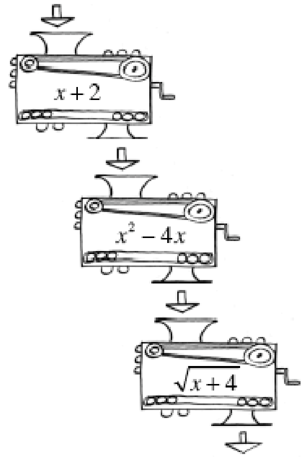
b. $2ax + 3ax = b$

c. $x + ax = b$

$(-2, 5, 1)$	$(6, 0, 1)$	$(1, 1, 1)$	$(\frac{1}{2}, 3, \frac{7}{2})$
$(-4, 5, 2)$	$(5, 8, 10)$	$(8, 0, 0)$	$(0, 0, \frac{7}{2})$
$(0, 0, 10)$	$(4, \frac{5}{2}, 0)$	$(2, 5, -1)$	$(10, 0, -1)$
$(\frac{5}{2}, 1, \frac{3}{2})$	$(6, 5, -3)$	$(2, 1, 4)$	$(7, 1, 0)$
$(0, 0, 4)$	$(\frac{1}{2}, 6, 3)$	$(3, 3, \frac{7}{2})$	$(0, \frac{15}{2}, -2)$
$(\frac{34}{5}, 2, -1)$	$(3, 1, 4)$	$(3, \frac{5}{2}, \frac{1}{2})$	$(3, 3, 4)$
$(8, 5, 4)$	$(0, \frac{25}{4}, -1)$	$(0, 6, 0)$	$(-\frac{6}{5}, 7, -1)$

$(8, \frac{5}{2}, -2)$	$(0, 0, 0)$	$(2, \frac{15}{4}, 0)$	$(\frac{1}{2}, 1, \frac{9}{2})$
$(0, \frac{15}{8}, \frac{5}{2})$	$(3, \frac{1}{2}, 2)$	$(0, 6, 1)$	$(0, \frac{15}{4}, 1)$
$(3, 2, 1)$	$(-2, 4, 2)$	$(5, 0, \frac{3}{2})$	$(0, 5, 0)$
$(\frac{7}{2}, 0, \frac{1}{2})$	$(6, \frac{5}{2}, -1)$	$(0, \frac{5}{2}, 2)$	$(-5, 8, 10)$
$(4, 1, 2)$	$(6, \frac{5}{4}, 0)$	$(5, 1, 1)$	$(8, 2, -\frac{5}{2})$
$(\frac{5}{2}, 4, 5)$	$(4, 0, 2)$	$(1, -\frac{7}{2}, 5)$	$(-2, \frac{25}{4}, 0)$
$(-4, 5, \frac{5}{2})$	$(3, 0, \frac{5}{2})$	$(-4, \frac{15}{2}, 0)$	$(2, 0, 3)$

7-26. Mark claims to have created a sequence of three function machines that always gives him the same number he started with.



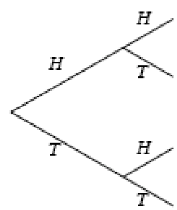
a. Test his machines. Do you think he is right?

b. Be sure to test negative numbers. What happens for negative numbers?

c. Mark wants to get his machines patented but has to prove that the set of machines will always do what he says it will, at least for positive numbers. Show Mark how to prove that his machines work for positive numbers by dropping in a variable (for example, *n*) and writing out each step the machines must take.

d. Why do the negative numbers come out positive?

7-27. Flipping a penny and a nickel can be shown in a **tree diagram**. Use the tree diagram at right to calculate the following probabilities.



a. $P(H,T)$
(Note: This means the probability of getting one head and then one tail.)

b. $P(T,H)$

c. $P(H,H)$ or $P(T,T)$

7-28. Given $f(x)=-2x^2-4$ and $g(x)=5x+3$, calculate:

a. $g(-2)$

b. $f(-7)$

c. $f(g(-2))$

d. $f(g(1))$

7.1.3 What can I discover about 3-D systems?

Systems of Three-Variable Equations

You know a lot about systems of two-variable equations, their solutions, and their graphs. Today you will **investigate** systems of three-variable equations.

7-29. THREE-DIMENSIONAL SYSTEM INVESTIGATION

Consider the following systems of equations:

System I

$$20x+12y+15z=60$$
$$20x+12y+15z=120$$

System II

$$20x+15y+12z=60$$
$$10x+30y+12z=60$$

Your task: With your team, find out as much as you can about each of these systems of equations, their graphs, and their solutions. Be sure to record all of your work carefully and be prepared to share your summary statements with the class.

Discussion Points

What does the graph of a three-variable equation look like?

What does it mean to be a solution to a system of equations?

What does a solution to a three-variable system of equations look like on a graph?

Is there always a solution to a system of equations?

Further Guidance

7-30. Using isometric dot paper, graph both equations in *System I* from problem 7-29 on a single three-dimensional coordinate system. Use different colors to help identify each graph.

a. Describe the graph of the system in as much detail as you can.

b. Looking at the graph, can you tell what the solution to this system is? Explain.

7-31. Using isometric dot paper, graph both equations in *System II* from problem 7-29 on a single three-dimensional coordinate system. Use different colors to help identify each graph.

a. Describe the graph of the system in as much detail as you can.

b. Looking at the graph, can you tell what the solution to this system is? Explain.

7-32. Now compare the graphs of the two systems. How are they similar? How are they different?

Further Guidance section ends here.

7-33. Look closely at your graph of *System II*. Can you see the intersection of the two planes clearly? If not, make a new set of axes and graph the systems carefully.

a. What does the intersection of two planes look like?

b. Work with your team to find the coordinates of as many points as you can that lie in both planes. Show your work and describe your **strategies**. Be prepared to share your ideas with the class.

c. Can you add a third equation to the system that will share the same intersection with the original two graphs?

Review & Preview

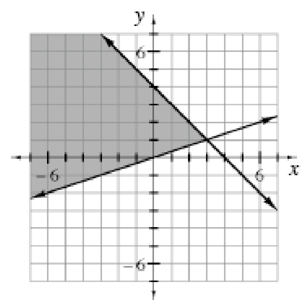
7-34. On isometric dot paper, graph the system of equations at right. What shape is the intersection? Use color to show the intersection clearly on your graph.

$$10x + 6y + 5z = 30$$
$$6x + 15y + 5z = 30$$

7-35. Verify that $2^7 = 128$. Is it true that $\log 2^7 = \log 128$?

7-36. If $24 = y$, is it true that $\log 24 = \log y$? **Justify** your answer.

7-37. Write the system of inequalities that would give the graph at right.



7-38. Solve $1 - \frac{b}{x} = a$ for x.

7-39. Cheri has forgotten how to change a quadratic equation from standard form to graphing form. She remembers *some* things about averaging intercepts and completing the square, but she is really confused. Clearly show Cheri *both* methods to change $y = x^2 + 4x - 7$ into graphing form.

7-40. Solve $\sqrt{x+2} = 8$ and check your solution.

7-41. Factor each expression completely.

a. $25x^2 - 1$

b. $5x^3 - 125x$

c. $x^2 + x - 72$

d. $x^3 - 3x^2 - 18x$

7-42. Solve for x, y, and z: $(2^x)(3^y)(5^z) = (2^3)(3^{x-2})(5^{2x-3y})$.

7.1.4 What is a solution in three dimensions?

Solving Systems of Three Equations with Three Unknowns

Today you will extend what you know about systems of equations to examine how to solve systems of equations with three variables. As you work with your team, look for connections to previous work. The focus questions below can help generate mathematical discussion.

What does a solution to a system in three variables mean?

What **strategies** can we use?

What does the intersection look like?

7-43. Review the **strategies** for solving systems that you already have as you solve the following two-variable system of equations. Use any method. Do not hesitate to change **strategies** if your first **strategy** seems cumbersome. If there is no solution, explain what that indicates about the graph of this system. Leave your solution in (x, y) form.

$$12x - 2y = 16$$
$$30x + 2y = 68$$

7-44. Solve the following three-variable system of equations by graphing it with your graphing tool or on isometric dot paper. Give your solution in (x, y, z) form. Then test your solution in the equations and describe your results.

$$2x+3y+3z=6$$
$$6x-3y+4z=12$$
$$2x-3y+2z=6$$

7-45. FINDING AN EASIER WAY

As you saw in problem 7-44, using a graph to solve a system of three equations with three variables can lead to inconclusive results. What other **strategies** should be considered? Discuss this with your team and be prepared to share your ideas with the class.

7-46. Looking at the equations in problem 7-44, Elissa wanted to see if she could apply some of her solving techniques from two-variable equations to this 3-D system.

a. Elissa noticed that the first two equations could be combined to form the new equation $8x + 7z = 18$. How did she accomplish this? Explain.

b. Now that Elissa has an equation with only x and z, she needs to find another equation with only x and z to be able to solve the system. Choose a different pair of equations to combine and find a way to eliminate y so that the new equation only has x and z. Then solve the system to find x and z.

c. For which variable do you still need to solve? Work with your team to solve for this variable. Then write the solution as a point in (x, y, z) form.

d. Is your solution reasonable? Does it make sense? Does it agree with your graph?

7-47. Practice using your algebraic **strategies** by solving the systems below, if possible. If there is no solution or if the solution is different than you expected, use the graphing tool to help you figure out why.

a.
$$x + y + 3z = 3$$
$$2x + y + 6z = 2$$
$$2x - y + 3z = -7$$

b.
$$20x + 12y + 15z = 60$$
$$20x + 12y + 15z = 120$$
$$10x + 20z = 30$$

c.
$$5x - 4y - 6z = -19$$
$$-2x + 2y + z = 5$$
$$3x - 6y - 5z = -16$$

d.
$$6x + 4y + z = 12$$
$$6x + 4y + 2z = 12$$
$$6x + 4y + 3z = 12$$

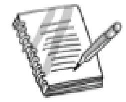
7-48. Today you developed a way to solve a system of three equations with three variables. But what does the solution of a system like those provided in problem 7-47 represent? Consider this as you answer the questions below with your graphing tool.

a. One of the systems in problem 7-47 should have had no solution. Graph this system with your graphing tool. Describe how the planes are positioned and why there is no common point on all three planes.

b. In what other ways could three planes be positioned so that there is no solution? Use paper or cardboard to help you communicate your ideas with others.

c. Graph the system in part (d) of problem 7-47 with your graphing tool and examine the result. How can you describe the intersection of these planes?

7-49. LEARNING LOG



In your Learning Log, describe your algebraic **strategy** to solve a system of three equations with three variables. Give enough details to help you later when you need to refer to it. Title this entry "Systems of Three Equations with Three Variables" and include today's date.

METHODS AND MEANINGS

MATH NOTES

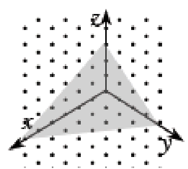
Graphing Planes in Three Dimensions

To graph a plane, it is easiest to use the intercepts to draw the trace lines (the intersections of the plane with the *xy*-, *xz*-, and *yz*-planes) that will represent the plane.

To find the intercepts, let two of the variables equal zero. Then solve to find the intercept corresponding to the remaining variable.

For example, for $2x+3y+4z=12$, the *x*-intercept is found by letting *y* and *z* equal zero, which gives $2x=12$. Therefore the *x*-intercept is (6, 0, 0). Similarly, the *y*-intercept is (0, 4, 0), and the *z*-intercept is (0, 0, 3).

Drawing the line between two intercepts gives the trace line for the plane. For example, connecting the *x*- and *y*-intercepts, you would get the equation $2x+3y=12$, which is the trace line in the *xy*-plane when $z=0$ in the equation $2x+3y+4z=12$. Connecting the *x*- and *z*-intercepts gives the trace line in the *xz*-plane.



Review & Preview

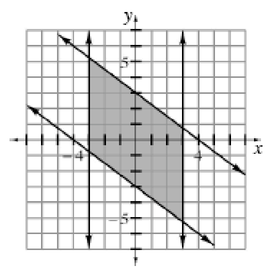
7-50. Use the algebraic **strategies** you developed in today's lesson to solve the system of equations at right. Be sure to check your solution.

$$2x + y - 3z = -12$$
$$5x - y + z = 11$$
$$x + 3y - 2z = -13$$

7-51. Use each pair of points given below to write a system of equations in $y = mx + b$ form to find the equation of a line that passes through the points.

a. (20, 2) and (32, –4)   b. (–3, –17) and (12, –7)

7-52. Write a system of inequalities that could be represented by the graph at right.



7-53. Solve $\sqrt{5x-1} = \sqrt{6+4x}$ and check your solution.

7-54. If two quantities are equal, are their logarithms also equal? Consider the questions below.

a. Is it true that 4^2 is equal to 2^4? Is this a special case, or is a^b equal to b^a for any values of a and b?

b. Is $\log 4^2$ equal to $\log 2^4$? How can you be sure?

c. Are the equations $x = 5$ and $\log x = \log 5$ equivalent? **Justify** your answer.

d. Is the equation $\log 7 = \log x^2$ equivalent to the equation $7 = x^2$? How can you be sure?

7-55. Use the ideas from problem 7-54 to help you solve the following equations.

a. $\log 10 = \log(2x - 3)$ b. $\log 25 = \log(4x^2 - 5x - 50)$

7-56. Find an equation for each of the lines described below.

a. The line with slope $\frac{1}{3}$ that goes through the point (0, 5).

b. The line parallel to $y = 2x - 5$ that goes through the point (1, 7).

c. The line perpendicular to $y = 2x - 5$ that goes through the point (1, 7).

d. The line that goes through the point (0, 0) so that the tangent of the angle it makes with the x-axis is 2.

7-57. Solve each equation below for y so that it could be entered in the graphing calculator.

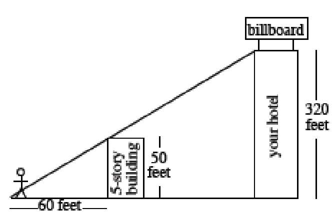
a. $x^2 = x(2x - 4) + y$ b. $x = 3 + (y - 5)^2$

7-58. Sketch the graph of each equation or inequality below.

a. $(x-2)^2 + (y+3)^2 = 9$

b. $(x-2)^2 + (y+3)^2 \geq 9$

7-59. You are standing 60 feet away from a five-story building in Los Angeles, looking up at its rooftop. In the distance you can see the billboard on top of your hotel, but the building is completely obscured by the one in front of you. If your hotel is 32 stories tall and the average story is 10 feet high, how far away from your hotel are you?



7.1.5 How can I apply systems of equations?

Using Systems of Three Equations for Curve Fitting

In this lesson you will work with your team to find the equation of a quadratic function that passes through three specific points. You will be challenged to extend what you know about writing and solving a system of equations in *two variables* to solving a system of equations in *three variables*.

7-60. In your work with parabolas, you have developed two forms for the general equation of a quadratic function: $y = ax^2 + bx + c$ and $y = a(x-h)^2 + k$. What information does each equation give you about the graph of a parabola? Be as detailed in your explanation as possible. When is each form most useful?

7-61. Suppose the graph of a quadratic function passes through the points (1, 0), (2, 5), and (3, 12). Sketch its graph. Then work with your team to develop an algebraic method to find the equation $y = ax^2 + bx + c$ of this specific quadratic function.

Discussion Points

What does the graph of any quadratic function look like?

What does it mean for the graph of $y = ax^2 + bx + c$ to pass through the point (3, 12)?

What solving method can we use to find *a*, *b*, and *c*?

How can we check our equation?

Would this method allow us to find the equation of a quadratic using *any* three points?

Would this method work if we only had two points?

Further Guidance

7-62. How many points does it take to determine the equation of a linear function $y = ax + b$? Discuss this with your team and include at least one sketch to support your answer.

Now think about the graph of a quadratic function $y = ax^2 + bx + c$. How many points do you think it would take to determine this graph? Why? Does there need to be any restriction on the points you use? Discuss these questions with your team and **justify** your answers before moving on to part (a).

a. Suppose you wanted the graph of a quadratic function $y = ax^2 + bx + c$ to pass through the points (1, 0), (2, 5), and (3, 12). How would these points be useful in finding the specific equation of this function? If your team has not already done so, include a sketch of the parabola going through these points to support your answer.

b. It is often useful to label points with the variable they represent. For instance, for the point (3, 12), which variable does the 3 represent? Which variable does the 12 represent?

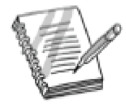
c. Using the general equation of a quadratic, $y = ax^2 + bx + c$, substitute the x- and y-values from your first point into the equation. Then do the same for the other two points to create three equations where the unknowns are a, b, and c.

d. Now use the **strategies** you developed in Lesson 7.1.4 to solve the system of equations for a, b, and c.

e. Use your results to write the equation of the quadratic function that passes through the points (1, 0), (2, 5), and (3, 12). How can you check your answer? That is, how can you make sure your equation would actually go through the three points? Using the method your team decides on, check your equation.

Further Guidance section ends here.

7-63. LEARNING LOG



In your Learning Log, summarize the method you used in problem 7-61 (or problem 7-62) to find the equation $y = ax^2 + bx + c$ of the quadratic function whose graph passes through three given points. Title this entry "Finding the Equation of a Parabola Given Three Points" and label it with today's date.

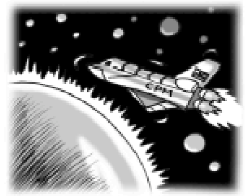
7-64. Find the equation $y = ax^2 + bx + c$ of the function that passes through the three points given in parts (a) and (b) below. Be sure to check your answers.

a. (3, 10), (5, 36), and (–2, 15)

b. (2, 2), (–4, 5), and (6, 0)

7-65. What happened in part (b) of problem 7-64? Why did this occur? (If you are not sure, plot the points on graph paper.)

7-66. Space Shuttle CPM is approaching a star and is caught in its gravitational pull. When the shuttle's engines are fired, the shuttle will slow down, stop momentarily, and then pick up speed and move away from the star, avoiding its gravitational field. Space Shuttle CPM engaged its engines when it was 750 thousand miles from the star. After one full minute, the shuttle was 635 thousand miles from the star. After two minutes, the ship was 530 thousand miles from the star.



a. Name the three points given in the information above if x = the time since the engines were engaged and y = the distance (in thousands of miles) from the star.

b. Based on the points in part (a), make a rough sketch of a graph that shows the distance reaching a minimum and then increasing again, over time. What kind of function could follow this pattern?

c. Find the equation of a graph that fits the three points you found in part (a).

d. If the ship comes within 50 thousand miles of the star, the shields will fail and the ship will burn up. Use your equation to determine whether the space ship has failed to escape the gravity of the star.

7-67. Sickly Sid has contracted a serious infection and has gone to the doctor for help. The doctor takes a blood sample and finds 900 bacteria per cc (cubic centimeter) and gives Sid a shot of a strong antibiotic. The bacteria will continue to grow for a period of time, reach a peak, and then decrease as the medication succeeds in overcoming the infection. After ten days, the infection has grown to 1600 bacteria per cc. After 15 days it has grown to 1875.

a. Name three data points given in the problem statement.

b. Make a rough sketch that will show the number of bacteria per cc over time.

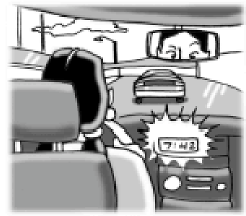
c. Find the equation of the parabola that contains the three data points.

d. Based on the equation, how long will it take until the bacteria are eliminated?

e. Based on the equation, how long had Sid been infected before he went to the doctor?

7-68. THE COMMUTER

Sensible Sally has a job that is 35 miles from her home and needs to be at work by 8:15 a.m. She wants to get as much sleep as she can, leave as late as possible, and still get to work on time. Sally discovered that if she leaves at 7:10, it takes her 40 minutes to get to work. If she leaves at 7:30, it takes her 60 minutes to get work. If she leaves at 7:40, it takes her 50 minutes to get to work. Since her commute time increases and then decreases, Sally decided to use a parabola to model her commute, assuming the time it takes her to get to work varies quadratically with the number of minutes after 7:00 that she leaves her house.



a. If x = the number of minutes after 7:00 that Sally leaves, and y = the number of minutes it takes Sally to get to work, what three ordered pairs can you determine from the problem?

b. Use the three points from part (a) to find the equation of a parabola in standard form that can be used to model Sally's commute.

c. Will Sally make it to work on time if she leaves at 7:20?

7-69. PAIRS PARABOLA CHALLENGE

Your challenge will be to work with a partner to create a parabola puzzle for another pair of students to solve. Follow the directions below to create a puzzle that will make them think and allow them to show off their algebra skills. When you are ready, you will trade puzzles with another pair and attempt to solve theirs.

a. With your partner, decide on an equation for a parabola and then identify three points that lie on its graph. Keep track of how you came up with your equation and how you chose your points. Be ready to share **strategies**.

b. Write the coordinates of the three points on an index card or small slip of paper to give to another pair of students. Be sure you keep a copy of your equation so you can check their work later.

c. Trade points with another pair and work with your partner to solve their puzzle. When you are confident of your equation, have the writers check your work.

7-70. Make a conjecture about how you would find the equation $y = ax^3 + bx^2 + cx + d$ of a cubic function that passes through a given set of points when graphed. How many points do you think you would need to be given to be able to determine a unique equation? How could you extend the method you developed for solving a quadratic to solving a cubic?

Review & Preview

7-71. Solve the system of equations at right and then check your solution in each equation. Be sure to keep your work well organized.

$$x - 2y + 3z = 8$$
$$2x + y + z = 6$$
$$x + y + 2z = 12$$

7-72. Find the equation in $y = ax^2 + bx + c$ form of the parabola that passes through the points (1, 5), (3, 19), and (–2, 29).

7-73. This problem is a checkpoint for graphing linear inequalities. It will be referred to as Checkpoint 12.

Complete parts (a) through (c) below for the system of inequalities at right.

$$y \le -2x + 3$$
$$y \ge x$$
$$x \ge -1$$

a. Draw the graph.

b. Find the area of the shaded region.

c. Check your graph by referring to the Checkpoint 12 materials located at the back of your book.

If you needed help graphing this system of inequalities correctly, then you need more practice graphing linear inequalities and systems of inequalities. Review the Checkpoint 12 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to graph systems of inequalities such as this one easily and accurately.

7-74. Simplify each expression in parts (a) through (c) below. Then complete part (d).

a. $xy(\frac{1}{x} + \frac{1}{2y})$ b. $ab(\frac{2}{a} + \frac{4a}{b})$ c. $2x(3 - \frac{1}{2x})$

d. What expression would go in the box to make the equation $\square(\frac{2}{x} + \frac{7}{y}) = 2y + 7x$ true?

7-75. Change each of the following equations from logarithmic form to exponential form, or vice versa.

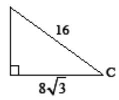
a. $y = \log_{12} x$ b. $x = \log_y 17$

c. $y = 1.75^{2x}$ d. $3y = x^7$

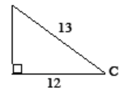
7-76. Solve $\sqrt{3x-6}+6=12$ and check your solution.

7-77. Find $m\angle C$ in each triangle below.

a.



b.



7-78. Rewrite each expression below as an exponential expression with a base of 2.

a. 16 b. $\frac{1}{8}$ c. $\sqrt{2}$ d. $\sqrt[3]{4}$

7-79. Solve the system of equations at right and then check your solution in each equation. Be sure to keep your work well organized.

$$x+2y-z=-1$$
$$2x-y+3z=13$$
$$x+y+2z=14$$

7-80. Find an equation for the parabola that passes through the points $(-1,10)$, $(0,5)$, and $(2,7)$.

7-81. Change each of the following equations from logarithmic form to exponential form, or vice versa.

a. $a=\log_b 24$ b. $3x=\log_{2y} 7$

c. $3y=2^{5x}$ d. $4p=(2q)^6$

7-82. Consider the function defined as follows: The inputs are the numbers on a normal, fair die, and the outputs are the probability of that number coming up when the die is rolled. **Investigate** this function.

7-83. On their team test, Raymond, Sarah, Hannah, and Aidan were given $y=4x^2-24x+7$ to change into graphing form. Raymond noticed that the leading coefficient was a 4 and not a 1. His team agreed on a way to start rewriting, but then they worked in pairs and got two different solutions, shown below.

Raymond and Hannah	Aidan and Sarah
(1) $y=4x^2-24x+7$	(1) $y=4x^2-24x+7$
(2) $y=4(x^2-6x)+7$	(2) $y=4(x^2-6x+9)+7-9$
(3) $y=4(x^2-6x+3^2)+7-36$	(3) $y=4(x-3)^2+7-3^2$
(4) $y=4(x-3)^2-29$	(4) $y=4(x-3)^2-2$

Hannah says, *"Aidan and Sarah made a mistake in Step 3. Because of the factored 4 they really added 4(9) to complete the square, so they should subtract 36, not just 9."*

Is Hannah correct? **Justify** your answer by showing whether the results are equivalent to the original equation.

7-84. Use the correct method from problem 7-83 to change each of the following equations to graphing form. Then, without graphing, find the vertex and equation of the line of symmetry for each.

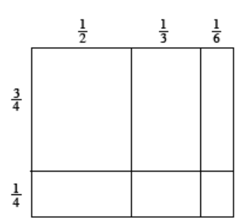
a. $y=2x^2-8x+7$ b. $y=5x^2-10x-7$

7-85. Given $f(x)=2x^2-4$ and $g(x)=5x+3$, find the value of each expression below.

a. $f(a)$ b. $f(3a)$ c. $f(a+b)$

d. $f(x+7)$ e. $f(5x+3)$ f. $g(f(x))$

7-86. Complete the area model shown at right, then complete parts (a) through (c) below.

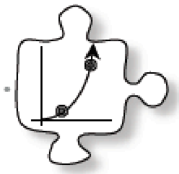


a. What is the sum of the areas of each of the rectangles that makes up the larger one?

b. What are the length and width of the large rectangle?

c. Write a statement of the form $l\cdot w = Area\ as\ a\ sum$ for this rectangle.

7.2.1 How can I solve exponential equations?



Using Logarithms to Solve Exponential Equations

In Chapter 6, you learned what a logarithm was, and you also learned several important facts about logs. In this lesson, you will learn about a property of logarithms that will prove very useful for solving problems that involve exponents.

7-87. LOGARITHMS SO FAR

There are three important log facts you have worked with so far. Discuss these questions with your team to ensure everyone remembers these ideas. For each problem, make up an example to illustrate your ideas.

a. What is a logarithm? How can log equations be converted into another form?

b. What do you know about the logarithm key on your calculator?

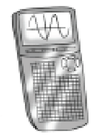
c. What does the graph of $y = \log(x)$ look like? Write a general equation for $y = \log(x)$.

7-88. Marta was convinced that there had to be some way to graph $y = \log_2 x$ on her graphing calculator. She typed in $y = \log(2^x)$ and pressed GRAPH.

"It WORKED!" Marta yelled in triumph.

"Whaaaat?" said Celeste. *"I think* $y = \log_2 x$ *and* $y = \log(2^x)$ *are totally different, and I bet we can prove it by converting both of them to exponential form."*

"Yeah, I think you're wrong, Marta," said Sophia. *"I think we can prove* $y = \log_2 x$ *and* $y = \log(2^x)$ *are totally different by looking at the graphs."*



a. Show that the two equations are different by using what you learned in previous lessons to sketch the graph of $y = \log_2 x$. Then sketch what your graphing calculator shows to be the graph of $y = \log(2^x)$.

b. Now show that $y = \log_2 x$ and $y = \log(2^x)$ are different by converting both of them to exponential form.

7-89. The work you did in problem 7-88 is a **counterexample**, which shows that in general, the statement $\log_b x = \log(b^x)$ is *false*. For each of the following log statements, use the **strategies** from problem 7-88 to determine whether they are true or false, and **justify** your answer. Be ready to present your conclusions and **justifications**.

a. $\log_5(25) \stackrel{?}{=} \log_{25}(5)$

b. $\log(x^2) \stackrel{?}{=} (\log x)^2$

c. $\log(7^x) \stackrel{?}{=} x\log(7)$

d. $\log(2x) \stackrel{?}{=} \log_2 x$

7-90. In the previous problem only *one* of the statements was true.

a. Use different numbers to make up four more statements that follow the same pattern as the one true statement, and test each one to see whether it appears to be true.

b. Use your results to complete the following statement, which is known as the **Power Property of Logs**: $\log(b^x) =$ ________ .

7-91. Do you remember solving problems like $1.04^x = 2$ in your homework? What method(s) did you and your teammates use to find x? In tonight's homework there are several more of these problems. (You probably wish there were a more efficient way!)

7-92. THERE MUST BE AN EASIER WAY

It would certainly be helpful to have an easier method than Guess and Check to solve equations like $1.04^x = 2$. Complete parts (a) through (c) below to discover an easier way.

a. What makes the equation $1.04^x = 2$ so hard to solve?

b. Surprise! In the first part of this lesson, you already found a method for getting rid of inconvenient exponents! Talk with your team about how your results from problems 7-89 and 7-90 can help you rewrite the equation $1.04^x = 2$. Be prepared to share your ideas with the class.

c. Solve $1.04^x = 2$ using this new method. Be sure to check your answer.

7-93. Solve the following equations. Be sure your answers are accurate to three decimal places, and also be sure to check your answers.

a. $5 = 2.25^x$

b. $3.5^x = 10$

c. $2(8^x) = 128$

d. $2x^8 = 128$

Review & Preview

7-94. Complete the table at right and find its equation.

x	y
	1
	3
2	9
	27
4	
	243
6	
7	
8	

7-95. Margee thinks she can use logs to solve $56 = x^8$, since logs seem to make exponents disappear. Unfortunately, Margee is wrong. Explain the difference between equations like $2 = 1.04^x$, in which you can use logs, and $56 = x^8$, in which it does not make sense to use logs.

7-96. What values must x have so that $\log(x)$ is greater than 2? **Justify** your answer.

7-97. Simplify each expression below. If you are stuck, the ideas in problem 7-74 should be helpful.

a. $\dfrac{x}{1-\frac{1}{x}}$

b. $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{b}-a}$

7-98. Consider the questions below.

a. What can you multiply 8 by to get 1?

b. What can you multiply x by to get 1?

c. Using the rules of exponents, find a way to solve $m^8 = 40$. Remember that logarithms will not be useful here, but the exponent key on your calculator *will* be. (Obtain the answer as a decimal approximation using your calculator. Check your result by raising it to the 8th power.)

d. Now solve $n^6 = 300$.

e. Describe a method for solving $x^a = b$ for x with a calculator.

7-99. Adam keeps getting negative exponents and fractional exponents confused. Help him by explaining the difference between $2^{1/2}$ and 2^{-1}.

7-100. Solve each inequality and graph its solution on a number line.

a. $|x| < 3$ b. $|2x+1| < 3$ c. $|2x+1| \geq 3$

7-101. What is the equation of the line of symmetry for the graph of $y = (x-17)^2$? **Justify** your answer.

7-102. Solve each system of equations below.

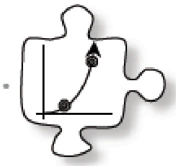
a.
$$-4x = z - 2y + 12$$
$$y + z = 12 - x$$
$$8x - 3y + 4z = 1$$

b.
$$3x + y - 2z = 6$$
$$x + 2y + z = 7$$
$$6x + 2y - 4z = 12$$

c. What does the solution in part (b) tell you about the graphs?

7.2.2 How can I rewrite it?

Investigating the Properties of Logs



You already know the basic rules for working with exponents. Since logs are the inverses of exponential functions, they also have properties that are similar to the ones you already know. In this lesson, you will explore these properties.

7-103. Marta now knows that if she wants to find $\log_2(30)$, she cannot just type $\log(2^{30})$ into her calculator, since her calculator's log key cannot directly calculate logs with base 2. But she still wants to be able to find what $\log_2(30)$ equals.

a. First, use your knowledge of logs to estimate $\log_2(30)$.

b. Now use what you learned in the previous lesson to get a better estimate. Since you want to determine what $\log_2(30)$ equals, you can write $\log_2(30) = x$. When working with a log equation, it is often easier to convert it to exponential form. Rewrite this equation in exponential form.

c. Use the methods you developed in class to solve this equation. Refer back to your work on problem 7-92 if you need help.

d. Congratulations! You are smarter than your calculator. You have just evaluated a log with base 2, even though your calculator does not know how to do that. Now you will have more practice. First estimate an answer, then apply the method you have just developed to evaluate $\log_5(200)$.

e. Apply the process you used in part (d) to evaluate the expression $\log_a b = x$.

7-104. Since logs and exponentials are inverses, the properties of exponents (which you already know) translate to logs. The problems below will help you discover these new log properties.

a. Complete the two exponent rules below. In part (b), you will find the equivalent properties of logs.

$$x^a x^b = \underline{\qquad\qquad} \quad \text{and} \quad \frac{x^b}{x^a} = \underline{\qquad\qquad}$$

b. To help you find the equivalent log properties, use your calculator to solve for x in each problem below. Note that x is a whole number in parts (i) through (v). Look for patterns that would make your job easier and allow you to **generalize** in part (vi).

i. $\log(5)+\log(6)=\log(x)$ *ii.* $\log(5)+\log(2)=\log(x)$

iii. $\log(5)+\log(5)=\log(x)$ *iv.* $\log(10)+\log(100)=\log(x)$

v. $\log(9)+\log(11)=\log(x)$ *vi.* $\log(a)+\log(b)=\log(\underline{\quad\quad})$

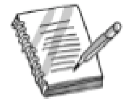
c. What if the log expressions are being subtracted instead of added? Solve for x in each problem below. Note that x will not always be a whole number.

i. $\log(20)-\log(5)=\log(x)$ *ii.* $\log(30)-\log(3)=\log(x)$

iii. $\log(5)-\log(2)=\log(x)$ *iv.* $\log(17)-\log(9)=\log(x)$

v. $\log(375)-\log(17)=\log(x)$ *vi.* $\log(b)-\log(a)=\log(\underline{\quad\quad})$

7-105. LEARNING LOG



The two properties you found in problem 7-104 work for logs in *any* base, not just base 10. (You will officially prove this later.) You now know three different log properties and you have developed a process for solving log problems that are not in base 10. Write and explain each of the log properties in your Learning Log. Be sure to include examples and add an example of a problem where you need to change to base 10. Title this entry “Log Properties” and label it with today’s date.

7-106. LOG PROPERTY PUZZLES

Obtain the Lesson 7.2.2 Resource Page from your teacher or copy the table below. Use the log properties to fill in the missing parts. Be sure to remember that in every row, each expression is equivalent to every other expression.

	Product Property		Quotient Property	
$\log_3 60$	$= \log_3 6 +$ ___	$= \log_3 3 +$ ___	$= \log_3 120 -$ ___	$= \log_3 240 -$ ___
$\log_7 36$	=	=	=	=
	$= \log_6 9 + \log_6 2$	=	=	=
	=	=	$= \log_{25} 75 - \log_{25} 1.5$	=
	=	=	$= \log 160 - \log 4$	=

7-107. Use the properties of logs to write each of the following expressions as a single logarithm, if possible.

a. $\log_{1/2}(4)+\log_{1/2}(2)-\log_{1/2}(5)$

b. $\log_2(M)+\log_3(N)$

c. $\log(k)+x\log(m)$

d. $\frac{1}{2}\log_5 x+2\log_5(x+1)$

e. $\log(4)-\log(3)+\log(\pi)+3\log(r)$

f. $\log(6)+23$

7-108. What values must x have so that $\log(x)$ has a negative value? **Justify** your answer.

7-109. The fact that for any base m (when $m > 0$), $\log_m a + \log_m b = \log_m ab$ is called the **Product Property of Logarithms**. To prove that this property is true, follow the directions below.

a. Since logarithms are the inverses of exponential functions, each of their properties can be derived from a similar property of exponents. Here, you are trying to prove that "logs turn products into sums." First, recall similar properties of exponents. If $a = m^x$ and $b = m^y$, write $a \cdot b$ as a power of m.

b. Rewrite $a = m^x$, $b = m^y$, and your answer to part (a) in logarithmic form.

c. In the third equation you wrote for part (b), substitute for x and y to obtain a log equation of base m that involves only the variables a and b.

d. The property $\log_m a - \log_m b = \log_m \frac{a}{b}$ is called the **Quotient Property of Logarithms**. Use $a = m^x$ and $b = m^y$ to express $\frac{a}{b}$ as a power of m. Then use a similar process to rewrite each into log form and prove the Quotient Property of Logs.

7-110. The **Power Property of Logs** is a little trickier to prove. A proof is given below. As you copy each step onto your paper, work with your team to make sense of what was done. Give a reason for each step.

To prove that $\log_m a^n = n\log_m a$,

Let $\log_m a^n = p$ and $n\log_m a = q$

Convert to $m^p = a^n$

First rewrite to $\log_m a = \frac{q}{n}$ and then convert to $m^{q/n} = a$.

Using the two resulting equations, substitute for a and then simplify:

$$m^p = (m^{q/n})^n$$

$$m^p = m^q$$

Therefore, $p = q$.

Remember that $p = \log_m a^n$ and $q = n\log_m a$, so $\log_m a^n = n\log_m a$, which was the goal of the proof.

MATH NOTES

METHODS AND MEANINGS

Logarithm Properties

The following definitions and properties hold true for all positive $m \neq 1$.

Definition of logs:	$\log_m(a) = n$ means $m^n = a$
Product Property:	$\log_m(a \cdot b) = \log_m(a) + \log_m(b)$
Quotient Property:	$\log_m(\frac{a}{b}) = \log_m(a) - \log_m(b)$
Power Property:	$\log_m(a^n) = n \cdot \log_m(a)$
Inverse relationship:	$\log_m(m)^n = n$ and $m^{\log_m(n)} = n$

March 03, 2011

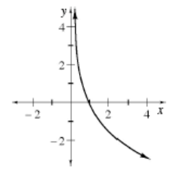
Review & Preview

7-111. Solve each of the following equations to the nearest 0.001.

a. $(5.825)^{(x-3)} = 120$

b. $18(1.2)^{(2x-1)} = 900$

7-112. At right is a graph of $y = \log_b x$. Describe the possible values for b.



7-113. Use the definition of a logarithm to change $\log_2 7$ into a logarithmic expression of base 5.

7-114. Sketch the graph of $y = \log_3(x+4)$ and describe the transformation from its parent graph.

7-115. The economy has worsened to the point that the merchants in downtown Hollywood cannot afford to replace their outdoor light bulbs when the bulbs burn out. On average, about thirteen percent of the light bulbs burn out every month. Assuming there are now about one million outside store lights in Hollywood, how long will it take until there are only 100,000 bulbs lit? Until there is only one bulb lit?

7-116. Raymond, Hannah, Aidan, and Sarah were working together to change $y = 3x^2 - 15x - 5$ into graphing form. They started by rewriting it as $y = 3(x^2 - 5x) - 5$, when Raymond said, *"Will this one work? Look, the perfect square would have to be* $(x-2.5)^2$*."*

After thinking about it for a while, Sarah said, *"That's OK. Negative 2.5 squared is 6.25, but because of the 3 we factored out, we are really adding* $3(6.25)$*."*

"Yes," Aidan added, *"So we have to subtract 18.75 to get an equivalent equation."*

Hannah summarized with the work shown at right.

$$y = 3x^2 - 15x - 5$$
$$y = 3(x^2 - 5x) - 5$$
$$y = 3(x-2.5)^2 - 5 - 18.75$$
$$y = 3(x-2.5)^2 - 23.75$$

What do you think? Did they rewrite the equation correctly? If so, find the vertex and the line of symmetry of the parabola. If not, explain their mistakes and show them how to do it correctly.

7-117. Use the ideas developed in problem 7-116 to change each of the following quadratic equations into graphing form. Identify the vertex and the line of symmetry for each one.

a. $f(x) = 4x^2 - 12x + 6$

b. $g(x) = 2x^2 + 14x + 4$

7-118. Consider the function $y = 3(x+2)^2 - 7$ as you complete parts (a) through (c) below.

a. How could you restrict the domain to show "half" of the graph?

b. Find the equation for the inverse function for your "half" graph.

c. What are the domain and range for the inverse function?

7-119. Eniki has a sequence of numbers given by the formula $t(n) = 4(5^n)$.

a. What are the first three terms of Eniki's sequence?

b. Chelita thinks the number 312,500 is a term in Eniki's sequence. Is she correct? **Justify** your answer by either giving the term number or explaining why it is not in the sequence.

c. Elisa thinks the number 94,500 is a term in Eniki's sequence. Is she correct? Explain.

7-120. Solve each of the following equations.

a. $\frac{x}{3} = x + 4$

b. $\frac{x+6}{3} = x$

c. $\frac{x+6}{x} = x$

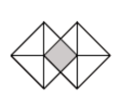
d. $\frac{2x+3}{6} + \frac{1}{2} = \frac{x}{2}$

7-121. Which of the following statements are true? If true, explain how you know, and if not, show why not.

a. $\frac{x+3}{5} = \frac{x}{5} + \frac{3}{5}$

b. $\frac{5}{x+3} = \frac{5}{x} + \frac{5}{3}$

7-122. Two congruent overlapping squares are shown at right. If a point inside the figure is chosen at random, what is the probability that it will *not* be in the shaded region?



Log Property Puzzles

		Product Property				Quotient Property		
$\log_3 60$	=	$\log_3 6 +$ _____	=	$\log_3 3 +$ _____	=	$\log_3 120 -$ _____	=	$\log_3 240 -$ _____
$\log_7 36$	=		=		=		=	
	=	$\log_6 9 + \log_6 2$	=		=		=	
	=		=		=	$\log_{25} 75 - \log_{25} 1.5$	=	
	=		=		=	$\log 160 - \log 4$	=	

Log Property Puzzles

		Product Property				Quotient Property		
$\log_3 60$	=	$\log_3 6 +$ _____	=	$\log_3 3 +$ _____	=	$\log_3 120 -$ _____	=	$\log_3 240 -$ _____
$\log_7 36$	=		=		=		=	
	=	$\log_6 9 + \log_6 2$	=		=		=	
	=		=		=	$\log_{25} 75 - \log_{25} 1.5$	=	
	=		=		=	$\log 160 - \log 4$	=	

7.2.3 How can I find an exponential function?

Writing Equations of Exponential Functions

You have worked with exponential equations throughout this chapter. Today you will look at how you can find the equation for an exponential function using data.

7-123. DUE DATE

Brad's mother has just learned that she is pregnant! Brad is very excited that he will soon become a big brother. However, he wants to know when his new sibling will arrive and decides to do some research. On the Internet, he finds the following article:

> **Hormone Levels for Pregnant Women**
>
> When a woman becomes pregnant, the hormone HCG (human chorionic gonadotropin) is produced to enable the baby to develop.
>
> During the first few weeks of pregnancy, the level of HCG hormone grows exponentially, starting with the day the embryo is implanted in the womb. However, the rate of growth varies with each pregnancy. Therefore, doctors cannot use just a single test to determine how long a woman has been pregnant. They must test the levels over time. Commonly, the HCG levels are measured two days apart to look for this rate of growth.

Brad's mother says she was tested for HCG during her last two doctor visits. On March 21, her HCG level was 200 mIU/ml. Two days later, her HCG level was 392 mIU/ml.

a. Assuming that the model for HCG levels is of the form $y = ab^x$, find an equation that models the growth of HCG for Brad's mother's pregnancy.

b. Assuming that Brad's mother's level of HCG on the day of implantation was 5 mIU/ml, on what day did the baby most likely become implanted? How many days after implantation was his mother's first doctor visit?

c. Brad learned that a baby is born approximately 37 weeks after implantation. When can Brad expect his new sibling to be born?

7-124. SOLVING STRATEGIES

In problem 7-123, you and your team developed a **strategy** to find the equation of an exponential equation of the form $y = ab^x$ when given two points on the curve.

a. What different **strategies** were generated by the other teams in your class? If no one shares your solving method with the class, be sure to share yours. Take notes on the different **strategies** that are presented.

b. Did any team use a system of exponential equations to solve for a and b? If not, examine this **strategy** as you answer the questions below.

i. The doctor visits provide two data points that can help you find an exponential model: $(21, 200)$ and $(23, 392)$. Use each of these points to substitute for x and y into $y = ab^x$. You should end up with two equations in terms of a and b.

ii. Consider the **strategies** you already have for solving systems of equations. Are any of those **strategies** useful for this problem? Discuss a way to solve your system from part (*i*) for a and b with your team. Be ready to share your method with the class.

7-125. The context in problem 7-123 required you to assume that the exponential model had an asymptote at $y = 0$ to find the equation of the model. But what if the asymptote is not at the x-axis? Consider this situation below.

a. Assume the graph of an exponential function passes through the points $(3, 12.5)$ and $(4, 11.25)$. Is the exponential function increasing or decreasing? **Justify** your answer.

b. If the horizontal asymptote for this function is the line $y = 10$, make a sketch of its graph showing the horizontal asymptote.

c. If this function has the equation $y = ab^x + c$, what would be the value of c? Use what you know about this function to find its equation. Verify that as x increases, the values of y get closer to $y = 10$.

d. Find the y-intercept of the function. What is the connection between the y-intercept and the asymptote?

7-126. Janice would like to have \$40,000 to help pay for college in 8 years. Currently, she has \$1000. What interest rate, when compounded yearly, would help her reach her goal?

a. What type of function would best model this situation? Explain how you know and write the general form of this function.

b. If y represents the amount of money and x represents the number of years after today, find an equation that models Janice's financial situation. What interest rate does she need to earn?

c. Janice's friend Sarah starts with \$7800 and wants to have \$18,400 twenty years from now. What interest rate does she need (compounded yearly)?

d. Is Janice's goal or Sarah's goal more realistic? **Justify** your response.

Review & Preview

7-127. Ryan has the chickenpox! He was told that the number of pockmarks on his body would grow exponentially until his body overcomes the illness. He found that he had 60 pockmarks on November 1, and by November 3 the number had grown to 135. To find out when the first pockmark appeared, he will need to find the exponential function that will model the number of pockmarks based on the day.

a. Ryan decides to find the exponential function that passes through the points (3, 135) and (1, 60). Use these points to write the equation of his function of the form $f(x) = ab^x$.

b. According to your model, what day did Ryan get his first chickenpox pockmark?

7-128. Give an example of an equation that requires the use of logarithms to solve it.

7-129. Write three different, but equivalent, expressions for each of the following logs. For example: $\log(7^{3/2})$ can be written as $\frac{3}{2}\log(7)$, $\frac{1}{2}\log(7^3)$, $3\log(\sqrt{7})$, etc.

a. $\log(8^{2/3})$ b. $-2\log(5)$ c. $\log(na)^{bo}$

7-130. Kendra just made a cup of hot chocolate that was too hot for her to drink. She set it aside so it could cool off. While she was waiting, her friend Lara called and Kendra forgot about her hot chocolate. Sketch a graph that shows the temperature of the hot chocolate since Kendra first set it aside. How cold will the hot chocolate get?

7-131. Simplify the following fractions.

a. $\dfrac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a}+\frac{1}{b}}$ b. $\dfrac{x+y}{\frac{1}{x}+\frac{1}{y}}$

7-132. Use $f(x) = 3+\sqrt{2x-1}$ to complete parts (a) through (e) below.

a. What are the domain and range of $f(x)$?

b. What is the inverse of $f(x)$? Call it $g(x)$.

c. What are the domain and range of $g(x)$?

d. Find an expression for $f(g(x))$.

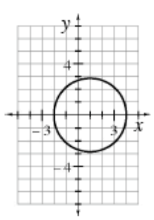
e. Find an expression for $g(f(x))$. What do you notice? Why does this happen?

7-133. Solve each of the following equations for x.

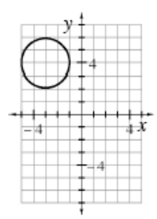
a. $x^3 = 243$ b. $3^x = 243$

7-134. Write the equation of each circle graphed below

a.



b.



7-135. Add or subtract each expression below. Be sure to simplify.

a. $\frac{x^2}{x-5}-\frac{25}{x-5}$ b. $\frac{a^2}{a+5}+\frac{10a+25}{a+5}$

c. $\frac{x^2}{x-y}-\frac{2xy-y^2}{x-y}$ d. $\frac{x}{x+1}+\frac{1}{x-1}$

7-136. If $f(x) = x^4$ and $g(x) = 3(x+2)$, find the value of each expression below.

a. $f(2)$ b. $g(2)$ c. $f(g(2))$ d. $g(f(2))$

e. Are $f(x)$ and $g(x)$ inverses of each other? **Justify** your answer.

7.2.4 Who killed Dr. Dedman?

An Application of Logarithms

7-137. THE CASE OF THE COOLING CORPSE

The coroner's office is kept at a cool 17°C. Agent 008 kept pacing back and forth trying to keep warm as he waited for any new information about his latest case. For more than three hours now, Dr. Dedman had been performing an autopsy on the Sideroad Slasher's latest victim, and Agent 008 could see that the temperature of the room and the deafening silence were beginning to irritate even Dr. Dedman. The Slasher had been creating more work than Dr. Dedman cared to investigate.

"Dr. Dedman, don't you need to take a break?"* Agent 008 queried. *"You've been examining this body for hours! Even if there were any clues, you probably wouldn't see them at this point."

"I don't know,"* Dr. Dedman replied, *"I just have this feeling something is not quite right. Somehow the Slasher slipped up with this one and left a clue. We just have to find it."

"Well, I have to check in with headquarters,"* 008 stated. *"Do you mind if I step out for a couple of hours?"

"No, that's fine,"* Dr. Dedman responded. *"Maybe I'll have something by the time you return."

***"Sure,"* 008 thought to himself. *"Someone always wants to be the hero and solve everything himself. The doctor just does not realize how big this case really is. The Slasher has left a trail of dead bodies through five states!"* Agent 008 left, closing the door quietly. As he walked down the hall, he could hear the doctor's voice describing the victim's gruesome appearance into the tape recorder fade away.**

The hallway from the coroner's office to the elevator was long and dark. This was the only way to Dr. Dedman's office. Didn't this frighten most people? Well, it didn't seem to bother old Ajax Boraxo who was busy mopping the floor, thought 008.

He stopped briefly to use the restroom and bumped into one of the deputy coroners, who asked, *"Dedman still at it?"*

"Sure is, Dr. Quincy. He's totally obsessed. He's certain there is a clue." As usual, when leaving the courthouse, 008 had to sign out.

"How's it going down there, Agent 008?" Sergeant Foust asked. Foust spent most of his shifts monitoring the front door, forcing all visitors to sign in, while he recorded the time next to the signature. Agent 008 wondered if Foust longed for a more exciting aspect of law enforcement. He thought if he were doing Foust's job he would get a little stir-crazy sitting behind a desk most of the day. Why would someone become a cop to do this?

"Dr. Dedman is convinced he will find something soon. We'll see!" Agent 008 responded. He noticed the time: ten minutes before 2:00. Would he make it to headquarters before the chief left?

"Well, good luck!" Foust shouted as 008 headed out the door.

Some time later, Agent 008 sighed deeply as he returned to the coroner's office. Foust gave his usual greeting: *"Would the secret guest please sign in?"* he would say, handing a pen to 008 as he walked through the door. *"Sign in again,"* he thought to himself. *"Annoying!"* 5:05 PM. Agent 008 had not planned to be gone so long, but he had been caught up in what the staff at headquarters had discovered about that calculator he had found. For a moment he saw a positive point to having anyone who came in or out of the courthouse sign in: He knew by quickly scanning the list that Dr. Dedman had not left. In fact, the old guy must still be working on the case.

As he approached the coroner's office, he had a strange feeling that something was wrong. He could not hear or see Dr. Dedman. When he opened the door, the sight inside stopped him in his tracks. Evidently, Dr. Dedman was now the *newest* victim of the Slasher. But wait! The other body, the one the doctor had been working on, was gone! Immediately, the security desk with its annoying sign-in sheet came to mind. Yes, there were lots of names on that list, but if he could determine the time of Dr. Dedman's death, he might be able to scan the roster to find the murderer! Quickly, he grabbed the thermometer to measure the Doctor's body temperature. He turned around and hit the security buzzer. The bells were deafening. He knew the building would be sealed off instantly and security would be there within seconds.

"Oh no!" Foust cried as he rushed in, *"How could this happen? I spoke to the Doctor less than an hour ago!"*

As the security officers crowded into the room, Agent 008 explained what he knew, which was almost nothing. He had stopped long enough to check the doctor's body temperature: 27°C. That was 10°C below normal. Then he remembered: the tape recorder! Dr. Dedman had been taping his observations; that was standard procedure. They began looking everywhere. The Slasher must have realized that the doctor had been taping and taken the tape recorder as well. Exactly an hour had passed during the search, and Agent 008 noticed that the thermometer still remained in Dr. Dedman's side. The thermometer clearly read 24°C. Agent 008 knew he could now determine the time of death.

Problem continues on next page. →

7-137. *Problem continued from previous page.*

Coroner's Office – Please Sign In		
Name	**Time In**	**Time Out**
Lt. Borman	12:08	2:47
Alice Bingham	12:22	1:38
Chuck Miranda	12:30	2:45
Harold Ford	12:51	1:25
Ajax Boraxo	1:00	2:30
D. C. Quincy	1:10	2:45
Agent 008	1:30	1:50
Ronda Ripley	1:43	2:10
Jeff Dangerfield	2:08	2:48
Stacy Simmons	2:14	2:51
Brock Ortiz	2:20	2:43
Pierce Bronson	3:48	4:18
Max Sharp	3:52	5:00
Maren Ezaki	3:57	4:45
Caroline Cress	4:08	4:23
Milly Osborne	4:17	4:39
D.C. Quincy	4:26	4:50
Vinney Gumbatz	4:35	
Cory Delphene	4:48	4:57
Max Crutchfield	5:04	
Agent 008	5:05	
Security	5:12	

a. Make a sketch showing the relationship between body temperature and time. What type of function is it? **Justify** your answer.

b. Is there an asymptote for this relationship? If so, what does it represent? If not, explain why not.

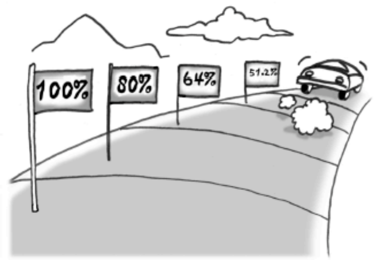
c. Use your data and the general equation $y = km^x + b$ to find the equation that represents the temperature of the body at any certain time.

d. At approximately what time did Dr. Dedman die?

e. Who is the murderer?

Review & Preview

7-138. A rule-of-thumb used by car dealers is that the trade-in value of a car decreases by 20% of its value each year.



a. Explain how the phrase "decreases by 20% of its value each year" tells you that the trade-in value varies exponentially with time (i.e., can be represented by an exponential function).

b. Suppose the initial value of your car is $23,500. Write an equation expressing the trade-in value of your car as a function of the number of years from now.

c. How much will your car be worth in four years?

d. In how many years will the trade-in value of your car be $6000?

e. If your car is really 2.7 years old now, what was its trade-in value when it was new?

7-139. Solve for x without using a calculator.

a. $x = \log_{25}(5)$

b. $\log_x(1) = 0$

c. $23 = \log_{10}(x)$

7-140. Using your calculator, solve the equations below. Round answers to the nearest 0.001.

a. $x^6 = 125$

b. $x^{3.8} = 240$

c. $x^{-4} = 100$

d. $(x+2)^3 = 65$

e. $4(x-2)^{12.5} = 2486$

7-141. Find the inverse of each of the functions below. Write your answers in function notation.

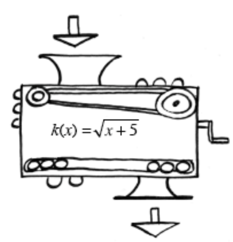
a. $p(x) = 3(x^3 + 6)$

b. $k(x) = 3x^3 + 6$

c. $h(x) = \frac{x+1}{x-1}$

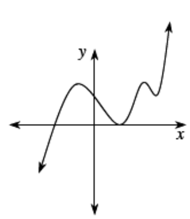
d. $j(x) = \frac{2}{3-x}$

7-142. Kirsta was working with the function machine shown at right, but when she turned her back, her little brother Caleb dropped in a number. She didn't see what he dropped, but she did see what fell out: 9. What operations must she perform on 9 to undo what her machine did? Use this to find out what Caleb dropped in.



7-143. Write a rule for a machine that will undo Kirsta's machine. Call it $c(x)$.

7-144. Consider the graph at right.



a. Is the graph a function? Explain.

b. Make a sketch of the inverse of this graph. Is the inverse a function? **Justify** your answer.

c. Must the inverse of a function be a function? Explain.

d. Describe what is characteristic about functions that do have inverse functions.

e. Could the inverse of a non-function be a function? Explain or give an example.

7-145. This problem is a checkpoint for solving rational equations. It will be referred to as Checkpoint 13.

Solve each of the following rational equations.

a. $\frac{x}{3}=\frac{4}{x}$

b. $\frac{x}{x-1}=\frac{4}{x}$

c. $\frac{1}{x}+\frac{1}{3x}=6$

d. $\frac{1}{x}+\frac{1}{x+1}=3$

e. Check your answers by referring to the Checkpoint 13 materials located at the back of your book.

If you needed help solving these equations correctly, then you need more practice solving rational equations. Review the Checkpoint 13 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to solve equations such as these easily and accurately.

7-146. The half-life of an isotope is 1000 years. A 50-gram sample of the isotope is sealed in a box.

a. How much is left after 10,000 years?

b. How long will it take to reduce to 1% of the original amount?

c. How long will it take until all of the original sample of the isotope is gone? Support your answer.

7-147. Suppose that a two-bedroom house in Nashville is worth \$110,000 and appreciates at a rate of 2.5% each year.

a. How much will it be worth in 10 years?

b. When will it be worth \$200,000?

c. In Homewood, houses are depreciating at a rate of 5% each year. If a house is worth \$182,500 now, how much will it be worth two years from now?

The Case of the Cooling Corpse:
A Mathematical Mystery

The Cast: Narrator, Agent 008, Dr. Dedman, Ajax Boraxo, Dr. Quincy, Sergeant Foust

Setting: The coroner's office inside the courthouse.

Narrator: The coroner's office is kept at a cool 17°C. Agent 008 kept pacing back and forth trying to keep warm as he waited for any new information about his latest case. For more than three hours now, Dr. Dedman had been performing an autopsy on the Sideroad Slasher's latest victim, and Agent 008 could see that the temperature of the room and the deafening silence were beginning to irritate even Dr. Dedman. The Slasher had been creating more work than Dr. Dedman cared to investigate.

008: Dr. Dedman, don't you need to take a break? You've been examining this body for hours! Even if there were any clues, you probably wouldn't see them at this point.

Dedman: I don't know. I just have this feeling something is not quite right. Somehow the Slasher slipped up with this one and left a clue. We just have to find it.

008: Well, I have to check in with headquarters. Do you mind if I step out for a couple of hours?

Dedman: No, that's fine. Maybe I'll have something by the time you return.

008 (*to himself*): Sure! Someone always wants to be the hero and solve everything himself. The doctor just does not realize how big this case really is. The Slasher has left a trail of dead bodies through five states!

Narrator: Agent 008 left, closing the door quietly. As he walked down the hall, he could hear the doctor's voice describing the victim's gruesome appearance into the tape recorder fade away.

008 *(to himself):* The hallway from the coroner's office to the elevator is long and dark, but this was the only way to Dr. Dedman's office. Doesn't this frighten most people? Well, it doesn't seem to bother old Ajax Boraxo. There he is mopping the floor.

(008 shakes his head in wonder as he passes Ajax Boraxo mopping.)

Narrator: Agent 008 stopped briefly to use the restroom and bumped into one of the deputy coroners, who asked…

Quincy: Dedman still at it?

008: Sure is, Dr. Quincy. He's totally obsessed. He's certain there is a clue.

Narrator: As usual, when leaving the courthouse, 008 had to sign out.

(008 goes to sign out, where he encounters Sergeant Foust.)

Foust: How's it going down there, Agent 008?

Narrator: Sergeant Foust asked. Foust spent most of his shifts monitoring the front door, forcing all visitors to sign in, while he recorded the time next to the signature. Agent 008 wondered if Foust longed for a more exciting aspect of law enforcement.

008 (*to himself*): If I were doing Foust's job, I would get a little stir-crazy sitting behind a desk most of the day. Why would someone become a cop to do this?

008 (*to Foust*): Dr. Dedman is convinced he will find something soon. We'll see!

008 (*to himself, looking at his watch*): It's already 10 minutes before 2:00. Will I make it back to headquarters before the chief leaves?

Foust: Well, good luck!

(*008 heads out the door.*)

Narrator: It was later in the afternoon when Agent 008 returned to the courthouse, sighing deeply.

Foust: Would the secret guest please sign in?

008 (*to himself*): Sign in again. Annoying! 5:05 pm. Wow, I hadn't planned to be gone so long.

Narrator: For a moment, Agent 008 saw a positive point to having anyone who came in or out of the courthouse sign in: He knew by quickly scanning the list that Dr. Dedman had not left. In fact, the old guy must still be working on the case.

As he approached the coroner's office, he had a strange feeling that something was wrong. He could not hear or see Dr. Dedman. When he opened the door, the sight inside stopped him in his tracks. Evidently, Dr. Dedman was now the *newest* victim of the Slasher. But wait! The other body, the one the doctor had been working on, was gone! Immediately, the security desk with its annoying sign-in sheet came to mind. Yes, there were lots of names on that list, but if he could determine the time of Dr. Dedman's death, he might be able to scan the roster to find the murderer! Quickly, he grabbed the thermometer to measure the Doctor's body temperature. He turned around and hit the security buzzer. The bells were deafening. He knew the building would be sealed off instantly and security would be there within seconds.

Foust (*rushing in*): Oh no! How could this happen? I spoke to the doctor less than an hour ago!

Narrator: As the security officers crowded into the room, Agent 008 explained what he knew, which was almost nothing. He had stopped long enough to check the doctor's body temperature.

008 (*looking at the thermometer*): 27°C. That's 10°C below normal.

Narrator: Then 008 remembered: the tape recorder! Dr. Dedman had been taping his observations; that was standard procedure. They began looking everywhere. The Slasher must have realized that the doctor had been taping and taken the tape recorder as well. Some time later…

008: The thermometer is still in Dr. Dedman's side. Exactly an hour has passed during the search and the thermometer clearly reads 24°C. Now I can determine the time of death!

7.3.1 What is a matrix?

Introduction to Matrices

A matrix is a type of table that is used to keep information organized in lists. With the rise of modern computing, matrices have become increasingly important for solving problems that involve many variables in science, economics, computer science, and mathematics. Matrices have their own notation and form, and their own unique arithmetic, which you will be learning in the next few lessons. Once you know how to represent problems using matrices and understand how they work, your calculator or computer can do the computations for you. At the end of this section, you will see how matrices can be used to solve complicated systems of equations.

7-148. Otto Toyom's toy factory has become extremely successful. His cars and trucks are selling like crazy. He now has three major stores selling his toys: Bull's-Eye Discount Outlet, JC Nickles, and Marcey's Department Store. He needs an efficient way to keep track of his profits, the number of cars ordered, and the number of parts needed to complete his orders. He begins by writing down a simple matrix (shown above) to represent the number of wheels, seats, and gas tanks needed to build his cars and trucks.

$$\overset{\text{parts}}{\text{vehicles}\begin{bmatrix} 4 & 2 & 1 \\ 6 & 1 & 3 \end{bmatrix}}$$

a. Otto has asked for your help. Copy the matrix and the labels "parts" and "vehicles" and title it the "Vehicle-by-Parts Matrix." Use the words *cars* and *trucks* to label the vehicle **rows** (rows run across), and use the words *wheels*, *seats*, and *gas tanks* to label the parts **columns** (columns run up and down).

b. This matrix is called a **2-by-3 matrix** (sometimes written "2 × 3") because it has 2 rows and 3 columns. If this matrix represents the special parts needed to build each car and truck, what do you think the matrix sum shown at right would represent, and what matrix would it equal?

$$\begin{bmatrix} 4 & 2 & 1 \\ 6 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 1 \\ 6 & 1 & 3 \end{bmatrix}$$

c. Just as letters are used to represent numbers, letters are also used to represent matrices, only for matrices capital letters are always used. Use A to represent the original vehicles-by-parts matrix. How could you represent the matrix of parts needed to build 5 cars and 5 trucks?

d. Write a matrix that represents the numbers of parts needed for five cars and five trucks.

7-149. Otto has a question for you: *"Bull's-Eye just sent in their order for this week. They want 20 cars and 25 trucks. How many special parts will I need to complete that order?"*

a. Make a 1-by-2, store-by-vehicles matrix to represent the number of cars and trucks Otto wants to build. Label this matrix *B*. Make sure you label the row *BE* for Bull's-Eye and use cars and trucks to label the columns.

b. Write matrix *A* to the right of matrix *B* and find the total number of wheels Otto needs to build the cars and trucks requested by Bull's-Eye.

c. Now use the information in the two matrices to calculate the total number of seats and the total number of gas tanks Otto needs to complete the order.

d. Record the numbers of wheels, seats, and gas tanks Otto needs to complete the order in a 1-by-3 matrix (a store-by-parts matrix). Call this matrix *C* and label the row and columns appropriately.

7-150. In the last problem, you performed a new operation called **matrix multiplication**. To find the total supplies needed, you multiplied matrices B and A together to get matrix C. Algebraically, this is written as $BA = C$.

Write the problem again as an equation that shows the product of the store matrix times the parts matrix equal to the complete order matrix, and *carefully* describe each step of what you had to take to find each of the entries in matrix C. Pay close attention to the labels on the numbers you multiply, "Number of cars times number of wheels on a car," etc.

7-151. Suppose a store requests x cars and y trucks. Show both the matrix multiplication representing the number of parts needed and the store-by-parts product matrix in terms of x and y.

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 6 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \_ & \_ & \_ \end{bmatrix}$$

This process of multiplying-then-adding that you used to get each entry in the product matrix is the fundamental operation of matrix multiplication. It will be referred to as **multiplying a row into a column**.

7-152. You have impressed Otto with your use of matrices, but he has more questions. JC Nickels Department store has called to place an order this week. They would like 15 cars and 30 trucks by the end of the week. Otto would like you to show him how to represent this information with matrices.

a. Add a row to matrix B to include the quantities requested by JC Nickles. Be sure to label rows and columns appropriately.

b. Calculate the number of each special part (wheels, seats, gas tanks) that Otto needs to build the toys for JC Nickels and write a new well-labeled matrix C that shows the number of each special part (wheels, seats, gas tanks) for each store. Write the new C matrix with all appropriate labels.

c. With the expanded versions of B and C, it can still be said that $BA = C$. Rewrite this equation using the complete matrices. This example, in which each matrix has more than one row, shows a more general case of matrix multiplication. Describe or show how each entry in matrix C is calculated from B and A.

7-153. Using your new matrix, C, what does the entry in the second row, third column mean?
Note: A shorthand notation is $c_{2,3}$.

7-154. With your team, determine which of the following are matrices and which are not. For each matrix that you find, specify the dimensions. For each that is not a matrix, explain why not.

$A = \begin{bmatrix} -6 & 12 & 0.4 \\ 3.9 & 0 & -2x \end{bmatrix}$ $B = \begin{bmatrix} 5 & 9 \\ 1 & \end{bmatrix}$ $C = \begin{bmatrix} 8 & x & 3y \end{bmatrix}$

$D = \begin{bmatrix} Fred & Barney \\ Wilma & Betty \end{bmatrix}$ $E = \begin{bmatrix} 5 & -9 \\ -\frac{3}{7} & 12 \end{bmatrix}$ $F = \begin{bmatrix} 6 & 1 & \\ & 3 & -2 \end{bmatrix}$

$G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $H = \begin{bmatrix} 2 \end{bmatrix}$ $I = \begin{matrix} 1 \\ 0 \end{matrix}$

MATH NOTES

METHODS AND MEANINGS

Matrices

A **matrix** is a rectangular array of numbers or algebraic expressions enclosed in square brackets. (Some math texts use big parentheses instead.) Usually a matrix is represented by a capital letter. The plural of matrix is **matrices**. Each matrix has **rows** and **columns**. The rows are horizontal, and columns are vertical. If matrix *M* has 2 rows and 3 columns, it is said that the **dimensions** of *M* are 2 by 3, or *M* is a 2-by-3 matrix. $m_{2,1}$ is the **entry** in the second row and first column of matrix *M*; in general, $m_{r,c}$ is the entry in the r^{th} row and c^{th} column. When you multiply a matrix by a number, as you did when you needed to find the numbers of special parts for 5 cars and 5 trucks, the number is called a **scalar**. In the expression 5*A*, *A* represents a matrix, and 5 is a scalar.

In problem 7-149 you performed a new operation, **matrix multiplication**. To find the total supplies needed, you multiplied matrices *B* and *A* together to get matrix *C*. Algebraically, it is written $BA = C$.

To find the total number of wheels needed, you had to use the row of matrix *B* that contained the number of cars and trucks requested by Bull's-Eye and the column of matrix *A* that contained the number of wheels needed per car and per truck. For this to work, the row of matrix *B* had to match (i.e., have the same number of entries as) the columns of matrix *A*.

In this example, the product matrix gives you the number of wheels, seats, and gas tanks needed to complete the Bull's-Eye order. The store-by-vehicle matrix multiplied by the vehicle-by-parts matrix gives a store-by-parts matrix, as illustrated in the example below.

store-by-vehicle matrix | vehicle-by-parts matrix | store-by-parts matrix

$$\begin{matrix} & c \quad t \\ BE & \begin{bmatrix} 20 & 25 \end{bmatrix} \end{matrix} \cdot \begin{matrix} & w \quad s \quad g \\ \begin{matrix} c \\ t \end{matrix} & \begin{bmatrix} 4 & \_ & \_ \\ 6 & \_ & \_ \end{bmatrix} \end{matrix} = \begin{matrix} & w \quad s \quad g \\ BE & \begin{bmatrix} 230 & \_ & \_ \end{bmatrix} \end{matrix}$$

Review & Preview

7-155. Let G be the matrix shown at right. $G=\begin{bmatrix}16 & 3 & -4 & 21\\ 19 & 31 & 12 & 17\\ 25 & -6 & 8 & 11\end{bmatrix}$

a. What is $g_{1,3}$?

b. Two matrices can be added only if they have the same dimensions. If matrix H can be added to matrix G, what must be the dimensions of H?

c. All the entries of the zero matrix are 0. Write the zero matrix with the same dimensions as G.

d. Create a matrix to add to matrix G to get the zero matrix. This is called the **additive inverse of a matrix**. What could you name your new matrix to show its relationship to G?

7-156. Shola's Bakery uses sugar, eggs, and butter in all of its cakes, as well as in the frosting. Matrix C shows how many eggs, cups of sugar, and ounces of butter are used in each angel food cake and in each devil's food cake. Matrix F shows how many eggs, cups of sugar, and ounces of butter are used in the frosting for each cake.

$$C=\begin{matrix} \\ af \\ df\end{matrix}\begin{matrix} e & s & b\\ \begin{bmatrix}6 & 1 & 5\\ 3 & 1.5 & 4\end{bmatrix}\end{matrix}$$

$$F=\begin{matrix} \\ af \\ df\end{matrix}\begin{matrix} e & s & b\\ \begin{bmatrix}2 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}\end{matrix}$$

a. Write the matrix $C + F$, being sure to label the rows and columns, and explain what it represents.

b. Write the matrix $3C$, with labels, and explain what it represents.

c. Leora orders three angel food cakes and two devil's food cakes without frosting, as represented by the matrix L at right. Use matrix multiplication to write a matrix that shows how much sugar, eggs, and butter Shola will need to fill Leora's order.

$$L=\begin{matrix} af & df\\ [3 & 2]\end{matrix}$$

7-157. Solve this system of equations:

$$\begin{aligned}5x-4y-6z&=-19\\ -2x+2y+z&=5\\ 3x-6y-5z&=-16\end{aligned}$$

7-158. Complete the square to change each equation below to graphing form. Find the domain and range of each relation and determine if it is a function.

a. $y=2x^2-14x+13$

b. $x=2y^2-6y-11$

7-159. Graph each system below and shade the solution region.

a. $y\ge x^2-4$
$y<-3x+1$

b. $y<2x+5$
$y\ge|x+1|$

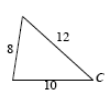
7-160. Consider $\sqrt{5-2x}+7=4$.

a. Solve the equation and check your solution.

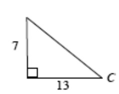
b. Did you *really* check your solution? If not, do it now. What happened?

7-161. Find $m\angle C$ in each triangle below.

a.



b.



7-162. Solve $\sqrt{3x+1}-x=-3$ and check your solution.

a. You should have gotten two values for x when you solved. Did you? If not, rework the problem.

b. Did you check *both* solutions? What happened?

7-163. Consider the function $f(x)=\sqrt{x+3}$.

a. What are the domain and range of $f(x)$?

b. If $g(x)=x-10$, what is $f(g(x))$?

c. What are the domain and range of $f(g(x))$?

d. Is $f(g(x))=g(f(x))$? **Justify** why or why not.

7-164. Find the equation of the exponential function of the form $y=ab^x$ that passes through each of the following pairs of points.

a $(1,18)$ and $(4, 3888)$

b. $(-2,-8)$ and $(3,-0.25)$

7.3.2 How can I multiply matrices?

Matrix Multiplication

In Lesson 7.3.1 you multiplied matrices together. In this lesson, you will continue to work with matrix multiplication as you learn which types of matrices can be multiplied together and which cannot.

7-165. Congratulations! Otto has offered you a full-time position with his company. He would like you to expand the previous lesson's store-by-vehicle matrix (matrix *B*) to include his third retailer, Marcey's Department Store, which has requested 12 cars and 24 trucks.

a. Show an expanded matrix *B* to include the information for Marcey's Department Store.

b. Multiply the new matrix *B* with matrix *A*. What does the product matrix represent?

c. In matrix *B*, the rows represent stores, and the columns represent vehicles. Therefore, matrix *B* is referred to as a stores-by-vehicles matrix. What would you call matrix A? What about matrix *C*? Describe the relationships of the labeling of the rows and columns in *B* and *A* to the labeling in the result *C*.

7-166. Because each store sells its toys for a different amount, Otto has instructed you to keep all of the ordering information sorted by stores. He would like you to figure out his cost to fill each store's order. Each car costs Otto $2.75 to make, and each truck costs him $3.10. Make two new matrices to represent these costs, one a 1-by-2 matrix and the other a 2-by-1 matrix.

a. Using the descriptive word labels for row-by-column, what does the 1-by-2 matrix represent?

b. What does the 2-by-1 matrix represent?

c. You need to multiply one of these matrices, call it matrix *D*, with the expanded matrix *B* from problem 7-165 to find Otto's cost to supply each store, but it is not always possible to multiply two matrices together. Which matrix would you use and in what order would you have to write the multiplication problem to make matrix multiplication possible? Discuss this with your team and be prepared to share your conclusions with the class.

d. Find this product and label the matrix appropriately. Call this matrix *E*.

e. What does the entry in the third row, first column of matrix *E* represent?

7-167. Otto wants to figure out his profit on this week's order. Because he has expanded his business, he now charges an 86% markup beyond his costs on the vehicles he sells to Bull's-Eye, 74% on the vehicles he sells to JC Nickles, and a whopping 93% on the items he sells to Marcey's.

a. Make a matrix to represent Otto's profit percentages by store. Label it appropriately.

b. Multiply this profit-by-stores matrix with matrix *E*. What does this product matrix represent?

7-168. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$.

a. Let C be the product of matrix A with matrix B (in symbols, $C = AB$). Find C.

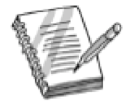
b. Now find BA. Be careful. This time use the rows of B with the columns of A.

c. Since $ab = ba$ for any numbers a and b, it is said that multiplication of numbers is *commutative*. Is multiplication of matrices commutative? How do you know?

March 03, 2011

7-169. LEARNING LOG

Create a Learning Log entry that answers the following question: What has to be true about two matrices for it to be possible to multiply them? Be sure to use examples to illustrate your ideas. Title this entry "Conditions for Matrix Multiplication" and label it with today's date.



7-170. With your team, make up a two matrices and find their product.

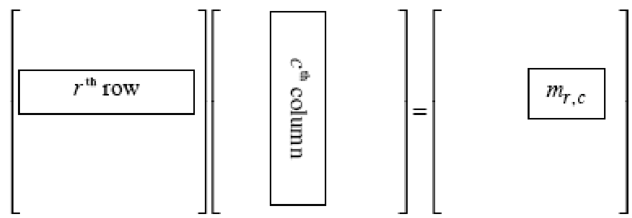
Methods and Meanings

Math Notes

More Matrix Multiplication

Matrices of any size can be multiplied as long as they fit correctly. That is, the number of columns in the first matrix must match the number of rows in the second. The entries of the product matrix can be obtained by finding sums of products of rows of the first matrix with columns of the second.

To get the $m_{r,c}$ entry of the product matrix, multiply the corresponding entries in the r^{th} row of the first matrix and the c^{th} column of the second matrix and add the products. Visually, it looks like this:



Review & Preview

7-171. Abe, Barbara, and Cassie work at Budding Success Flower Shop. Each day, they plan to make bouquets in three styles in the quantities indicated by matrix E, an employees-by-bouquets matrix, shown at right.

Bouquet Styles

$$\text{Employees}\quad \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \#1 & \#2 & \#3 \end{array} \\ \begin{bmatrix} 6 & 4 & 7 \\ 4 & 8 & 5 \\ 5 & 6 & 6 \end{bmatrix} \end{array}$$

a. How many #2 bouquets will Cassie make each day?

b. Who makes the most bouquets?

c. If all employees make their quota each day for a full workweek (Monday through Friday), write a matrix that shows how many bouquet styles each worker made.

d. Represent your answer to part (c) in terms of E.

7-172. Each #1 bouquet has 5 lilies, 4 roses, and 3 daisies. Each #2 bouquet has 4 lilies, 3 roses, and 3 daisies. Each #3 bouquet has 4 lilies, 6 roses, and 6 daisies.

a. Arrange this information in a new matrix with bouquet styles as rows. What will be the columns of this new matrix? Label the rows and columns of the matrix and call it matrix B.

b. Using the row-by-column labels, what will matrix B represent?

7-173. Which matrix product makes sense: BE or EB? Show this matrix. What does it represent?

7-174. Use your knowledge of shifting parent graphs to graph each equation below.

a. $y = -2(x-3)^2 + 4$

b. $y = \frac{1}{2}(x+2)^3 - 3$

c. $y = 2|x-5|$

d. $y = \sqrt{x-2} - 3$

7-175. Change each equation to graphing form. Sketch the graph and label each vertex and axis of symmetry.

a. $y = 2x^2 + 7x - 7$

b. $y = 3x^2 - x - 8$

7-176. Solve each of the following equations. Be sure to check your solutions.

a. $\frac{3}{x} + \frac{2}{x+1} = 5$

b. $x^2 + 6x + 9 = 2x^2 + 3x + 5$

c. $8 - \sqrt{9-2x} = x + 3$.

7-177. Solve each of the following equations.

a. $(x+4)(2x-5) = 0$

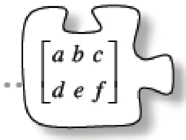
b. $(x+4)(x^2 - 5x + 6) = 0$

c. $3x(x+1)(2x-7)(3x+4)^2(x-13)(x+7) = 0$

d. Describe how to solve an equation made up of any number of factors all multiplied to equal zero.

7-178. Find the equation of the parabola that passes through the points (–2, 24), (3, 1), and (–1, 15).

7.3.3 How can I use a graphing calculator?



Matrix Multiplication with a Graphing Calculator

In today's lesson, you will learn how to use a graphing calculator to perform operations on matrices. Obtain a Lesson 7.3.3A or 7.3.3B resource page from your teacher, which contains instructions about using the calculator for matrix multiplication.

7-179. Look back to problems 7-171 and 7-172 for matrices B and W.

a. Enter W and B into your graphing calculator.

b. Use your calculator to find $W \cdot B$. Compare this to your answer for problem 7-173.

c. Find $B \cdot W$. Write down the result and explain what it means to the problem.

7-180. Let matrix $A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$. Enter A into your graphing calculator.

a. Find $A \cdot W$.

b. What does this product represent?

7-181. Suppose A is an m-by-n matrix and B is a p-by-q matrix.

a. If AB is a valid matrix product, what do you know about m, n, p, and q?

b. What will be the dimensions of AB?

7-182. Suppose you needed to multiply the matrices at right. Is it faster to multiply with or without a calculator? Have members of your team try it each way so that you can compare. What did you decide?

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

7-183. At Budding Success Flower Shop, three different bouquet styles, each consisting of lilies, roses, and daisies, are assembled as shown in matrix *B* at right.

Flowers

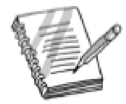
Boquet Styles	l	r	d
#1	5	4	3
#2	4	3	3
#3	4	6	6

a. Suppose lilies cost \$0.30 each, roses cost \$0.45 and daisies cost \$0.60. Write this information as a matrix in such a way that it makes sense to be multiplied by matrix *B* on the left. (That is, $B \cdot$(your matrix) must make sense.) Title and label it as usual.

b. Do the matrix multiplication $B \cdot$(your matrix from part (a)) and explain the meaning of the resulting matrix.

7-184. LEARNING LOG

In your Learning Log, write directions and clear examples for how to multiply a matrix by a scalar, how to add two matrices, and how to multiply two matrices. Title this entry "Matrix Operations" and label it with today's date.



Review & Preview

7-185. **Perform the matrix multiplication at right without using a calculator.**

$$\begin{bmatrix} 2 & 3 & 7 \\ 5 & 1 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \\ 1 \end{bmatrix}$$

7-186. **In the previous problem, the first matrix is a 2-by-3 matrix, and the second is a 3-by-1 matrix. What are the dimensions of the product matrix?**

7-187. **Perform each of the following matrix operations without using a calculator. If the operation is impossible, explain why.**

a. $\begin{bmatrix} 4 & 9 & 2 \\ 6 & 0 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

b. $2\begin{bmatrix} 4 & 9 & 2 \\ 6 & 0 & -5 \end{bmatrix}+\begin{bmatrix} 1 & 0 & -3 \\ 0 & 4 & 1 \end{bmatrix}$

c. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \end{bmatrix}$

d. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}+\begin{bmatrix} e & f \end{bmatrix}$

7-188. **Suppose you want to multiply a 5-by-11 matrix by a *c*-by-*d* matrix.**

a. **For what value of *c* will the multiplication be defined?**

b. **Given this value of *c*, what will be the dimensions of the product matrix?**

7-189. Given that $\log 2 \approx 0.3010$, $\log 3 \approx 0.4771$ and $\log 5 \approx 0.6990$, calculate each of the following logarithms without using a calculator.

a. $\log 6$

b. $\log 15$

c. $\log 9$

d. $\log 50$

7-190. Several times throughout this course, you have solved equations involving radicals (square-root signs). You have found that it is important to check the solutions to these equations because sometimes solutions are **extraneous** (that is, they don't work when you substitute them back into the equation to check). Now you will see what else can happen with this type of equation. Solve $\sqrt{x}-\sqrt{5}=\sqrt{x-5}$.

a. What happened when you squared both sides? Did you remember to use the distributive property correctly when you squared the left side? Did squaring both sides eliminate the radicals?

b. After squaring both sides, the equation should look like previous equations involving radicals. Simplify, square again, and solve. Check your solution.

7-191. Show how the Zero Product Property can be used to solve each of the following equations.

a. $x(2x-1)(x-3)=0$

b. $2x^3+x^2-3x=0$

7-192. Graph each equation below.

a. $x^2+(y-3)^2=9$

b. $(x-5)^2+(y-1)^2=4$

7-193. Simplify each of the following expressions.

a. $\frac{2x^3+5x^2-3x}{4x^3-4x^2+x}$

b. $\frac{3x^2-5x-2}{2x^2-11x+15}\cdot\frac{2x^2-5x}{3x^3-5x^2-2x}$

MATRIX FUNCTION RESOURCE PAGE
(for use with a TI graphing calculator)

TO ENTER DATA INTO A MATRIX:

To set the size of the matrix [A]:

Go to MATRX (2nd Function of the x^{-1} key)
--> --> EDIT (Right arrow twice to select "EDIT")
ENTER

Set the correct number of rows and columns and then enter the appropriate data.

To enter data into matrices [B], [C], etc., choose **MATRIX --> --> EDIT** and then select the matrix you wish to edit before **ENTER**. Set the number of rows and columns and enter the data.

TO PERFORM COMPUTATIONS WITH MATRICES:
(assuming the sizes are compatible)

2nd QUIT to return to home screen for calculations.

Matrix arithmetic is done just like arithmetic with numbers, except the matrix menu must be used to print the matrix names to the home screen. For example:

To add [A] + [B]: **MATRIX [A] ENTER + MATRIX [B] ENTER**
([A] + [B] will appear on the screen) **ENTER**

To save the answer to matrix [C]: **ST0> MATRIX [C] ENTER ENTER**

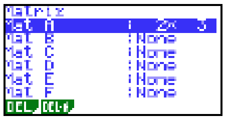
TO FIND THE (MULTIPLICATIVE) INVERSE OF A MATRIX:
(it must be a square matrix)

To find $[A]^{-1}$: **MATRIX [A] ENTER x^{-1} ENTER**
($[A]^{-1}$ will appear on the screen) **ENTER**

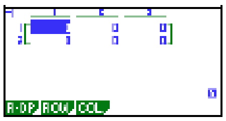
If you want the inverse matrix elements to appear as fractions:
MATH Frac ENTER ENTER

MATRIX FUNCTION RESOURCE PAGE
(for use with a Casio 9850G or 9850G+ graphing calculator)

ENTERING DATA INTO A MATRIX:
(Your calculator must be in MAT MODE.)



Step 1. You must first set the size of your matrix. Suppose you want to work with a matrix A: Highlight Mat A, then specify its dimensions: # of rows, [EXE], # of columns, [EXE].



Step 2. Your calculator should now be prompting you to begin entering your data.

Repeat steps 1 and 2 for each of your remaining matrices.

PERFORMING COMPUTATIONS WITH MATRICES:
(Your calculator must be in RUN MODE, and the sizes of your matrices must be compatible.)

∑ For example, to add matrix A + matrix B:

Press [OPTN], [F2](MAT) to bring up the matrix menu choices. Then press [F1](Mat), [ALPHA], [X,θ,T] (A), [+], [F1](Mat), [ALPHA] , [X,θ,T] (B). Your screen should look as shown at right.

Now, press [EXE] and your sum should appear.

Matrix subtraction, matrix multiplication and multiplication by a scalar can be completed by making simple adjustments to the series of keystrokes given above.

∑ Sometimes you will need to use the **inverse of a matrix**. To tell your calculator you want to use the inverse of matrix A, for example, you must do the following:

Press [OPTN], [F2](MAT) to bring up the matrix menu choices, if it isn't already in your window. Then press [F1](Mat), [ALPHA], [X,θ,T] (A) to tell your calculator you want matrix A. And finally press [SHIFT], [ )] (x^{-1}). Your screen should look as shown at right:

7·3·4 How can I use matrices?

Writing Systems as Matrix Equations

In the past, you have solved systems of three equations with three variables. As you noticed, the algebraic manipulations can get complicated. Many applications in advanced mathematics and technology (such as computer drawing programs or switching and circuitry problems in telephone communications) can lead to the need to solve systems of one hundred equations with one hundred unknowns. That would take a lot of time and a lot of paper! In the next two lessons you will learn how matrices, along with calculators, can make solving these systems much simpler.

7-194. Consider matrices A and X at right.

$$A = \begin{bmatrix} 9 & -3 & 1 \\ 1 & 1 & 1 \\ 16 & 4 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

a. Find the product AX.

b. Explain how this product could be related to asystem of equations.

7-195. Write the system at right as a matrix equation.

$$\begin{aligned} 2x + 7y + 5z &= -12 \\ 3x + y + 4z &= -22 \\ 6x + \quad\; 9z &= -57 \end{aligned}$$

This matrix equation can be abbreviated as $AX = B$, where A is the 3-by-3 matrix of coefficients, X is the single-column matrix of variables, and B is the single-column matrix of constants.

7-196. Discuss with your team what you need to be able to do to solve the matrix equation $AX = B$ in problem 7-195.

7-197. In the equation $AX = B$, AX represents a matrix multiplication problem. To solve this problem, you need to answer some questions. Is there a way to undo *AX* to find *X*? Is there an A^{-1}?

Remember that when you first learned to solve an equation such as $\frac{2}{3}x = 14$, you used the reciprocal or multiplicative inverse of $\frac{2}{3}$ to write:

$$(\tfrac{3}{2})\tfrac{2}{3}x = 14(\tfrac{3}{2})$$

Because $\frac{3}{2}(\frac{2}{3}) = 1$, which is the identity element for multiplication, the equation is solved and $x = 21$.

To solve $AX = B$, there are two questions that have to be answered. First, what is the identity element? Second, what is the matrix multiplication inverse (A^{-1}) for matrix *A*? Parts (a) through (d) below address the first question.

a. Is the multiplicative identity for 2-by-2 matrices just a 2-by-2 matrix full of ones? That is, is the matrix product at right correct? $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

b. Find a **multiplicative-identity** matrix for 2-by-2 matrices. Try out some possibilities, and work with your team to find a matrix that you can multiply by any 2-by-2 matrix (on the left or on the right) without changing its value. When you have found the identity matrix, name it *I*. Be prepared to **justify** your conclusions.

c. Based on your previous results, guess the multiplicative identity for 3-by-3 matrices.

d. In the matrix equations below, replace the blank matrix with your guess. Then check whether the matrix equations are true.

$$\begin{bmatrix} 2 & 7 & 5 \\ 3 & 1 & 4 \\ 6 & 0 & 9 \end{bmatrix}\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} = \begin{bmatrix} 2 & 7 & 5 \\ 3 & 1 & 4 \\ 6 & 0 & 9 \end{bmatrix} \qquad \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}\begin{bmatrix} 2 & 7 & 5 \\ 3 & 1 & 4 \\ 6 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 5 \\ 3 & 1 & 4 \\ 6 & 0 & 9 \end{bmatrix}$$

7-198. Rewrite the system of equations at right into a matrix equation of the form $AX = B$. With your team, discuss what you could do to solve this equation using matrices. In other words, how could you get X by itself?

$$\begin{aligned} 9x - 3y + z &= -7 \\ x + y + z &= -3 \\ 16x + 4y + z &= 21 \end{aligned}$$

7-199. What matrix could you multiply by $M = \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$ to get a result of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$?

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Review & Preview

7-200. Let A and B be the matrices defined at right. Complete each of the following matrix-multiplication problems.

$$A = \begin{bmatrix} 4 & -1 \\ 7 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -5 & 0 \\ 3 & -8 \end{bmatrix}$$

a. $A - B$ b. $5A$

c. AB d. BA

7-201. What matrix is the **additive identity** for 2-by-2 matrices?

7-202. Represent each system of equations below with matrices. You do not have to solve the systems.

a. $\begin{aligned} p + 2q &= 7 \\ 3p + 4q &= 11 \end{aligned}$

b. $\begin{aligned} 4m - 5n &= -2 \\ -3m + 4n &= 9 \end{aligned}$

c. $\begin{aligned} 4x - y + z &= -5 \\ 2x + 2y + 3z &= 10 \\ 5x - 2y + 6z &= 1 \end{aligned}$

d. $\begin{aligned} 7w - 3x + 2z &= 41 \\ -2w + x - z &= -13 \\ 4w + y - 2z &= 12 \\ 2 - 3x &= 1 \end{aligned}$

7-203. Juan, Huang, and Danusha have pencils and pens in their backpacks. Juan has 3 pencils and 2 pens, Huang has 4 pencils and 5 pens, and Danusha has 6 pencils and 4 pens.

a. Represent this information in a 3-by-2 matrix. Label rows and columns.

b. Each pencil is worth 10 cents, and each pen is worth 25 cents. Represent this information in a matrix in such a way that you can multiply the first matrix by this one. Label rows and columns.

c. Find the product matrix and interpret the 2, 1 entry.

7-204. Suppose the entries in a 3-by-2 matrix are *a, b, c, d, e, f* (reading across the rows), and the entries in a second 2-by-1 matrix are *g, h*. Write the product matrix in terms of these letters.

7-205. If x is a number, represent its reciprocal using exponents (not fractions).

7-206. Add or subtract and simplify each of the following expressions. **Justify** that each step of your process makes sense.

a. $\frac{3}{(x-4)(x+1)} + \frac{6}{x+1}$

b. $\frac{x+2}{x^2-9} - \frac{1}{x+3}$

7-207. Solve $\sqrt{2x+7} + \sqrt{x+48} = 10$ and check your solution. If your solution is incorrect, refer to parts (a) and (b) below.

a. Did you remember you had to square twice?

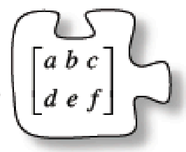
b. To solve this problem, you will have to square both sides twice, but you can make this problem a little easier first. Having one radical on each side of the equation simplifies the algebra slightly. If you did not rewrite the equation this way when you solved, try it now and solve the equation. Did you get the same solution? You should!

7-208. Solve each of the following equations. Be sure to check your answers.

a. $3|2x-5| - 8 = -5$

b. $\sqrt{3x^2 + 11x} = 2$

7.3.5 How can I use matrices?



Using Matrices to Solve Systems of Equations

In Lesson 7.3.4 you learned that a system of equations can be represented by a single *matrix equation* of the form $AX = B$. If you can find the numbers for the matrix X that make this equation true, you will have found the values of x, y, and z that solve the original system of equations. Solving matrix equations is very similar to solving other equations: a goal is to get X by itself. One complication is that there is no way to divide matrices, so you will need to learn how to use the **multiplicative inverse of a matrix**.

7-209. Think about what you have learned about additive and multiplicative inverses of numbers.

a. What should you get when you multiply a number or a matrix by its multiplicative inverse?

b. Why would the multiplicative inverse of a matrix be useful in solving the matrix equation $AX = B$?

c. How can you determine whether two matrices are multiplicative inverses? Decided whether the matrices at right are multiplicative inverses and **justify** your decision.

$$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

7-210. The process of finding the inverse of a matrix A (denoted A^{-1}) is very complicated. Fortunately, your graphing calculator can do it with ease! If you are using a TI or Casio graphing calculator, the Lesson 7.3.3A or 7.3.3B Resource Page will show you how. Otherwise, ask your teacher for directions. Use the graphing calculator to find the inverse of matrix A from problem 7-195.

7-211. To check the inverse, compute $[A]^{-1}[A]$. Do you see any strange entries? If so, talk with your team to figure out what they should be.

7-212. Now you are ready to solve the system of equations in problem 7-195 by solving the equivalent matrix equation $AX = B$.

a. With your team, use what you know about inverses and your calculator to solve the system of equations.

b. In general, if $AX = B$ is a matrix equation, then what does X equal?

c. In general, would $X = BA^{-1}$ give the solution to the matrix equation $AX = B$? Explain why or why not.

7-213. Write the linear system at right as a matrix equation of the form $AX = B$. Then enter matrices A and B in your graphing calculator and use the method of the previous problem to solve the system.

$$
\begin{aligned}
4x + 4y - 5z &= -2 \\
2x - 4y + 10z &= 6 \\
x + 2y + 5z &= 0
\end{aligned}
$$

7-214. Professor Zipthrough wants to prove that the method of problem 7-212 always works to solve matrix equations of the form $AX = B$. He has written the steps on the board, but has not **justified** them. Help Professor Z **justify** each step below.

Given: $AX = B$

Step 1: $A^{-1}(AX) = A^{-1}B$

Step 2: $(A^{-1}A)X = A^{-1}B$

Step 3: $IX = A^{-1}B$

Step 4: $X = A^{-1}B$

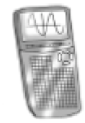
7-215. Use the matrix method to solve the system of equations shown at right.

$$-4x + 7y - 12z = -3.8$$
$$5x - 8y = -14.8$$
$$x - 4y + 9z = 7.6$$

7-216. The cubic function $g(x) = px^3 + qx^2 + rx + s$ passes through the points (2, –22), (1, 2), (2, –2), and (5, 118).

a. Set up four equations using p, q, r, and s as variables.

b. Write these four equations as a single matrix equation.



c. Use your graphing calculator to solve this matrix equation and write the equation for $g(x)$.

7-217. How many points would you need to know to be able to set up a system of equations and the corresponding matrix equation to find the exact equation of a graph of the form $g(x) = a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$?

a. Describe the matrix equation you would need to set up.

b. How many points would you need to know to be able to set up a system of equations and the corresponding matrix equation to find $g(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_0$?

METHODS AND MEANINGS

MATH NOTES

Inverse of a Matrix

If A is a matrix, then its **multiplicative inverse** is denoted A^{-1}. Note that A^{-1} does *not* mean $\frac{1}{A}$. (A number cannot be divided by a matrix.) Instead, A^{-1} is the matrix with the property that $A^{-1}(A) = I$.

The identity matrix for 3-by-3 matrices is shown at right. Only square matrices can have inverses, and A^{-1} must have the same dimensions as A.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In general, $AB \neq BA$ for matrices A and B. As a result, when referring to matrices, the word "multiply" alone is unclear; the terms **right-multiply** and **left-multiply** are used instead. To describe the product BA, one would say, "right-multiply B by A," or "left-multiply A by B."

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Review & Preview

7-218. Show that each of the following pairs of matrices are inverses.

a. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

b. $\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

7-219. Use matrices to solve these systems. Check your answers by substituting them in the original equations. You will not need a calculator if you use the results of the previous problem.

a. $p + 2q = 7$
$3p + 4q = 11$

b. $4m - 5n = -2$
$-3m + 4n = 9$

7-220. P is the product of two matrices, as shown at right. Show each result described below or explain why it cannot be shown.

$$P = \begin{bmatrix} 2 & -1 & 5 \\ -4 & 0 & 7 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 1 & -6 \\ 0 & 1 \end{bmatrix}$$

a. P b. $P_{1,2}$ c. $P_{2,3}$

7-221. If M is as shown below, find a matrix I such that $MI = IM = M$, or write "impossible" and explain why.

a. $M = \begin{bmatrix} 2 & -1 & 5 \\ -4 & 0 & 7 \end{bmatrix}$

b. $M = \begin{bmatrix} 2 & -1 \\ -4 & 0 \end{bmatrix}$

c. $M = \begin{bmatrix} 2 & -1 & 5 \\ -4 & 0 & 7 \\ 6 & -2 & 8 \end{bmatrix}$

7-222. Given that $\log_x 2 = a$, $\log_x 5 = b$, and $\log_x 7 = c$, write expressions using a, b, and/or c for each log expression below.

a. $\log_x 10$ b. $\log_x 49$ c. $\log_x 50$ d. $\log_x 56$

7-223. Solve each of the following equations.

a. $5^x = 72$ b. $2^{3x} = 7$ c. $3^{(2x+4)} = 17$

7-224. Solve $\sqrt{21x-3} - \sqrt{5x+1} = 6$ and check your solution. Remember: Square twice!

7-225. Add or subtract and simplify each of the following expressions. **Justify** that each step of your process makes sense.

a. $\frac{3x}{x^2+2x+1} + \frac{3}{x^2+2x+1}$

b. $\frac{3}{x-1} - \frac{2}{x-2}$

7-226. Graph each of the following systems of inequalities.

a. $y \ge 3(x-2)^2 - 4$
$y > -2|x-1| + 3$

b. $y > (x-1)^2 - 5$
$y > 3x - 5$
$y \le \frac{1}{2}(x-1)^2 + 1$

CL 7-227. Graph in three dimensions.

a. $(2, 3, 1)$

b. $(-2, 3, 0)$

c. $2x + y - z = 6$

CL 7-228. Determine the point of intersection of the three planes.

a.
$$\begin{aligned} x + y + z &= 3 \\ 2x - y + 2z &= 6 \\ 3x + 2y - z &= 13 \end{aligned}$$

b.
$$\begin{aligned} x + y + 4z &= 5 \\ -2x \quad + 2z &= 3 \\ 3x + y - 2z &= 0 \end{aligned}$$

CL 7-229. The parabola $y = ax^2 + bx + c$ passes through the points (2, 3), (–1, 6), and (0, 3). Determine:

a. The equation of the parabola.

b. The vertex of the parabola.

c. The x-intercepts of the parabola.

CL 7-230. Solve each equation to the nearest thousandth (0.001).

a. $2^x = 17$

b. $5x^3 = 75$

c. $5(3^{x+1}) = 85$

d. $\log_3(x+1) = -2$

CL 7-231. Compute the value of each expression. No calculator is necessary.

a. $\log(8) + \log(125)$

b. $\log_{25}(125)$

c. $\frac{1}{2}\log(25) + \log(20)$

d. $7^{\log_7(12)}$

CL 7-232. An exponential function $y = km^x + b$ passes through (3, 7.5) and (4, 6.25). It also has an asymptote at $y = 5$.

a. Find the equation of the function.

b. If the equation also passes through (8, w), what is the value of w?

CL 7-233. A gallon of propane costs \$3.59. Inflation has steadily increased 4% per year.

a. What did a gallon of propane cost ten years ago?

b. If this trend continues, how much longer will it be until it costs \$10?

CL 7-234. Find the inverse of this equation: $y = 2 + \sqrt{2x - 4}$.

CL 7-235. Factor the following expressions completely.

a. $y^2 + 2y - 35$

b. $2x^2 - x - 21$

c. $m(3m+2)(m-1)$

d. $8ab^2 - 28ab + 12a$

CL 7-236. Given the matrices defined below, compute the following operations, if possible.

$$A = \begin{bmatrix} 3 & -1 & 6 \\ 8 & 9 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 6 & -6 \\ 1 & 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

a. $A + B$

b. $B - A$

c. BC

d. CB

CL 7-237. Given the system of equations at right:

$$\begin{aligned} x + y + z &= 3 \\ 2x - y + 2z &= 6 \\ 3x + 2y - z &= 13 \end{aligned}$$

a. Write the equivalent matrix equation.

b. Write the equivalent matrix solution. (Do not actually solve.)

CL 7-238. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in previous math classes? Sort the problems into three groups: the ones you are confident you can do, the ones you need more practice with, and the ones you need further help to understand.

Chapter 7 Closure Resource Page: Multiple Representations of Logarithmic Functions GO

Create multiple representations of a logarithmic function. Write and **justify** summary statements using all of the representations. Use colors, arrows, and other tools to show connections between the representations.

Situation

Graph

Summary statements:

Table

Equation