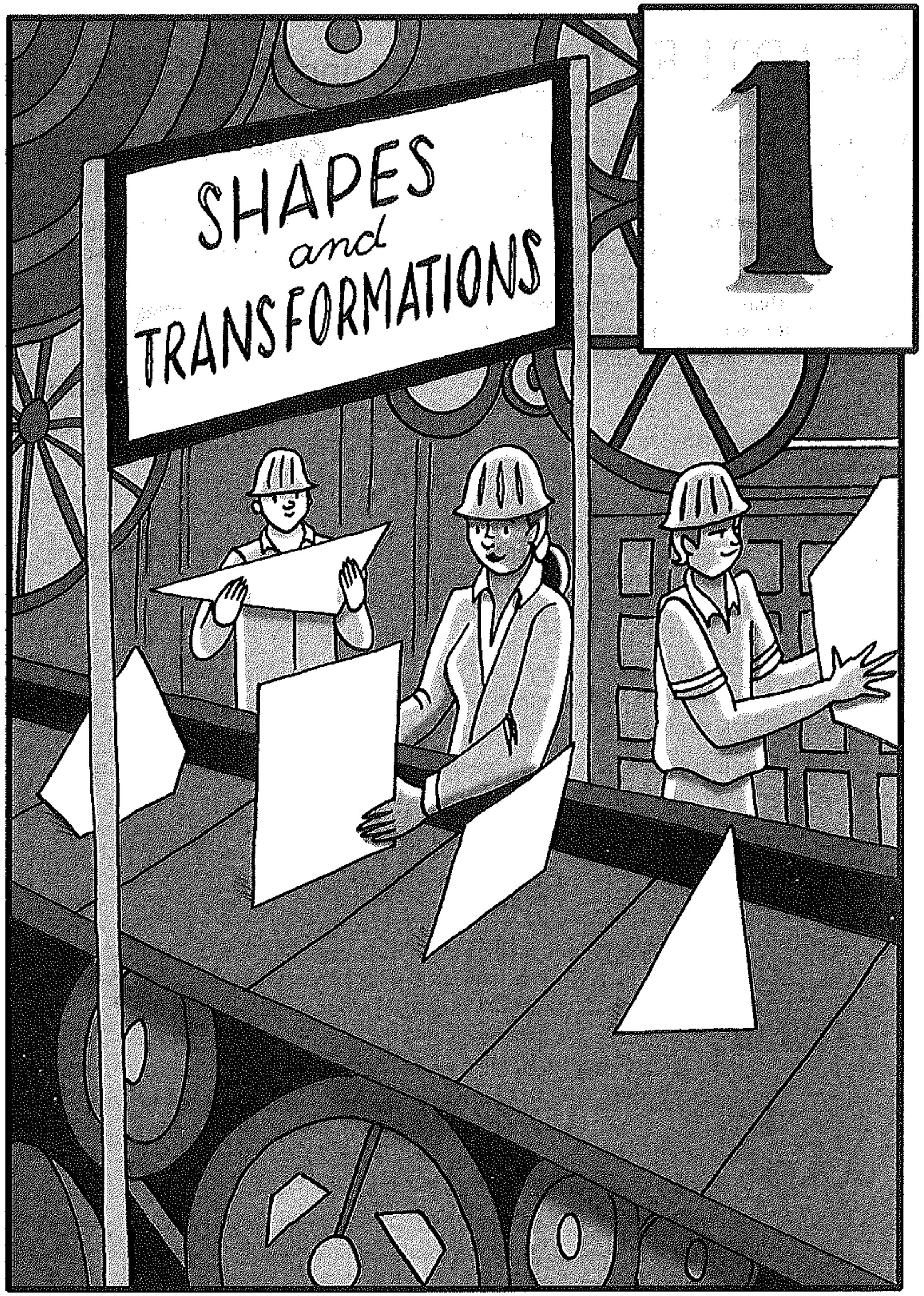

SHAPES
and
TRANSFORMATIONS
1

CHAPTER 1 Shapes and Transformations

Welcome to Geometry! *Geo* means Earth (*geo*graphy is mapping the Earth, for example) and *metry* means measurement. Geometry applies the arithmetic, algebra and **reasoning** skills you have learned to the objects you see all around you. During this course, you will ask and answer questions such as "How can I describe this shape?", "How can I measure this shape?", "Is this shape symmetrical?", and "How can I convince others that what I think about this shape is true?"

This chapter begins with some activities that will introduce you to the big ideas of the course. Then you will apply motions to triangles and learn how to specify a particular motion. Finally, you will explore attributes of shapes that can be used to categorize and name them and find the probabilities of selecting shapes with certain properties from a "shape bucket."

Guiding Questions

Think about these questions throughout this chapter:

What is geometry?

How can I describe this shape?

How can I move it?

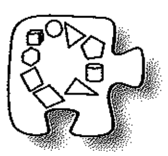
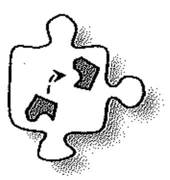
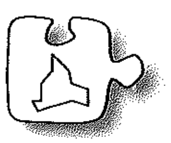
Is this shape symmetrical?

How can I communicate my ideas clearly?

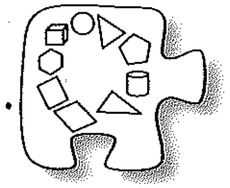
In this chapter, you will:

- become familiar with basic geometric shapes and learn how to describe each one using its attributes, such as parallel sides or rotation symmetry.
- investigate three basic rigid transformations: reflection (flip), rotation (turn), and translation (slide).
- be introduced to probability and learn how to use probability to make predictions.

Chapter Outline

	**Section 1.1**	Investigations involving quilts, twisted strips of paper, rug designs, precise reasoning, and kaleidoscopes will introduce you to some basic building-blocks of geometry: shapes, motions, measurements, patterns, reasoning and symmetry.
	**Section 1.2**	You will learn about transformations as you study how to flip, turn, and slide shapes. Then you will learn how to use these motions to build new shapes and to describe symmetry.
	**Section. 1.3**	A "shape bucket" will introduce you to a variety of basic shapes that you will describe, classify and name according to their attributes. You will also learn about probability.

1.1.1 How can I design it?



Creating a Quilt Using Symmetry

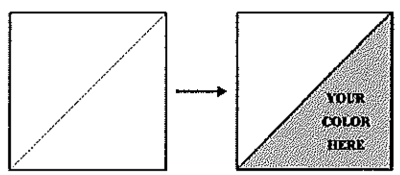
Welcome to Geometry! But what is geometry? At the end of this chapter you will have a better understanding of what geometry is. To start, you will focus on several activities that will hopefully challenge you and introduce you to important concepts in geometry that you will study in this course. While all of the problems are solvable with your current math skills, some will be revisited later in the course so that you may apply new geometric tools to solve and extend them.

Today you will consider an example of how geometry is applied in the world around you. A very popular American tradition is to create quilts by sewing together remnants of cloth in intricate geometric designs. These quilts often integrate geometric shapes in repeated patterns that show symmetry. For centuries, quilts have been designed to tell stories, document special occasions, or decorate homes.

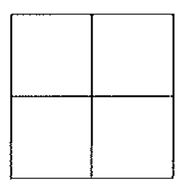
1-1. DESIGNING A QUILT, Part One

How can you use symmetry to design a quilt? Today you will work with your team to design a patch that will be combined with other team patches to make a class quilt. Before you start, review the team roles, which are outlined on the next page.

a. Each team member will receive four small squares. With a colored pencil or marker, shade in <u>half</u> of each square (one triangle) as shown at right. Each team member should use a different color.



b. Next arrange your squares to make a larger 2-by-2 square (as shown at right) with a design that has **reflection symmetry**. A design has reflection symmetry if it can be folded in half so that both sides match perfectly. Make sure that you have arranged your pieces into a different symmetrical pattern than the rest of your team.



c. Now, create a 4-by-4 square using the designs created by each team member as shown on the Lesson 1.1.1B Resource Page. Ask your teacher to verify that your designs are all symmetrical and unique. Then glue (or tape) all sixteen pieces carefully to the resource page and cut along the surrounding dashed square so that you have a blank border around your 4-by-4 square.

Problem continues on next page →

1-1. *Problem continued from previous page.*

d. Finally, discuss with your team what you all personally have in common. Come up with a team sentence that captures the most interesting facts. Write your names and this sentence in the border so they wrap around your 4-by-4 design.

To help you work together today, each member of your team has a specific job, assigned by your first name (or last name if team members have the same first name).

Team Roles

Resource Manager – If your name comes first alphabetically:

- Make sure the team has all of the necessary materials, such as colored pencils or markers and the Lesson 1.1.1A and 1.1.B Resource Pages.
- Ask the teacher when the **entire** team has a question. You might ask, *"No one has an idea? Should I ask the teacher?"*
- Make sure your team cleans up by delegating tasks. You could say, *"I will put away the ________ while you _________ ."*

Facilitator – If your name comes second alphabetically:

- Start the team's discussion by asking, *"What are some possible designs?"* or *"How can we make sure that all of our designs are symmetrical?"* or *"Are all of our designs different?"*
- Make sure that all of the team members get any necessary help. You don't have to answer all the questions yourself. A good facilitator regularly asks, *"Do you understand what you are supposed to do?"* and *"Who can answer ________'s question?"*

Recorder/Reporter – If your name comes third alphabetically:

- Coordinate the taping or gluing of the quilt pieces together onto the resource page in the orientation everyone agreed to.
- Take notes for the team. The notes should include phrases like, *"We found that we all had in common …"* and explanations like, *"Each of our designs was found to be unique and symmetrical because …"*
- Help the team agree on a team sentence: *"What do we all have in common?"* and *"How can I write that on our quilt?"*

Task Manager – If your name comes fourth alphabetically:

- Remind the team to stay on task and not to talk to students in other teams. You can suggest, *"Let's try coming up with different symmetrical patterns."*
- Keep track of time. Give your team reminders, such as *"I think we need to decide now so that we will have enough time to …"*

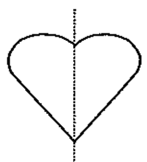
1-2. DESIGNING A QUILT, Part Two

Your teacher will ask the Recorder/Reporters from each team to bring their finished quilt patches up to the board one at a time and tape them to the other patches. Be prepared to explain how you came up with your unique designs and interesting ideas about symmetry. Also be prepared to read your team sentence to the class. As you listen to the presentations, look for relationships between your designs and the other team designs.

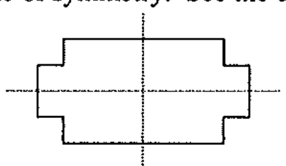
MATH NOTES — METHODS AND MEANINGS

Lines of Symmetry

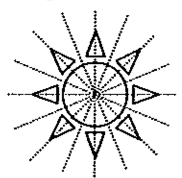
When a graph or picture can be folded so that both sides will perfectly match, it is said to have **reflection symmetry**. The line where the fold would be is called the **line of symmetry**. Some shapes have more than one line of symmetry. See the examples below.



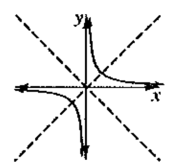
This shape has one line of symmetry



This shape has two lines of symmetry



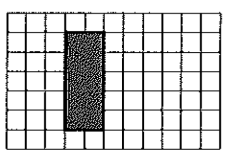
This shape has eight lines of symmetry

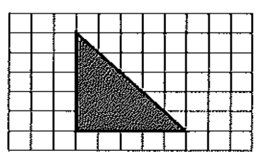


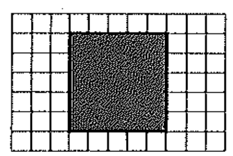
This graph has two lines of symmetry

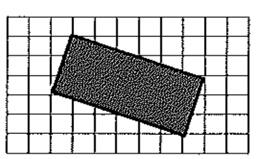
Review & Preview

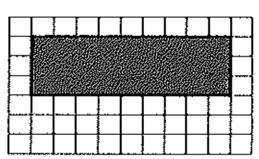
1-3. One focus of this Geometry course is to help you recognize and accurately identify a shape. For example, a **rectangle** is a four-sided shape with four right angles. Which of the shapes below can be called a rectangle? More than one answer is possible.

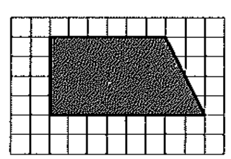
a. 

b. 

c. 

d. 

e. 

f. 

1-4. Calculate the values of the expressions below. Show all steps in your process. The answers are provided for you to check your result. If you miss two or more of these and cannot find your errors, be sure to seek help from your team or teacher.
[Answers: a: 40, b: –6, c: 7, d: 59]

a. $2 \cdot (3(5+2)-1)$

b. $6-2(4+5)+6$

c. $3 \cdot 8 \div 2^2+1$

d. $5-2 \cdot 3+6(3^2+1)$

1-5. Match each table of data on the left with its rule on the right and **briefly explain** why it matches the data.

a.

x	1	0	–4	2	–2	–1
y	4	3	–1	5	1	2

b.

x	–1	3	1	0	–2	2
y	–1	–9	–1	0	–4	–4

c.

x	3	–2	1	0	2	–3
y	12	7	4	3	7	12

d.

x	–3	4	2	–2	0	–10
y	–10	11	5	–7	–1	–31

(1) $y=x$

(2) $y=3x-1$

(3) $y=x+3$

(4) $y=x^2$

(5) $y=-x^2$

(6) $y=x^2+3$

1-6. Simplify the expressions below as much as possible.

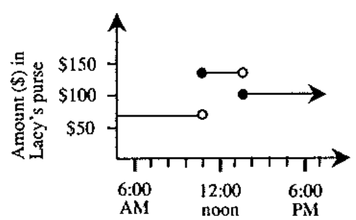
a. $2a+4(7+5a)$

b. $4(3x+2)-5(7x+5)$

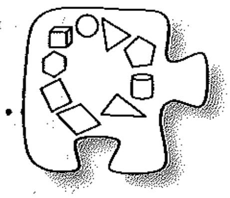
c. $x(x+5)$

d. $2x+x(x+6)$

1-7. **Examine** the graph at right. Then, in a sentence or two, suggest reasons why the graph rises at 11:00 AM and then drops at 1:15 PM.

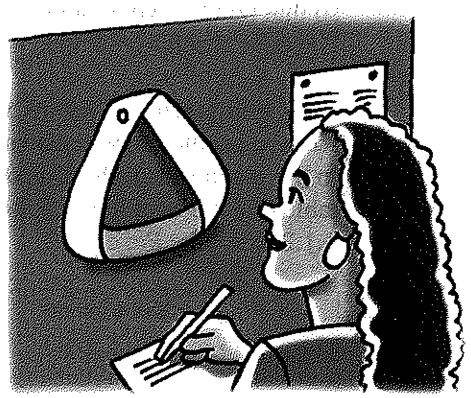


1.1.2 Can you predict the results?



Making Predictions and Investigating Results

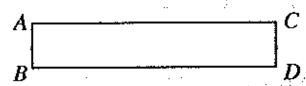
Today you will **investigate** what happens when you change the attributes of a Möbius strip. As you **investigate**, you will record data in a table. You will then analyze this data and use your results to brainstorm further experiments. As you look back at your data, you may start to consider other related questions that can help you understand a pattern and learn more about what is happening. This Way of Thinking, called **investigating**, includes not only generating new questions, but also rethinking when the results are not what you expected.



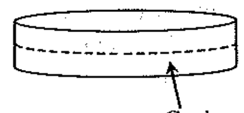
1-8. Working effectively with your study team will be an important part of the learning process throughout this course. Choose a member of your team to read aloud these Study Team Expectations:

> **STUDY TEAM EXPECTATIONS**
>
> Throughout this course you will regularly work with a team of students. This collaboration will allow you to develop new ways of thinking about mathematics, increase your ability to communicate with others about math, and help you strengthen your understanding by having you explain your thinking to someone else. As you work together,
>
> - You are expected to share your ideas and contribute to the team's work.
> - You are expected to ask your teammates questions and to offer help to your teammates. Questions can move your team's thinking forward and help others to understand ideas more clearly.
> - Remember that a team that functions well works on the same problem together and discusses the problem while it works.
> - Remember that one student on the team should not dominate the discussion and thinking process.
> - Your team should regularly stop and verify that everyone on the team agrees with a suggestion or a solution.
> - Everyone on your team should be consulted before calling on the teacher to answer a question.

1-9. On a piece of paper provided by your teacher, make a "bracelet" by taping the two ends securely together. Putting tape on both sides of the bracelet will help to make sure the bracelet is secure. In the diagram of the rectangular strip shown at right, you would tape the ends together so that point A would attach to point C, and point B would attach to point D.



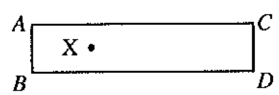
Now predict what you think would happen if you were to cut the bracelet down the middle, as shown in the diagram at right. Record your prediction in a table like the one shown below or on your Lesson 1.1.2 Resource Page.



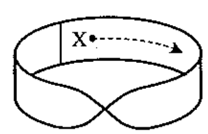
	Experiment	Prediction	Result
1-9	Cut bracelet in half as shown in the diagram.		
1-10			
1-11			
1-12a			
1-12b			
1-12c			
1-12d			

Now cut your strip as described above and record your result in the first row of your table. Make sure to include a short description of your result.

1-10. On a second strip of paper, label a point X in the center of the strip at least one inch away from one end.



Now turn this strip into a **Möbius strip** by attaching the ends together securely after making one twist. For the strip shown in the diagram at right, the paper would be twisted once so that point A would attach to point D. The result should look like the diagram at right.



A Möbius Strip

Predict what would happen if you were to draw a line down the center of the strip from point X until you ran out of paper. Record your prediction, conduct the experiment, and record your result.

1-11. What do you think would happen if you were to cut your Möbius strip along the central line you drew in problem 1-10? Record your prediction in your table.

Cut just one of your team's Möbius strips. Record your result in your table. Consider the original strip of paper drawn in problem 1-9 to help you explain why cutting the Möbius strip had this result.

1-12. What else can you learn about Möbius strips? For each experiment below, first record your expectation. Then record your result in your table after conducting the experiment. Use a new Möbius strip for each experiment.

a. What if the result from problem 1-11 is cut in half down the middle again?

b. What would happen if the Möbius strip is cut one-third of the way from one of the sides of the strip? Be sure to cut a constant distance from the side of the strip.

c. What if a strip is formed by 2 twists instead of one? What would happen if it were cut down the middle?

d. If time allows, make up your own experiment. You might change how many twists you make, where you make your cuts, etc. Try to generalize your findings as you conduct your experiment. Be prepared to share your results with the class.

1-13. LEARNING REFLECTION

Think over how you and your study team worked today, and what you learned about Möbius strips. What questions did you or your teammates ask that helped move the team forward? What questions do you still have about Möbius strips? What would you like to know more about?

MATH NOTES

METHODS AND MEANINGS

The Investigative Process

The **investigative process** is a way to study and learn new mathematical ideas. Mathematicians have used this process for many years to make sense of new concepts and to broaden their understanding of older ideas.

In general, this process begins with a **question** that helps you frame what you are looking for. For example, a question such as, "*What if the Möbius strip has 2 half-twists? What will happen when that strip is cut in half down the middle?*" can help start an investigation to find out what happens when the Möbius strip is slightly altered.

Once a question is asked, you can make an educated guess, called a **conjecture**. This is a mathematical statement that has not yet been proven.

Next, **exploration** begins. This part of the process may last awhile as you gather more information about the mathematical concept. For example, you may first have an idea about the diagonals of a rectangle, but as you draw and measure a rectangle on graph paper, you find out that your conjecture was incorrect. When this happens, you just experiment some more until you have a new conjecture to test.

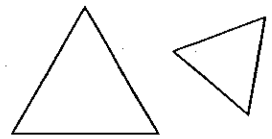
When a conjecture seems to be true, the final step is to **prove** that the conjecture is always true. A proof is a convincing logical argument that uses definitions and previously proven conjectures in an organized sequence.

Review & Preview

1-14. A major focus of this course is learning the **investigative process**, a process you used during the Möbius Strip activity in problems 1-9 through 1-12. One part of this process is asking mathematical questions.

Assume your teacher is thinking of a shape and wants you to figure out what shape it is. Write down three questions you could ask your teacher to determine more about his or her shape.

1-15. The shapes at right are examples of **equilateral triangles**. How can you describe an equilateral triangle? Make at least two statements that seem true for all equilateral triangles. Then trace these equilateral triangles on your paper and draw one more in a different orientation.



Examples of Equilateral Triangles

1-16. Match each table of data below with the most appropriate graph and **briefly explain** why it matches the data.

a. Boiling water cooling down.

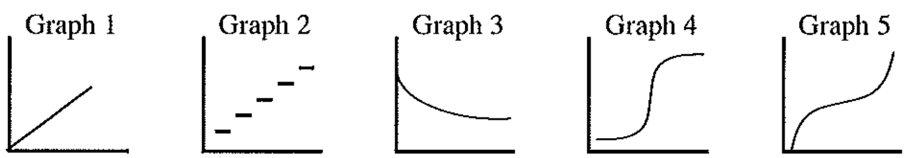
Time (min)	0	5	10	15	20	25
Temp (°C)	100	89	80	72	65	59

b. Cost of a phone call.

Time (min)	1	2	2.5	3	4	5	5.3	6
Cost (cents)	55	75	75	95	115	135	135	155

c. Growth of a baby in the womb.

Age (months)	1	2	3	4	5	6	7	8	9
Length (inches)	0.75	1.5	3	6.4	9.6	12	13.6	15.2	16.8



1-17. Solve for the given variable. Show the steps leading to your solution. Check your solution.

a. $-11x = 77$

b. $5c + 1 = 7c - 8$

c. $\frac{x}{8} = 2$

d. $-12 = 3k + 9$

1-18. Calculate the values of the expressions below. Show all steps in your process.

a. $\frac{3(2+6)}{2}$

b. $\frac{1}{2}(14)(5)$

c. $7^2 - 5^2$

d. $17 - 6 \cdot 2 + 4 \div 2$

1.1.3 How can I predict the area?

Perimeter and Area of Enlarging Tile Patterns

One of the core ideas of geometry is the measurement of shapes. Often in this course it will be important to find the areas and perimeters of shapes. How these measurements change as a shape is enlarged or reduced in size is especially interesting. Today your team will apply algebraic skills as you **investigate** the areas and perimeters of similar shapes.

1-19. CARPETMART

Your friend Alonzo has come to your team for help. His family owns a rug manufacturing company, which is famous for its unique and versatile designs. One of their most popular designs is shown at right. Each rug design has an "original" size as well as enlargements that are exactly the same shape.

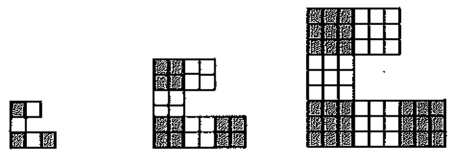


Figure #1: "The Original" Figure #2 Figure #3

Alonzo is excited because his family found out that the king of a far-away land is going to order an extremely large rug for one of his immense banquet halls. Unfortunately, the king is fickle and won't decide which rug he will order until the very last minute. The day before the banquet, the king will tell Alonzo which rug he wants and how big it will need to be. The king's palace is huge, so the rug will be VERY big!

Since the rugs are different sizes, and since each rug requires wool for the interior and fringe to wrap around the outside, Alonzo will need to quickly find the area and perimeter of each rug in order to obtain the correct quantities of wool and fringe.

Your Task: Your teacher will assign your team one of the rug designs to **investigate** (labeled (a) through (f) below). The "original" rug is shown in Figure 1, while Figures 2 and 3 are the next enlarged rugs of the series. With your team, create a table, graph, and rule for both the area and perimeter of your rug design. Then decide which representation will best help Alonzo find the area and perimeter for *any* figure number.

Problem continues on next page →

1-19. *Problem continued from previous page.*

Be ready to share your analysis with the rest of the class. Your work must include the following:

- Diagrams for the rugs of the next two sizes (Figures 4 and 5) following the pattern shown in Figures 1, 2, and 3.
- A description of Figure 20. What will it look like? What are its area and perimeter?
- A table, graph, and rule representing the perimeter of your rug design.
- A table, graph, and rule representing the area of your rug design.

Rug Designs:

a.

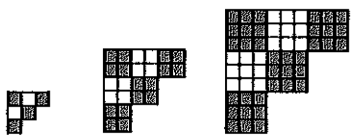


Figure 1 Figure 2 Figure 3

b.

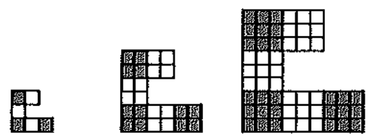


Figure 1 Figure 2 Figure 3

c.

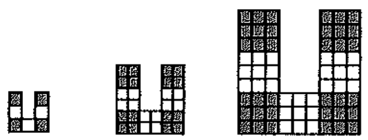


Figure 1 Figure 2 Figure 3

d.

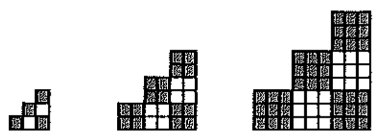


Figure 1 Figure 2 Figure 3

e.

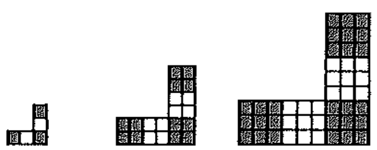


Figure 1 Figure 2 Figure 3

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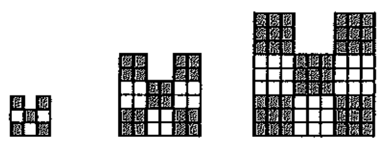


Figure 1 Figure 2 Figure 3

Further Guidance

1-20. To start problem 1-20, first analyze the pattern your team has been assigned on graph paper, draw diagrams of Figures 4 and 5 for your rug design. Remember to shade Figures 4 and 5 the same way Figures 1 through 3 are shaded.

1-21. Describe Figure 20 of your design. Give as much information as you can. What will it look like? How will the squares be arranged? How will it be shaded?

1-22. A table can help you learn more about how the perimeter changes as the rugs get bigger.

a. Organize your perimeter data in a table like the one shown below.

Figure number	1	2	3	4	5	20
Perimeter (in units)						

b. Graph the perimeter data for Figures 1 through 5. (You do not need to include Figure 20.) What shape is the graph?

c. How does the perimeter grow? **Examine** your table and graph and describe how the perimeter changes as the rugs get bigger.

d. Generalize the patterns you have found by writing an algebraic rule (equation) that will find the perimeter of any size rug in your design. That is, what is the perimeter of Figure n? Show how you got your answer.

1-23. Now analyze how the area changes with a table and graph.

a. Make a new table, like the one below, to organize information about the area of each rug in your design.

Figure number	1	2	3	4	5	20
Area (in square units)						

b. On a new set of axes, graph the area data for Figures 1 through 5. (You do not need to include Figure 20.) What shape is the graph?

c. How does the area grow? Does it grow the same way as the perimeter? **Examine** your table and graph and describe how the area changes as the rugs get bigger.

d. Write a rule that will find the area of Figure n. How did you find your rule? Be ready to share your **strategy** with the class.

Further Guidance section ends here.

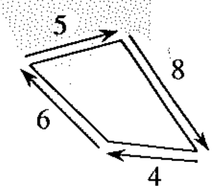
1-24. The King has arrived! He demands a Rug #100, which is Figure 100 in your design. What will its perimeter be? Its area? **Justify** your answer.

METHODS AND MEANINGS

MATH NOTES

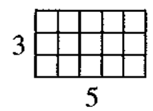
The Perimeter and Area of a Figure

The **perimeter** of a two-dimensional figure is the distance around its exterior (outside) on a flat surface. It is the total length of the boundary that encloses the interior (inside) region. See the example at right.



Perimeter = $5+8+4+6=23$ units

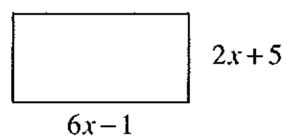
The **area** indicates the number of square units needed to fill up a region on a flat surface. For a rectangle, the area is computed by multiplying its length and width. The rectangle at right has a length of 5 units and a width of 3 units, so the area of the rectangle is 15 square units.



Area = $5 \cdot 3 = 15$ square units

Review & Preview

1-25. Read the Math Notes box for this lesson, which describes how to find the area and perimeter of a shape. Then **examine** the rectangle at right. If the perimeter of this shape is 120 cm, which equation below represents this fact? Once you have selected the appropriate equation, solve for x.



a. $2x+5+6x-1=120$

b. $4(6x-1)=120$

c. $2(6x-1)+2(2x+5)=120$

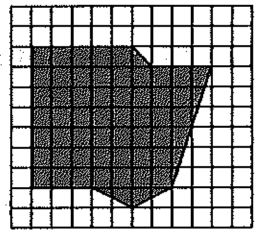
d. $(2x+5)(6x-1)=120$

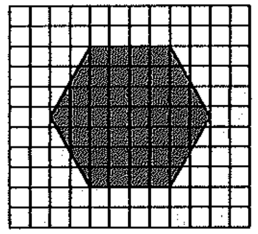
1-26. Delilah drew 3 points on her paper. When she connects these points, must they form a triangle? Why or why not? Draw an example on your paper to support your **reasoning**.

1-27. Copy the table below onto your paper. Complete it and write a rule relating x and y.

x	3	−1	0	2	−5	−2	1
y	0			−1			−2

1-28. Rebecca placed a transparent grid of square units over each of the shapes she was measuring below. Using her grid, determine the area of each shape.

a. 

b. 

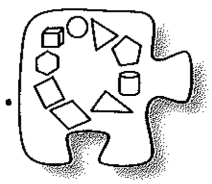
1-29. Evaluate each expression below if $a = -2$ and $b = 3$.

a. $3a^2 - 5b + 8$

b. $\frac{2}{3}b - 5a$

c. $\frac{a+2b}{4} + 4a$

1.1.4 Are you convinced?



Logical Arguments

"I don't have my homework today because..." Is your teacher going to be convinced? Will it make a difference whether you say that the dog ate your homework or whether you bring in a note from the doctor? Imagine your friend says, "I know that shape is a square because it has four right angles." Did your friend tell you enough to convince you?

Many jobs depend on your ability to convince other people that your ideas are correct. For instance, a defense lawyer must be able to form logical arguments to persuade the jury or judge that his or her client is innocent. Today you are going to use a Way of Thinking called **reasoning and justifying** to focus on what makes a statement convincing.

1-30. TRIAL OF THE CENTURY

The musical group Apple Core has accused your math teacher, Mr. Bosky, of stealing its newest pop CD, "Rotten Gala." According to the police, someone stole the CD from the BigCD Store last Saturday at some time between 6:00 PM and 7:00 PM. Because your class is so well known for only reaching conclusions when sufficient evidence is presented, the judge has made you the jury! You are responsible for determining whether or not there is enough evidence to convict Mr. Bosky.

Carefully listen to the evidence that is presented. As each statement is read, decide:

- *Does the statement convince you? Why or why not?*
- *What could be changed or added to the statement to make it more convincing?*

Testimony

Mr. Bosky: *"But I don't like that CD! I wouldn't take it even if you paid me."*

Mr. Bosky: *"I don't have the CD. Search me."*

Mr. Bosky: *"I was at home having dinner Saturday."*

Casey: *"There were several of us having dinner with Mr. Bosky at his house. He made us a wonderful lasagna."*

Mrs. Thomas: *"All of us at dinner with Mr. Bosky left his house at 6:10 PM."*

Police Officer Yates: *"Driving as quickly as I could, it took me 30 minutes to go from Mr. Bosky's house to the BigCD store."*

Coach Teller: *"Mr. Bosky made a wonderful goal right at the beginning of our soccer game, which started at 7:00 PM. You can check the score in the local paper."*

Police Officer Yates: *"I also drove from the BigCD store to the field where the soccer game was. It would take him at least 40 minutes to get there."*

1-31. THE FAMILY FORTUNE

You are at home when the phone rings. It is a good friend of yours who says, "Hey, your last name is Marston. Any chance you have a grandmother named Molly Marston who was REALLY wealthy? Check out today's paper." You glance at the front page:

Family Fortune Unclaimed

City officials are amazed that the county's largest family fortune may go unclaimed. Molly "Ol' Granny" Marston died earlier this week and it appears that she was survived by no living relatives. According to her last will and testament, "Upon my death, my entire fortune is to be divided among my children and grandchildren." Family members have until noon tomorrow to come forward with a written statement giving evidence that they are related to Ms. Marston or the money will be turned over to the city.

You're amazed – Molly is your grandmother, so your friend is right! However, you may not be able to collect your inheritance unless you can convince city officials that you are a relative. You rush into your attic where you keep a trunk full of family memorabilia.

a. You find several items that you think might be important in an old trunk in the attic. With your team, decide which of the items listed below will help prove that Ol' Molly was your grandmother.

Family Portrait — a photo showing three young children. On the back you see the date 1968.

Newspaper Clipping — an article from 1972 titled "Triplets Make Music History." The first sentence catches your eye: "Jake, Judy, and Jeremiah Marston, all eight years old, were the first triplets ever to perform a six-handed piano piece at Carnegie Hall."

Jake Marston's Birth Certificate — showing that Jake was born in 1964, and identifying his parents as Phillip and Molly Marston.

Your Learner's Permit — signed by your father, Jeremiah Marston.

Wilbert Marston's Passport — issued when Wilbert was fifteen.

b. Your team will now write a statement that will convince the city official (played by your faithful teacher!) that Ol' Molly was your grandmother. Be sure to support any claims that you make with appropriate evidence. Sometimes it pays to be convincing!

METHODS AND MEANINGS

MATH NOTES

Solving Linear Equations

In Algebra, you learned how to solve a linear equation. This course will help you apply your algebra skills to solve geometric problems. Review how to solve equations by reading the example below.

- **Simplify**. Combine like terms on each side of the equation whenever possible.
- **Keep equations balanced.** The equal sign in an equation tells you that the expressions on the left and right are balanced. Anything done to the equation must keep that balance.

$$3x-2+4=x-6 \quad \text{Combine like terms}$$
$$3x+2=x-6$$
$$-x=-x \quad \text{Subtract } x \text{ on both sides}$$
$$2x+2=-6$$
$$-2=-2 \quad \text{Subtract 2 on both sides}$$
$$\frac{2x}{2}=\frac{-8}{2} \quad \text{Divide both sides by 2}$$
$$x=-4$$

- **Move your x-terms to one side of the equation.** Isolate all variables on one side of the equation and the constants on the other.
- **Undo operations.** Use the fact that addition is the opposite of subtraction and that multiplication is the opposite of division to solve for x. For example, in the equation $2x=-8$, since the 2 and the x are multiplied, then dividing both sides by 2 will get x alone.

Review & Preview

1-32. One goal of this course will be to review and enhance your algebra skills. Read the Math Notes box for this lesson. Then solve for x in each equation below, show all steps leading to your solution, and check your answer.

a. $34x-18=10x-9$

b. $4x-5=4x+10$

c. $3(x-5)+2(3x+1)=45$

d. $-2(x+4)+6=-3$

1-33. The day before Gerardo returned from a two-week trip, he wondered if he left his plants inside his apartment or outside on his deck. He knows these facts:

- If his plants are indoors, he must water them at least once a week or they will die.
- If he leaves his plants outdoors and it rains, then he does not have to water them. Otherwise, he must water them at least once a week or they will die.
- It has not rained in his town for 2 weeks.

When Gerardo returns, will his plants be dead? Explain your **reasoning**.

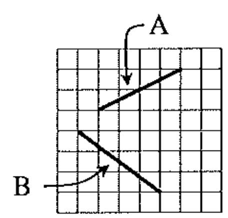
1-34. For each of the equations below, solve for y in terms of x.

a. $2x-3y=12$

b. $5x+2y=7$

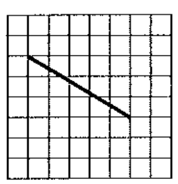
1-35. Find the area of the rectangle at right. Be sure to include units in your answer.

1-36. The **slope** of a line is a measure of its steepness and indicates whether it goes up or down from left to right. For example, the slope of the line segment A at right is $\frac{1}{2}$, while the slope of the line segment B is $-\frac{3}{4}$.

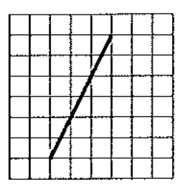


For each line segment below, find the slope. You may want to copy each line segment on graph paper in order to draw slope triangles.

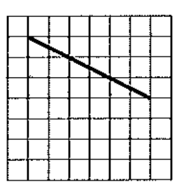
a.



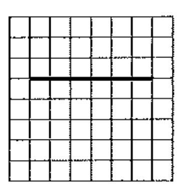
b.



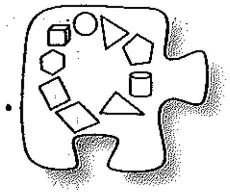
c.



d.



1.1.5 What shapes can you find?



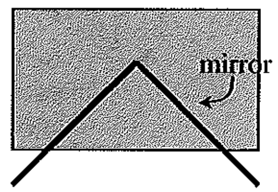
Building a Kaleidoscope

Today you will learn about angles and shapes as you study how a kaleidoscope works.

1-37. BUILDING A KALEIDOSCOPE

How does a kaleidoscope create the complicated, colorful images you see when you look inside? A hinged mirror and a piece of colored paper can demonstrate how a simple kaleidoscope creates its beautiful repeating designs.

Your Task: Place a hinged mirror on a piece of colored, unlined paper so that its sides extend beyond the edge of the paper as shown at right. Explore what shapes you see when you look directly at the mirror, and how those shapes change when you change the angle of the mirror. Discuss the questions below with your team. Be ready to share your responses with the rest of the class.



Discussion Points

What is this problem about? What is it asking you to do?

What happens when you change the angle (opening) formed by the sides of the mirror?

How can you describe the shapes you see in the mirror?

1-38. To complete your exploration, answer these questions together as a team.

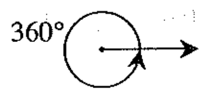
a. What happens to the shape you see as the angle formed by the mirror gets bigger (wider)? What happens as the angle gets smaller?

b. What is the smallest number of sides the shape you see in the mirror can have? What is the largest?

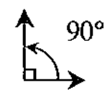
c. With your team, find a way to form a **regular hexagon** (a shape with six equal sides and equal angles).

d. How might you describe to another team how you set the mirrors to form a hexagon? What types of information would be useful to have?

1-39. A good way to describe an angle is by measuring how *wide* or *spread apart* the angle is. For this course, you should think of the measurement of an angle as representing the amount of rotation that occurs when you separate the two sides of the mirror from a closed position. The largest angle you can represent with a hinged mirror is 360°. This is formed when you open a mirror all the way so that the backs of the mirror touch. This is a called a **circular angle** and is represented by the diagram at right.

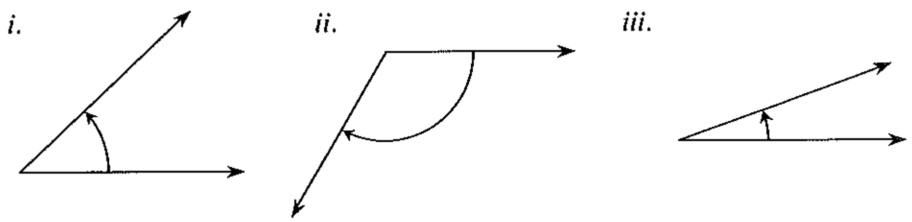


a. Other angles may be familiar to you. For example, an angle that forms a perfect "L" or a quarter turn is a 90° angle, called a **right angle** (shown at right). You can see that four of these angles would form a circular angle.

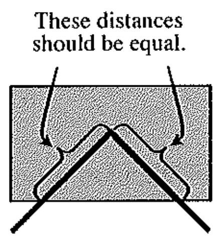


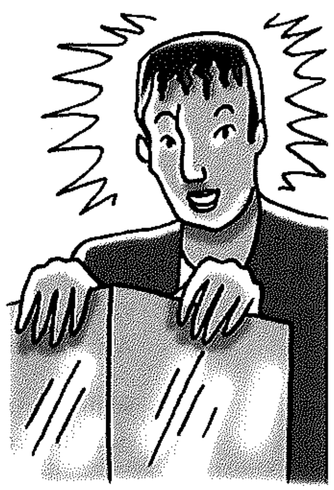
What if the two mirrors are opened to form a straight line? What measure would that angle have? Draw this angle and label its degrees. How is this angle related to a circular angle?

b. Based on the examples above, estimate the measures of these angles shown below. Then confirm your answer using a **protractor**, a tool that measures angles.



1-40. Now use your understanding of angle measurement to create some specific shapes using your hinged mirror. Be sure that both mirrors have the same length on the paper, as shown in the diagram at right.





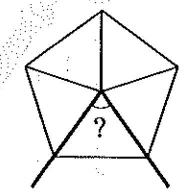
a. Antonio says he can form an **equilateral triangle** (a triangle with three equal sides and three equal angles) using his hinged mirror. How did he do this? Once you can see the triangle in your mirror, place the protractor on top of the mirror. What is the measure of the angle formed by the sides of the mirror?

Problem continues on next page →

1-40. *Problem continued from previous page.*

b. Use your protractor to set your mirror so that the angle formed is 90°. Be sure that the sides of the mirror intersect the edge of the paper at equal lengths. What is this shape called? Draw and label a picture of the shape on your paper.

c. Carmen's mirror shows the image at right, called a **regular pentagon**. She noticed that the five triangles in this design all meet at the hinge of her mirrors. She also noticed that the triangles must all be the same size and shape, because they are reflections of the triangle formed by the mirrors and the paper.



What must the sum of these five angles at the hinge be? And what is the angle formed by Carmen's mirrors? Test your conclusion with your mirror.

d. Discuss with your team and predict how many sides a shape would have if the angle that the mirror forms measures 40°. Explain how you made your prediction. Then check your prediction using the mirror and a protractor. Describe the shape you see with as much detail as possible.

1-41. Reflect on what you learned during today's activity.

a. Based on this activity, what are some things that you think you will be studying in Geometry?

b. This activity was based on the question, *"What shapes can be created using reflections?"* What ideas from this activity would you want to learn more about? Write a question that could prompt a different, but related, future **investigation**.

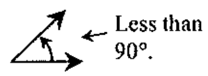
MATH NOTES

METHODS AND MEANINGS

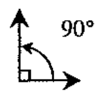
Types of Angles

When trying to describe shapes, it is convenient to classify types of angles. This course will use the following terms to refer to angles:

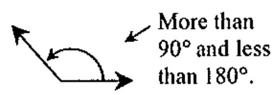
ACUTE: Any angle with measure **between** (but not including) 0° and 90°.



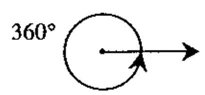
RIGHT: Any angle that measures 90°.



OBTUSE: Any angle with measure **between** (but not including) 90° and 180°.



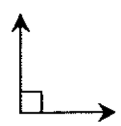
STRAIGHT: Straight angles have a measure of 180° and are formed when the sides of the angle form a straight line.



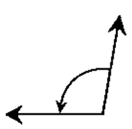
CIRCULAR: Any angle that measures 360°.

Review & Preview

1-42. Estimate the size of each angle below to the nearest 10°. A right angle is shown for reference so you should not need a protractor.

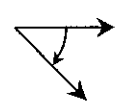


a.



b.

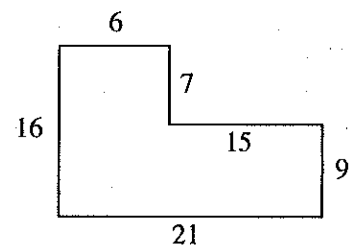
c.



1-43. Using graph paper, draw a pair of xy-axes and scale them for $-6 \le x \le 6$ (x-values between and including −6 and 6) and $-10 \le y \le 10$ (y-values between and including −10 and 10). Complete the table below, substituting the x-values (inputs) to find the corresponding y-values (outputs) for the rule $y = x + 2$. Plot and connect the resulting points.

x (input)	−4	−3	−2	−1	0	1	2	3	4
y (output)									

1-44. Angela had a rectangular piece of paper and then cut a rectangle out of a corner as shown at right. Find the area and perimeter of the resulting shape. Assume all measurements are in centimeters.



1-45. For each equation below, solve for the given variable. If necessary, refer to the Math Notes box in Lesson 1.1.4 for guidance. Show the steps leading to your solution and check your answer.

a. $75 = 14y + 5$

b. $-7r + 13 = -71$

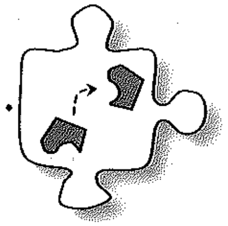
c. $3a + 11 = 7a - 13$

d. $2m + m - 8 = 7$

1-46. On graph paper, draw four different rectangles that each have an area of 24 square units. Then find the perimeter of each one.

1.2.1 How do you see it?

Spatial Visualization and Reflections



Were you surprised when you looked into the hinged mirror during the Kaleidoscope Investigation of Lesson 1.1.5? Reflection can create many beautiful and interesting shapes and can help you learn more about the characteristics of other shapes. However, one reason you may have been surprised is because it is sometimes difficult to predict what a reflection will be. This is where spatial visualization plays an important role. **Visualizing**, the act of "picturing" something in your mind, is often helpful when working with shapes. In order to be able to **investigate** and describe a geometric concept, it is first useful to **visualize** a shape or action.

Today you will be **visualizing** in a variety of ways and will develop the ability to find reflections. As you work today, keep the following focus questions in mind:

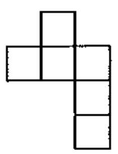
How do I see it?

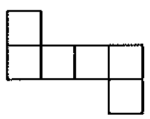
How can I verify my answer?

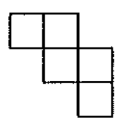
How can I describe it?

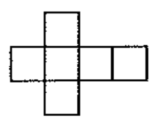
1-47. BUILDING BOXES

Which of the nets (diagrams) below would form a box with a lid if folded along the interior lines? Be prepared to defend your answer.

a. 

b. 

c. 

d. 

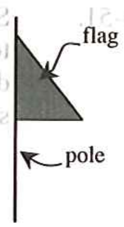
1-48. Have you ever noticed what happens when you look in a mirror? Have you ever tried to read words while looking in a mirror? What happens? Discuss this with your team. Then re-write the following words as they would look if you held this book up to a mirror. Do you notice anything interesting?

a. GEO

b. STAR

c. WOW

1-49. When Kenji spun the flag shown at right very quickly about its pole, he noticed a three-dimensional shape emerge.

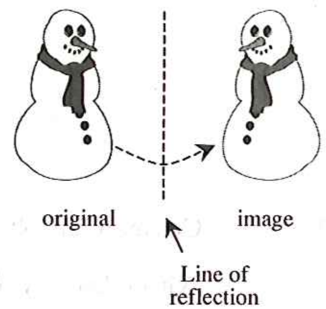


a. What shape did he see? Draw a picture of the three-dimensional shape on your paper and be prepared to defend your answer.

b. What would the flag need to look like so that a **sphere** (the shape of a basketball) is formed when the flag is rotated about its pole? Draw an example.

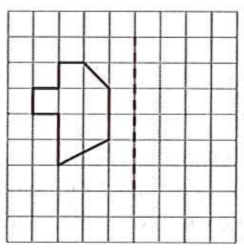
1-50. REFLECTIONS

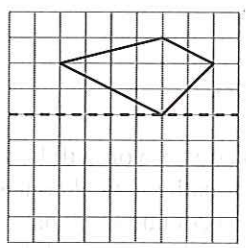
The shapes created in the Kaleidoscope Investigation in Lesson 1.1.5 were the result of reflecting a triangle several times in a hinged mirror. However, other shapes can also be created by a reflection. For example, the diagram at right shows the result of reflecting a snowman across a line.

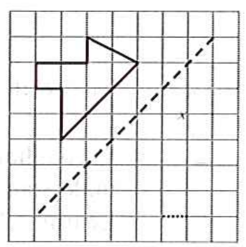


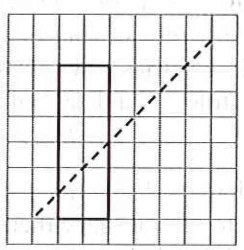
a. Why do you think the image is called a reflection? How is the image different from the original?

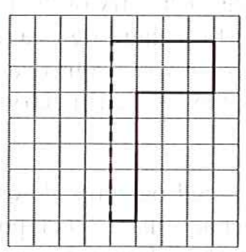
b. On the Lesson 1.2.1 Resource Page provided by your teacher, use your **visualization** skills to imagine the reflection of each shape across the given line of refection. Then draw the reflection. Check your work by folding the paper along the line of reflection.

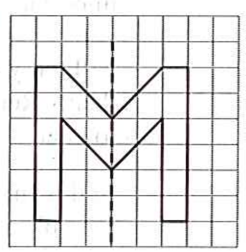
(1)


(2)


(3)


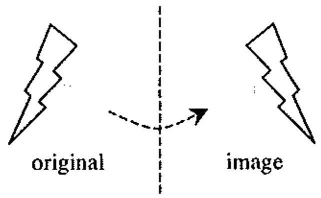
(4)


(5)


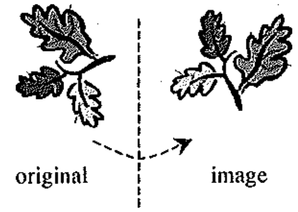
(6)


1-51. Sometimes, a motion appears to be a reflection when it really isn't. How can you tell if a motion is a reflection? Consider each pair of objects below. Which diagrams represent reflections across the given lines of reflection? Study each situation carefully and be ready to explain your thinking.

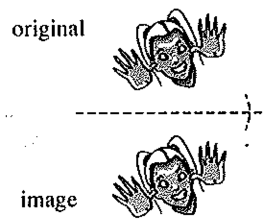
(1)



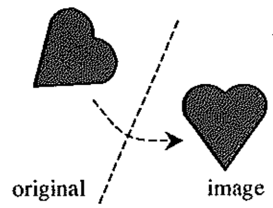
(2)



(3)



(4)



1-52. CONNECTIONS WITH ALGEBRA

What other ways can you use reflections? Consider how to reflect a graph as you answer the questions below.

a. On your Lesson 1.2.1 Resource Page, graph the parabola $y = x^2 + 3$ and the line $y = x$ for $x = -3, -2, -1, 0, 1, 2, 3$ on the same set of axes.

b. Now reflect the parabola over the line $y = x$. What do you observe? What happens to the x- and y-values of the original parabola?

1-53. LEARNING LOG

Throughout this course, you will be asked to reflect on your understanding of mathematical concepts in a Learning Log. Your Learning Log will contain explanations and examples to help you remember what you have learned throughout the course. It is important to write each entry of the Learning Log in your own words so that later you can use your Learning Log as a resource to refresh your memory. Your teacher will tell you where to write your Learning Log entries and how to structure or label them. Remember to label each entry with a title and a date so that it can be referred to later.

In this first Learning Log entry, describe what you learned today. For example, is it possible to reflect any shape? Is it possible to have a shape that, when reflected, doesn't change? How does reflection work? If it helps you to explain, sketch and label pictures to illustrate what you write. Title this entry "Reflections" and include today's date.

MATH NOTES

METHODS AND MEANINGS

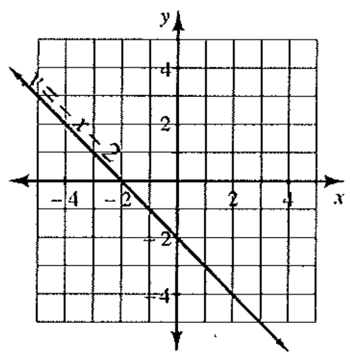
Graphing an Equation

In Algebra, you learned how to graph an equation. During this course, you will apply your algebra skills to solve geometric problems. Review how to graph an equation by reading the example below.

- **Create a table of x-values.** Choose x-values that will show you any important regions of the graph of the equation. If you do not know ahead of time what the graph will look like, use the values $-4 \le x \le 4$ as shown at right.

$y = -x - 2$

x	−4	−3	−2	−1	0	1	2	3	4
y	2	1	0	−1	−2	−3	−4	−5	−6



- **Use the equation to find y-values.** Substitute each value of x into your equation and find the corresponding y-value. For example, for $y = -x - 2$, when $x = -3$, $y = -(-3) - 2 = 1$.

- **Graph the points using the coordinates from your table onto a set of $x \rightarrow y$ axes.** Connect the points and, if appropriate, use arrows to indicate that the graph of the equation continues in each direction.

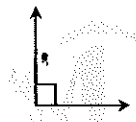
- **Complete the graph.** Be sure your axes are scaled and labeled. Also label your graph with its equation as shown in the example above.

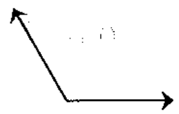
Review & Preview

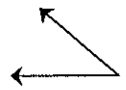
1-54. Review how to graph by reading the Math Notes box for this lesson. Then graph each line below on the same set of axes.

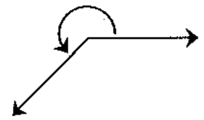
a. $y = 3x - 3$ b. $y = -\frac{2}{3}x + 3$ c. $y = -4x + 5$

1-55. Estimate the size of each angle below to the nearest 10°. A right angle is shown at right for reference so you do not need a protractor.

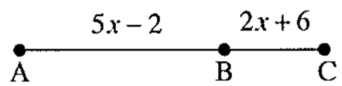


a. 

b. 

c. 

1-56. The distance along a straight road is measured as shown in the diagram below. If the distance between towns A and C is 67 miles, find the distance between towns A and B.



1-57. For each equation below, solve for x. Show all work. The answers are provided so that you can check them. If you are having trouble with any solutions and cannot find your errors, you may need to see your teacher for extra help (you can ask your team as well). **[Solutions: a: 3.75, b: 3, c: 0, d: 3, e: ≈ 372.25, f: –3.4]**

a. $5x-2x+x=15$

b. $3x-2-x=7-x$

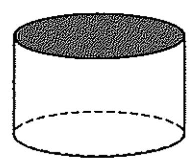
c. $3(x-1)=2x-3+3x$

d. $3(2-x)=5(2x-7)+2$

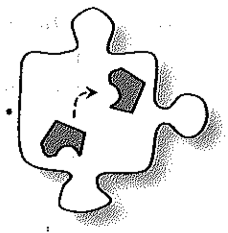
e. $\frac{26}{57}=\frac{849}{5x}$

f. $\frac{4x+1}{3}=\frac{x-5}{2}$

1-58. The three-dimensional shape at right is called a **cylinder**. Its bottom and top bases are both circles, and its side is perpendicular to the bases. What would the shape of a flag need to be in order to generate a cylinder when it rotates about its pole? (You may want to refer to problem 1-49 to review how flags work.)

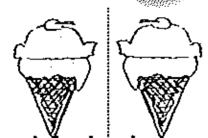


1.2.2 What if it is reflected more than once?



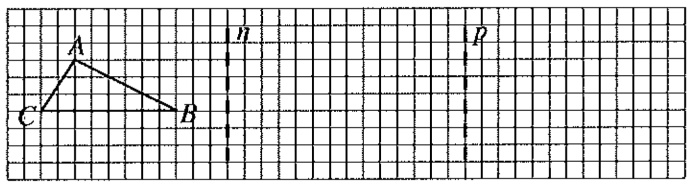
Rigid Transformations: Rotations and Translations

In Lesson 1.2.1, you learned how to change a shape by reflecting it across a line, like the ice cream cones shown at right. Today you will learn more about reflections and learn about two new types of transformations: rotations and translations.



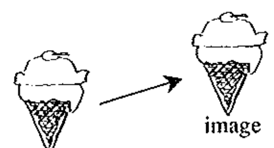
1-59. As Amanda was finding reflections, she wondered, "What if I reflect a shape twice over parallel lines?" **Investigate** her question as you answer the questions below.

a. On the Lesson 1.2.2 Resource Page provided by your teacher, find ΔABC and lines n and p (shown below). What happens when ΔABC is reflected across line n to form $\Delta A'B'C'$ and then $\Delta A'B'C'$ is reflected across line p to form $\Delta A''B''C''$? First **visualize** the reflections and then test your idea of the result by **drawing** both reflections.



b. **Examine** your result from part (a). Compare the original triangle ΔABC with the final result, $\Delta A''B''C''$. What single motion would change ΔABC to $\Delta A''B''C''$?

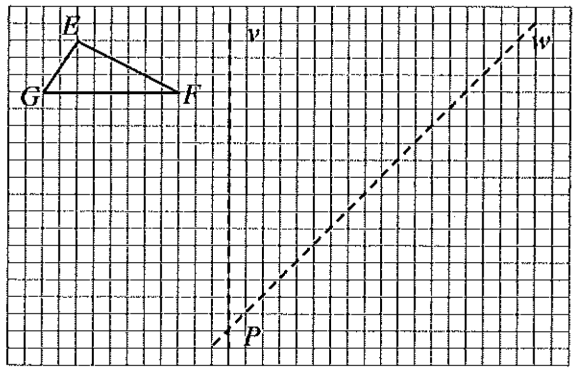
c. Amanda analyzed her results from part (a). "It looks like I could have just slid ΔABC over!" Sliding a shape from its original position to a new position is called **translating**. For example, the ice cream cone at right has been **translated**. Notice that the image of the ice cream cone has the same *orientation* as the original (that is, it is not turned or flipped). What words can you use to describe a translation?



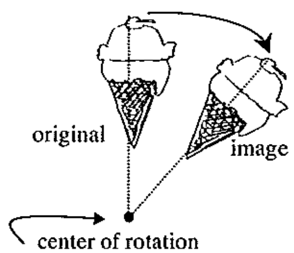
d. The words **transformation** and **translation** sound alike and can easily be confused. Discuss in your team what these words mean and how they are related to each other.

1-60. After answering Amanda's question in problem 1-59, her teammate asks, "What if the lines of reflection are not parallel? Is the result still a translation?" Find ΔEFG and lines v and w on the Lesson 1.2.2 Resource Page.

a. First **visualize** the result when ΔEFG is reflected over v to form $\Delta E'F'G'$, and then $\Delta E'F'G'$ is reflected over w to form $\Delta E''F''G''$. Then **draw** the resulting reflections on the resource page. Is the final image a translation of the original triangle? If not, describe the result.



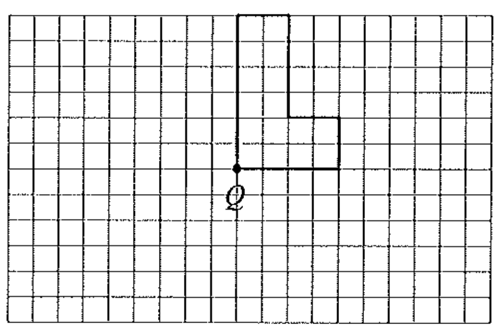
b. Amanda noticed that when the reflecting lines are not parallel, the original shape is **rotated**, or turned, to get the new image. For example, the diagram at right shows the result when an ice cream cone is rotated about a point.



In part (a), the center of rotation is at point P, the point of intersection of the lines of reflection. Use a piece of tracing paper to test that $\Delta E''F''G''$ can be obtained by rotating ΔEFG about point P. To do this, first trace ΔEFG and point P on the tracing paper. While the tracing paper and resource page are aligned, apply pressure on P so that the tracing paper can rotate about this point. Then turn your tracing paper until ΔEFG rests atop $\Delta E''F''G''$.

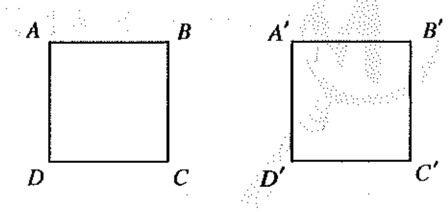
c. The rotation of ΔEFG in part (a) is an example of a 90° clockwise rotation. The term "clockwise" refers to a rotation that follows the direction of the hands of a clock, namely ↻. A rotation in the opposite direction (↺) is called "counter-clockwise."

On your resource page, rotate the "block L" 90° counter-clockwise (↺) about point Q.



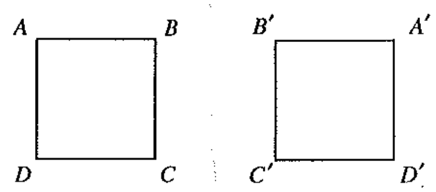
1-61. NOTATION FOR TRANSFORMATIONS

In the diagram at right, the original square *ABCD* on the left was *translated* to the image square on the right. (We label a shape by labeling each corner.) The image location is different from the original, so we use different letters to label its **vertices** (corners).

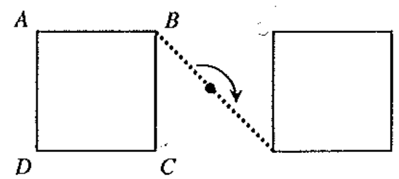


To keep track of how the vertices correspond, we call the image $A'B'C'D'$. The $'$ symbol is read as "prime," so the shape on the right is called, "*A* prime *B* prime *C* prime *D* prime." A' is the image of *A*, B' is the image of *B*, etc. This notation tells you which vertices correspond.

a. The diagram at right shows a different transformation of *ABCD*. Look carefully at the correspondence between the vertices. Can you rotate or reflect the original square to make the letters correspond as shown? If you can reflect, where would the line of reflection be? If you can rotate, where would the point of rotation be?



b. The diagram at right shows a rotation. Copy the diagram (both squares and the point) and label the corners of the image square on the right. If you have trouble, ask your teacher for tracing paper. Describe the rotation.



1-62. CONNECTIONS WITH ALGEBRA

Examine how translations and rotations can affect graphs. On graph paper, graph the equation $y=\frac{1}{2}x-3$.

a. Draw the result if $y=\frac{1}{2}x-3$ is translated up 4 units. You may want to use tracing paper. Write the equation of the result. What is the relationship of $y=\frac{1}{2}x-3$ and its image?

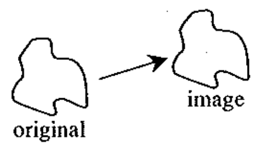
b. Now use tracing paper to rotate $y=\frac{1}{2}x-3$ 90° clockwise (↻) about the point (0, 0). Write the equation of the result. What is the relationship of $y=\frac{1}{2}x-3$ and this new image?

MATH NOTES

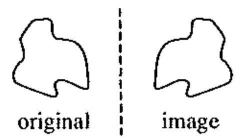
METHODS AND MEANINGS

Rigid Transformations

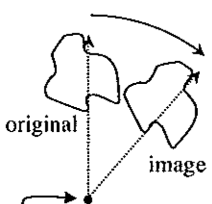
A transformation that preserves the size, shape, and orientation of a figure while *sliding* it to a new location is called a **translation.**



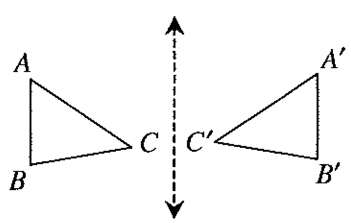
A transformation that preserves the size and shape of a figure across a line to form a mirror image is called a **reflection**. The mirror line is a **line of reflection**. One way to find a reflection is to *flip* a figure over the line of reflection.



A transformation that preserves the size and shape while *turning* an entire figure about a fixed point is called a **rotation**. Figures can be rotated either clockwise (↻) or counterclockwise (↺).

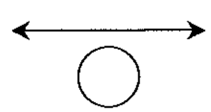


When labeling a transformation, the new figure (image) is often labeled with **prime notation**. For example, if $\triangle ABC$ is reflected across the vertical dashed line, its image can be labeled $\triangle A'B'C'$ to show exactly how the new points correspond to the points in the original shape. We also say that $\triangle ABC$ is **mapped** to $\triangle A'B'C'$.

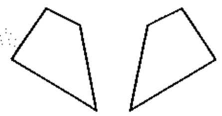


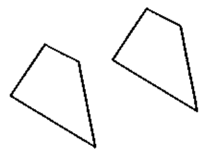
Review & Preview

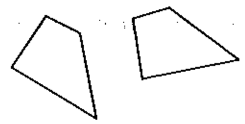
1-63. The diagram at right shows a flat surface containing a line and a circle with no points in common. Can you **visualize** moving the line and/or circle so that they intersect at exactly one point? Two points? Three points? Explain each answer and illustrate each with an example when possible.

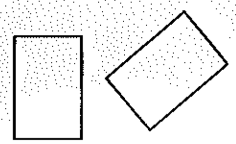


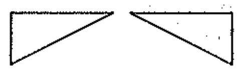
1-64. Decide which transformation was used on each pair of shapes below. Some may have undergone more than one transformation.

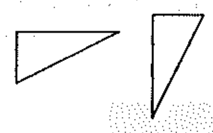
a. 

b. 

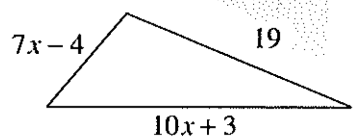
c. 

d. 

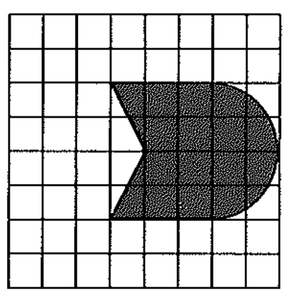
e. 

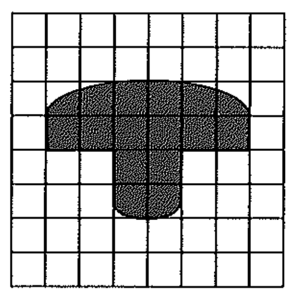
f. 

1-65. The **perimeter** (the sum of the side lengths) of the triangle at right is 52 units. Write and solve an equation based on the information in the diagram. Use your solution for x to find the measures of each side of the triangle. Be sure to confirm that your answer is correct.



1-66. Bertie placed a transparent grid made up of unit squares over each of the shapes she was measuring below. Using her grid, approximate the area of each region.

a. 

b. 

1-67. For each equation below, find y if $x = -3$.

a. $y = -\frac{1}{3}x - 5$

b. $y = 2x^2 - 3x - 2$

c. $2x - 5y = 4$

1.2.3 How can I move it?

Using Transformations

In Lesson 1.1.1, your class made a quilt using designs based on a geometric shape. Similarly, throughout American history, quilters have created quilts that use transformations to create intricate geometric designs. For example, the quilt at right is an example of a design based on rotation and reflection, while the quilt at left contains translation, rotation, and reflection.

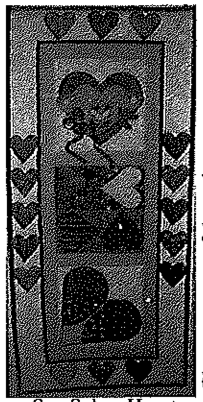


Photo courtesy of the artist.

Sue Sales, *Hearts*.

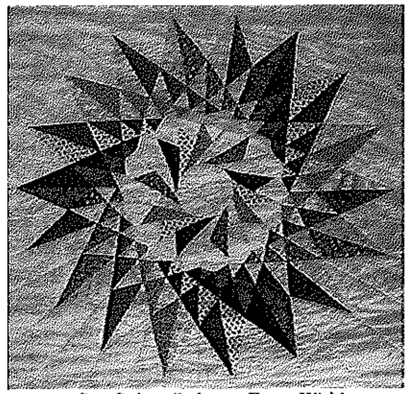


Photo courtesy of the artist.

Sue Sales, *Balance From Within*.

Today, you will work with your team to discover ways to find the image of a shape after it is rotated, translated, or reflected.

1-68. Here are some situations that occur in everyday life. Each one involves one or more of the basic transformations: reflection, rotation or translation. State the transformation(s) involved in each case.

a. You look in a mirror as you comb your hair.

b. While repairing your bicycle, you turn it upside down and spin the front tire to make sure it isn't rubbing against the frame.

c. You move a small statue from one end of a shelf to the other.

d. You flip your scrumptious buckwheat pancakes as you cook them on the griddle.

e. The bus tire spins as the bus moves down the road.

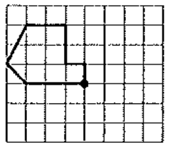
f. You examine the footprints made in the sand as you walked on the beach.

1-69. ROTATIONS ON A GRID

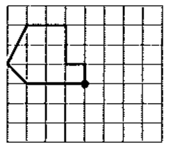
Consider what you know about rotation, a motion that turns a shape about a point. Does it make any difference if a rotation is clockwise (↻) versus counterclockwise (↺)? If so, when does it matter? Are there any circumstances when it does not matter? And are there any situations when the rotated image lies exactly on the original shape?

Investigate these questions as you rotate the shapes below about the given point on the Lesson 1.2.3 Resource Page. Use tracing paper if needed. Be prepared to share your answers to the questions posed above.

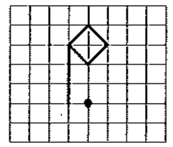
a. 180° ↺



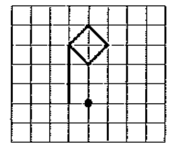
b. 180° ↺



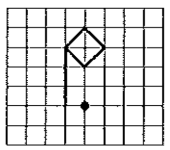
c. 90° ↺



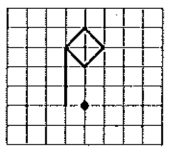
d. 90° ↺



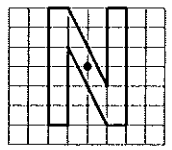
e. 270° ↺



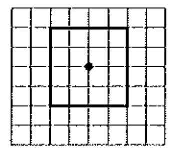
f. 360° ↺



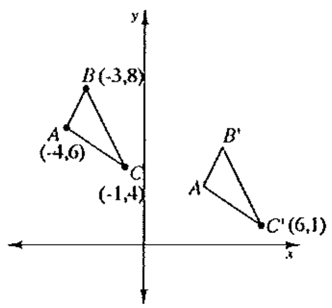
g. 180°



h. 90°



1-70. TRANSLATIONS ON A GRID



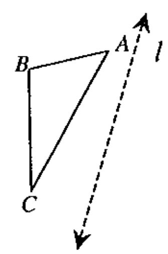
The formal name for a slide is a translation. (Remember that translation and transformation are different words.) $\Delta A'B'C'$ at right is the result of translating ΔABC.

a. Describe the translation. That is, how many units to the right and how many units down does the translation move the triangle?

b. On graph paper, plot ΔEFG if $E(4, 5)$, $F(1, 7)$, and $G(2, 0)$. Find the coordinates of $\Delta E'F'G'$ if $\Delta E'F'G'$ is translated the same way as ΔABC was in part (a).

c. $\Delta X'Y'Z'$ is the result of performing the same translation on ΔXYZ. If $X'(2, -3)$, $Y'(4, -5)$, and $Z'(0, 1)$, name the coordinates of X, Y, and Z.

1-71. Using tracing paper, reflect ΔABC across line l at right to form $\Delta A'B'C'$. One way to do this is to trace the triangle and the line of reflection. Fold the tracing paper along the line of reflection and trace the triangle on the other side of the line of reflection. Then unfold the tracing paper and observe the original triangle and its reflection. Label the reflection $\Delta A'B'C'$.

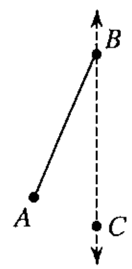


Problem continues on next page →

1-71. *Problem continued from previous page.*

Use your ruler to draw three dashed lines, each one joining one vertex of the original triangle with its image in the reflected triangle. Describe any geometric relationships you notice between each dashed line and the line of reflection.

1-72. How is the result from problem 1-71 useful? Consider reflecting a **line segment** (the portion of a line between two points) across a line that passes through one of its endpoints. For example, what would be the result when $\overline{AB}$ is reflected across $\overleftrightarrow{BC}$?



a. Copy $\overline{AB}$ and $\overleftrightarrow{BC}$ and draw $\overline{A'B}$, the reflection of $\overline{AB}$. When points A and A' are connected, what shape is formed?

b. Use what you know about reflection to make as many statements as you can about the shape from part (a). For example, are there any sides that must be the same length? Are there any angles that must be equal? Is there anything else special about this shape?

c. When two sides of a triangle have the same length, that triangle is called **isosceles**. In your Learning Log, describe all the facts you know about isosceles triangles. Be sure to include a diagram. Label this entry "Isosceles Triangles" and include today's date.

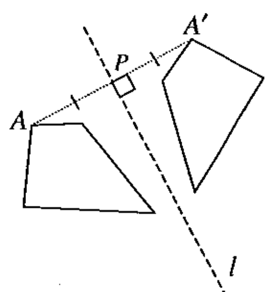
MATH NOTES

METHODS AND MEANINGS

More on Reflections

When a figure is reflected across a line of reflection, such as the quadrilateral at right, it appears that the figure is "flipped" over the line.

However, there are other interesting relationships between the original figure and its image.



For instance, the line segment connecting each image point with its corresponding point on the original figure is perpendicular to (meaning that it forms a right angle with) the line of reflection.

In addition, the line of reflection bisects (cuts in half) the line segment connecting each image point with its corresponding point on the original figure. For example, for reflection of the quadrilateral above, $AP = A'P$.

Review & Preview

1-73. Plot the following points on another sheet of graph paper and connect them in the order given. Then connect points A and D.

A(5, 3), B(5, –6), C(–4, –6), and D(–4, 3)

a. What is the resulting figure?

b. Find the area of shape *ABCD*.

c. If *ABCD* is rotated 90° clockwise (↻) about the origin to form $A'B'C'D'$, what are the coordinates of the vertices of $A'B'C'D'$?

1-74. Solve for the variable in each equation below. Show the steps leading to your answer.

a. $8x-22=-60$

b. $\frac{1}{2}x-37=-84$

c. $\frac{3x}{4}=\frac{6}{7}$

d. $9a+15=10a-7$

1-75. Graph the rule $y=-\frac{3}{2}x+6$ on graph paper. Label the points where the line intersects the x- and y-axes.

1-76. On graph paper, graph the line through the point (0, –2) with slope $\frac{4}{3}$.

a. Write the equation of the line.

b. Translate the graph of the line up 4 and to the right 3 units. What is the result? Write the equation for the resulting line.

c. Now translate the original graph down 5 units. What is the result? Write the equation for the resulting line.

1-77. Evaluate the expression $\frac{1}{4}k^5-3k^3+k^2-k$ for $k=2$.

1.2.4 What shapes can I create with triangles?

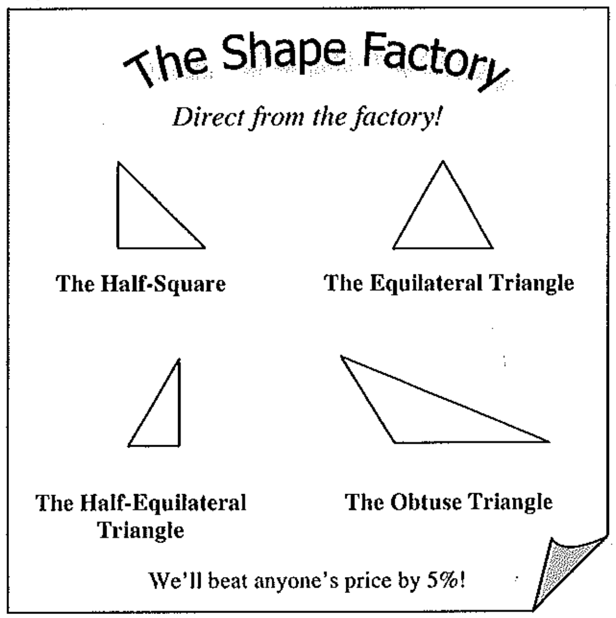
Using Transformations to Create Shapes

In Lesson 1.2.3, you practiced reflecting, rotating and translating figures. Since these are rigid transformations, the image always had the same size and shape as the original. In this lesson, you will combine the image with the original to make new, larger shapes from four basic "building-block" shapes.

As you create new shapes, consider what information the transformation gives you about the resulting new shape. By the end of this lesson, you will have generated most of the shapes that will be the focus of this course.

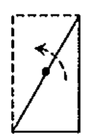
1-78. THE SHAPE FACTORY

The Shape Factory, an innovative new company, has decided to branch out to include new shapes. As Product Developers, your team is responsible for finding exciting new shapes to offer your customers. The current company catalog is shown at right.



Since your boss is concerned about production costs, you want to avoid buying new machines and instead want to reprogram your current machines.

The factory machines not only make all the shapes shown in the catalog, but they also are able to rotate or reflect a shape. For example, if the half-equilateral triangle is rotated 180° about the **midpoint** (the point in the middle) of its longest side, as shown at right, the result is a rectangle.



Your Task: Your boss has given your team until the end of this lesson to find as many new shapes as you can. Your team's reputation, which has suffered recently from a series of blunders, could really benefit by an impressive new line of shapes formed by a triangle and its transformations. For each triangle in the catalog, determine which new shapes are created when it is rotated or reflected so that the image shares a side with the original triangle. Be sure to make as many new shapes as possible. Use tracing paper or any other reflection tool to help.

Further Guidance

1-79. Since there are so many possibilities to test, it is useful to start by considering the shapes that can be generated just from one triangle. For example:

a. Test what happens when the half-square is reflected across each side. For each result (original plus image), draw a diagram and describe the shape that you get. If you know a name for the result, state it.

b. The point in the middle of each side is called its middle point, or **midpoint** for short. Try rotating the half-square 180° about the midpoint of each side to make a new shape. For each result, draw a diagram. If you know its name, write it near your new shape.

c. Repeat parts (a) and (b) with each of the other triangles offered by the Shape Factory.

Further Guidance section ends here.

1-80. EXTENSIONS

What other shapes can be created by reflection and rotation? Explore this as you answer the questions below. You can **investigate** these questions in any order. Remember that the resulting shape includes the original shape and all of its images. Remember to record and name each result.

- What if you reflect an equilateral triangle twice, once across one side and another time across a different side?
- What if an equilateral triangle is repeatedly rotated about one vertex so that each resulting triangle shares one side with another until new triangles are no longer possible? Describe the resulting shape.
- What if you rotate a trapezoid 180° around the midpoint of one if its non-parallel sides?

1-81. BUILDING A CATALOG

Your boss now needs you to create a catalog page that includes your shapes. Each entry should include a diagram, a name, and a description of the shape. List any special features of the shape, such as if any sides are the same length or if any angles must be equal. Use color and arrows to highlight these attributes in the diagram.

MATH NOTES

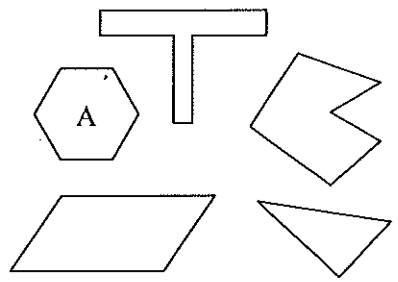
METHODS AND MEANINGS

Polygons

A **polygon** is defined as a two-dimensional closed figure made up of straight line segments connected end-to-end. These segments may not cross (intersect) at any other points.

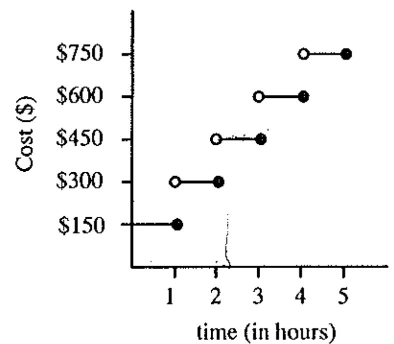
At right are some examples of polygons.

Shape A at right is an example of a **regular polygon** because its sides are all the same length and its angles have equal measure.



Review & Preview

1-82. According to the graph at right, how much money would it cost to speak to an attorney for 2 hours and 25 minutes?



1-83. Lourdes has created the following challenge for you: She has given you three of the four points necessary to determine a rectangle on a graph. She wants you to find the points that "complete" each of the rectangles below.

a. (–1, 3), (–1, 2), (9, 2)

b. (3, 7), (5, 7), (5, –3)

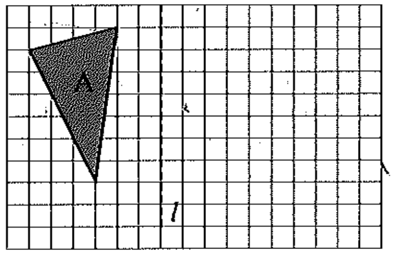
c. (–5, –5), (1, 4), (4, 2)

d. (–52, 73), (96, 73), (96, 1483)

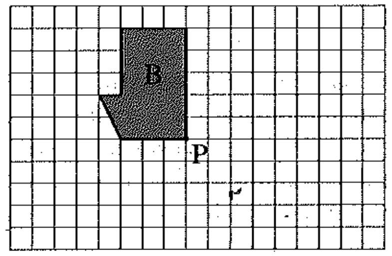
1-84. Find the area of the rectangles formed in parts (a), (b), and (d) of problem 1-83.

1-85. Copy the diagrams below on graph paper. Then find the result when each indicated transformation is performed.

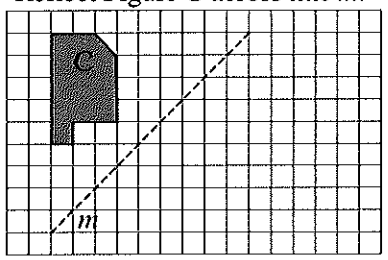
a. Reflect Figure A across line *l*.



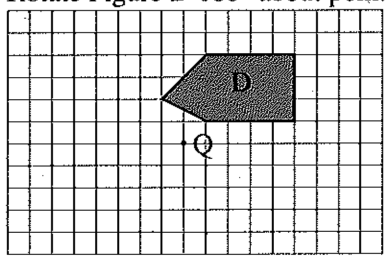
b. Rotate Figure B 90° clockwise (↻) about point P.



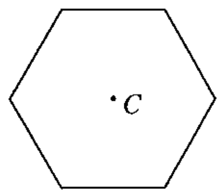
c. Reflect Figure C across line *m*.



d. Rotate Figure D 180° about point Q.

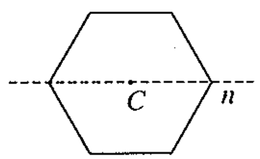


1-86. At right is a diagram of a **regular hexagon** with center *C*. A polygon is regular if all sides are equal and all angles are equal. Copy this shape on your paper, then answer the questions below.



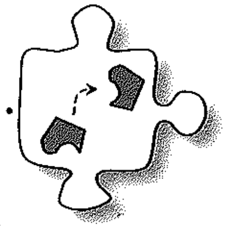
a. Draw the result of rotating the hexagon about its center 180°. Explain what happened. When this happens, the shape has **rotation symmetry**.

b. What is the result when the original hexagon is reflected across line *n*, as shown at right? A shape with this quality is said to have **reflection symmetry** and line *n* is a **line of symmetry** of the hexagon (not of the reflection).



c. Does a regular hexagon have any other lines of symmetry? That is, are there any other lines you could fold across so that both halves of the hexagon will match up? Find as many as you can.

1.2.5 What shapes have symmetry?



Symmetry

You have encountered symmetry several times in this chapter. For instance, the quilt your class created in Lesson 1.1.1 contained symmetry. The shapes you saw in the hinged mirrors during the kaleidoscope investigation were also symmetric. But so far, you have not developed a test for determining whether a polygon is symmetric. And since symmetry is related to transformations, how can we use transformations to describe this relationship? This lesson is designed to deepen your understanding of symmetry.

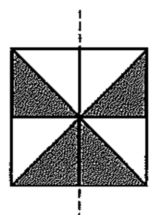
By the end of this lesson, you should be able to answer these target questions:

What is symmetry?

How can I determine whether or not a polygon has symmetry?

What types of symmetry can a shape have?

1-87. REFLECTION SYMMETRY



In problem 1-1, you created a quilt panel that had **reflection symmetry** because if the design were reflected across the line of symmetry, the image would be exactly the same as the original design. See an example of a quilt design that has reflection symmetry at right.

Obtain the Lesson 1.2.5 Resource Page. On it, **examine** the many shapes that will be our focus of study in this course. Which of these shapes have reflection symmetry? Consider this as you answer the questions below.

a. For each shape on the resource page, draw all the possible lines of symmetry. If you are not sure if a shape has reflection symmetry, trace the shape onto tracing paper and try folding the paper so that both sides of the fold match.

b. Which types of triangles have reflection symmetry?

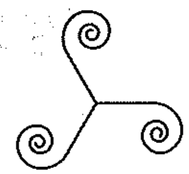
c. Which types of quadrilaterals (shapes with four sides) have reflection symmetry?

d. Which shapes on your resource page have more than three lines of symmetry?

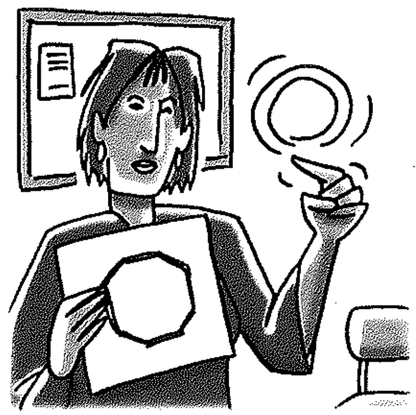
1-88. ROTATION SYMMETRY

In problem 1-87, you learned that many shapes have reflection symmetry. These shapes remain unaffected when they are reflected across a line of symmetry. However, some shapes remain unchanged when rotated about a point.

a. **Examine** the shape at right. Can this shape be rotated so that its image has the same position and orientation as the original shape? Trace this shape on tracing paper and test your conclusion. If it is possible, where is the point of rotation?



b. Jessica claims that she can rotate <u>all</u> shapes in such a way that they will not change. How does she do it?



c. Since all shapes can be rotated 360° without change, that is not a very special quality. However, the shape in part (a) above was special because it could be rotated less than 360° and still remain unchanged. A shape with this quality is said to have **rotation symmetry**.

But what shapes have rotation symmetry? **Examine** the shapes on your Lesson 1.2.5 Resource Page and identify those that have rotation symmetry.

d. Which shapes on the resource page have 90° rotation symmetry? That is, which can be rotated about a point 90° and remain unchanged?

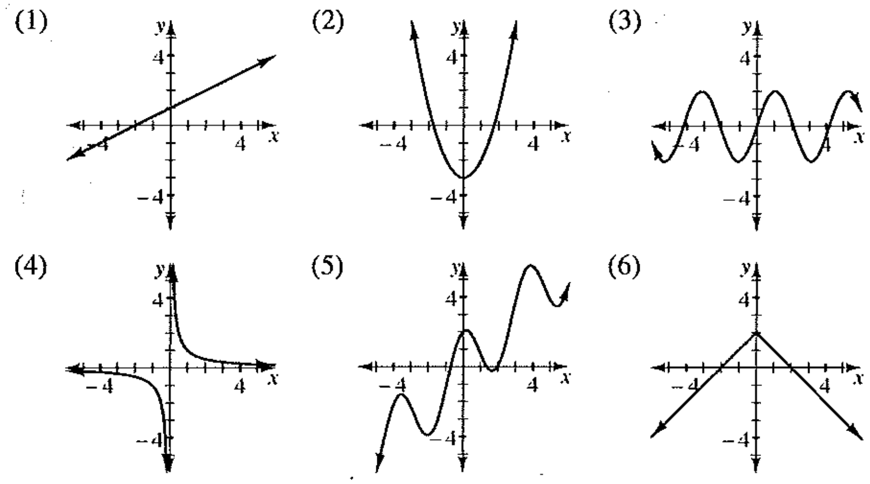
1-89. TRANSLATION SYMMETRY

In problems 1-87 and 1-88, you identified shapes that have reflection and rotation symmetry. What about translation symmetry? Is there an object that can be translated so that its end result is exactly the same as the original object? If so, draw an example and explain why it has **translation symmetry**.

1-90. CONNECTIONS WITH ALGEBRA

During this lesson, you have focused on the types of symmetry that can exist in geometric objects. But what about shapes that are created on graphs? What types of graphs have symmetry?

a. **Examine** the graphs below. Decide which have reflection symmetry, rotation symmetry, translation symmetry, or a combination of these.



b. If the y-axis is a line of symmetry of a graph, then its rule is referred to as **even**. Which of the graphs in part (a) have an even rule?

c. If the graph has rotation symmetry about the origin (0, 0), its rule is called **odd**. Which of the graphs in part (a) have an odd rule?

1-91. Reflect on what you have learned during this lesson. In your Learning Log, answer the questions posed at the beginning of this lesson, reprinted below. When helpful, give examples and draw a diagram.

What is symmetry?

How can I determine whether or not a polygon has symmetry?

What types of symmetry can a shape have?

METHODS AND MEANINGS

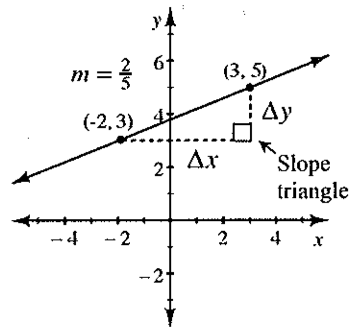
MATH NOTES

Slope of a Line and Parallel and Perpendicular Slopes

During this course, you will use your algebra tools to learn more about shapes. One of your algebraic tools that can be used to learn about the relationship of lines is slope. Review what you know about slope below.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x}$$

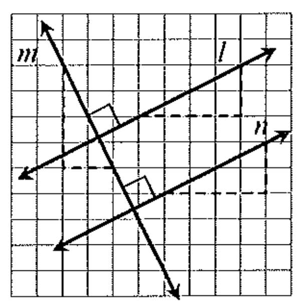
The **slope** of a line is the ratio of the change in y (Δy) to the change in x (Δx) between any two points on the line. It indicates both how steep the line is and its direction, upward or downward, left to right.



Lines that point upward from left to right have positive slope, while lines that point downward from left to right have negative slope. A horizontal line has zero slope, while a vertical line has undefined slope. The slope of a line is denoted by the letter m when using the $y = mx + b$ equation of a line.

One way to calculate the slope of a line is to pick two points on the line, draw a slope triangle (as shown in the example above), determine Δy and Δx, and then write the slope ratio. Be sure to verify that your slope correctly resulted in a negative or positive value based on its direction.

Parallel lines lie in the same plane (a flat surface) and never intersect. They have the same steepness, and therefore they grow at the same rate. Lines l and n at right are examples of parallel lines.



On the other hand, **perpendicular lines** are lines that intersect at a right angle. For example, lines m and n at right are perpendicular, as are lines m and l. Note that the small square drawn at the point of intersection indicates a right angle.

The **slopes of parallel lines** are the same. In general, the slope of a line parallel to a line with slope m is m.

The **slopes of perpendicular lines** are opposite reciprocals. For example, if one line has slope $\frac{4}{5}$, then any line perpendicular to it has slope $-\frac{5}{4}$. If a line has slope -3, then any line perpendicular to it has slope $\frac{1}{3}$. In general, the slope of a line perpendicular to a line with slope m is $-\frac{1}{m}$.

Review & Preview

1-92. On graph paper, graph each of the lines below on the same set of axes. What is the relationship between lines (a) and (b)? What about between (b) and (c)?

a. $y=\frac{1}{3}x+4$ b. $y=-3x+4$ c. $y=-3x-2$

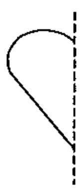
1-93. The length of a side of a square is $5x+2$ units. If the perimeter is 48 units, complete the following.

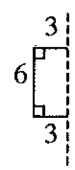
a. Write an equation to represent this information.

b. Solve for x.

c. What is the area of the square?

1-94. When the shapes below are reflected across the given line of reflection, the original shape and the image (reflection) create a new shape. For each reflection below, name the new shape that is created.

a. 

b. 

c. 

d. Use this method to create your own shape that has reflection symmetry. Add additional lines of symmetry. Note that the dashed lines of reflection in the figures above become lines of symmetry in the new shape.

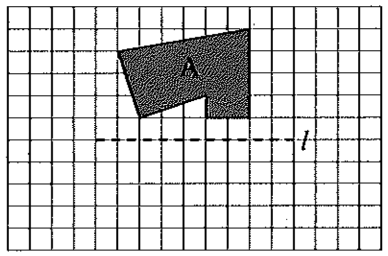
1-95. If a triangle has two equal sides, it is called **isosceles** (pronounced eye-SOS-a-lees). "Iso" means "same" and "sceles" is related to "scale." Decide whether each triangle formed by the points below is isosceles. Explain how you decided.

a. (6, 0), (0, 6), (6, 6) b. (–3, 7), (–5, 2), (–1, 2)

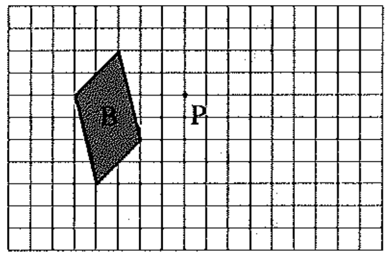
c. (4, 1), (2, 3), (9, 2) d. (1, 1), (5, –3), (1, –7)

1-96. Copy the diagrams below on graph paper. Then find each result when each indicated transformation is performed.

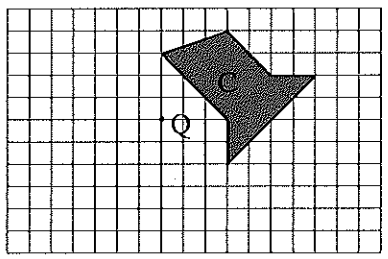
a. Reflect A across line *l*.



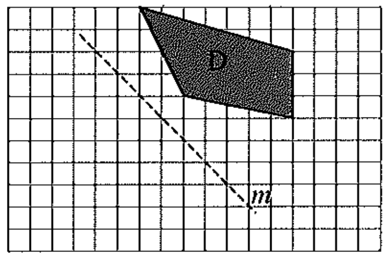
b. Rotate B 90° counterclockwise (↺) about point P.



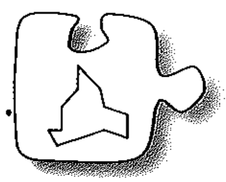
c. Rotate C 180° about point Q.



d. Reflect D across line *m*.



1.3.1 How can I classify this shape?



Attributes and Characteristics of Shapes

In Lesson 1.2.4, you generated a list of shapes formed by triangles and in Lesson 1.2.5, you studied the different types of symmetries that a shape can have. In Section 1.3, you will continue working with shapes to learn more about their attributes and characteristics. For example, which shapes have sides that are parallel? And which of our basic shapes are equilateral?

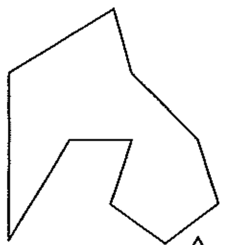
By the end of this lesson you should have a greater understanding about the attributes that make shapes alike and different. Throughout the rest of this course you will study these qualities that set shapes apart as well as learn how shapes are related.

1-97. INTRODUCTION TO THE SHAPE BUCKET

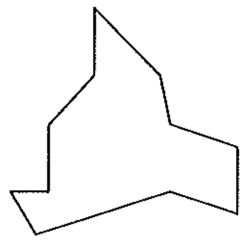
Obtain a Shape Bucket and a Lesson 1.3.1B Resource Page from your teacher. The Shape Bucket contains most of the basic geometric shapes you will study in this course. Count the items and verify that you have all 16 shapes. Take the shapes out and notice the differences between them. Are any alike? Are any strangely different?

Once you have examined the shapes in your bucket, work as a team to build the composite figures below (also shown on the resource page). Composite figures are made by combining two or more shapes to make a new figure. On the Lesson 1.3.1B Resource Page, show the shapes you used to build the composite shapes by filling in their outlines within each composite shape.

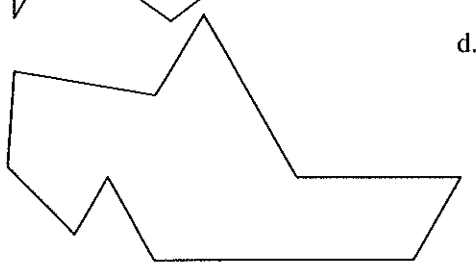
a.



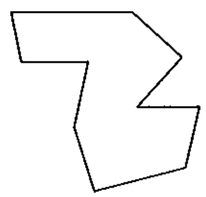
b.



c.



d.

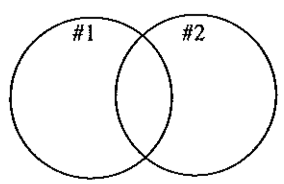


1-98. VENN DIAGRAMS

Obtain a Venn diagram from your teacher.

a. The **left** circle of the Venn diagram, Circle #1, will represent the attribute "has at least one pair of parallel sides" and the **right** side, Circle #2, will represent the attribute "has at least two sides of equal length" as shown above. Sort through the shapes in the Shape Bucket and decide as a team where each shape belongs. Be sure to record your solution on paper. As you discuss this problem with your teammates, **justify** your statements with reasons such as, "I think this shape goes here **because**…"

#1: Has at least one pair of parallel sides

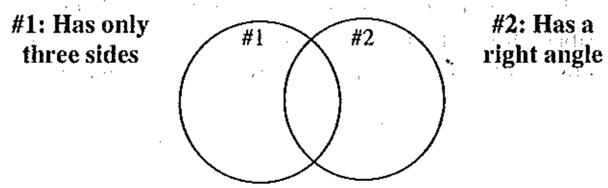


#2: Has at least two sides of equal length

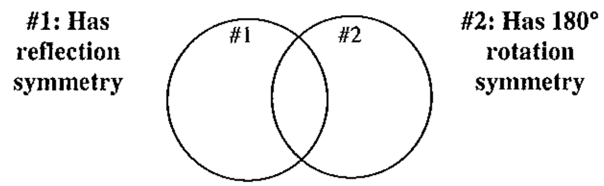
Problem continues on next page →

1-98. *Problem continued from previous page.*

b. Next, reclassify the shapes for the new Venn diagram shown below. Describe each region in a sentence.



c. Finally, reclassify the shapes for the new Venn diagram shown below. Describe each region in a sentence.

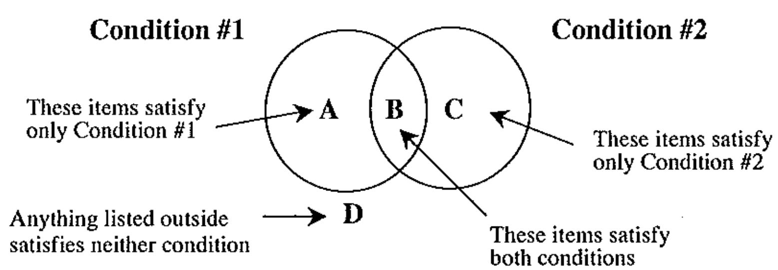


MATH NOTES

METHODS AND MEANINGS

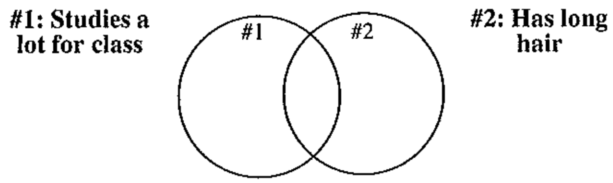
Venn Diagrams

A Venn diagram is a tool used to classify objects. It is usually composed of two or more circles that represent different conditions. An item is placed or represented in the Venn diagram in the appropriate position based on the conditions it meets. See the example below:



Review & Preview

1-99. Copy the Venn diagram below on your paper. Then show where each person described should be represented in the diagram. If a portion of the Venn diagram remains empty, describe the qualities a person would need to belong there.



a. Carol: *"I rarely study and enjoy braiding my long hair."*

b. Bob: *"I never do homework and have a crew cut."*

c. Pedro: *"I love joining after school study teams to prepare for tests and I like being bald!"*

1-100. Solve the equations below for x, if possible. Be sure to check your solution.

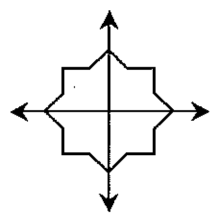
a. $\frac{3x-1}{4} = -\frac{5}{11}$

b. $(5-x)(2x+3)=0$

c. $6-5(2x-3)=4x+7$

d. $\frac{3x}{4}+2=4x-1$

1-101. **Examine** the figure graphed on the axes at right.

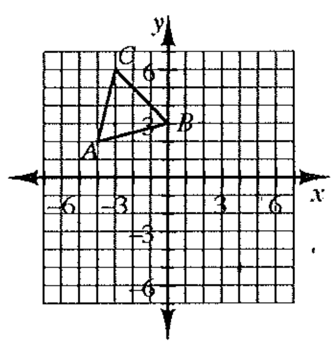


a. What happens when you rotate this figure about the origin 90°? 45°? 180°?

b. What other angle could the figure at right be rotated so that the shape does not appear to change?

c. What shape will stay the same no matter how many degrees it is rotated?

1-102. Copy ΔABC at right on graph paper.



a. Rotate ΔABC 90° counter-clockwise (↺) about the origin to create $\Delta A'B'C'$. Name the coordinates of C'.

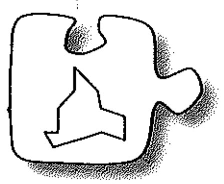
b. Reflect ΔABC across the vertical line $x=1$ to create $\Delta A''B''C''$.

c. Translate ΔABC so that A''' is at (4, –5). Name the coordinates of B'''.

1-103. Copy the table below, complete it, and write a rule relating x and y.

x	–3	–2	–1	0	1	2	3	4
y	–7			2	5			14

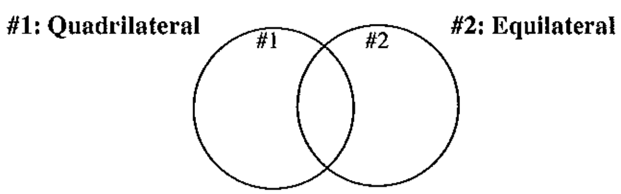
1.3.2 How can I describe it?



More Characteristics of Shapes

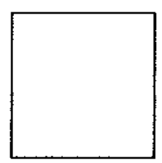
In Lesson 1.3.1, you used shapes to build new, unique, composite shapes. You also started to analyze the attributes (qualities) of shapes. Today you will continue to look at their attributes as you learn new vocabulary.

1-104. Using your Venn diagram Resource Page from Lesson 1.3.1, categorize the shapes from the Shape Bucket in the Venn diagram as shown below. Record your results on paper.



1-105. DESCRIBING A SHAPE

How can you describe a square? With your class, find a way to describe a square using its attributes (special qualities) so that anyone could draw a square based on your description. Be as complete as possible. You may not use the word "square" in the description.



Square

1-106. Each shape in the bucket is unique; that is, it differs from the others. You will be assigned a few shapes to describe **as completely as possible** for the class. But what is a complete description? As you work with your team to create a complete description, consider the questions below.

- What do you notice about your shape?
- What makes it different from other shapes?
- If you wanted to describe your shape to a friend on the telephone who cannot see it, what would you need to include in the description?

1-107. GEOMETRY TOOLKIT

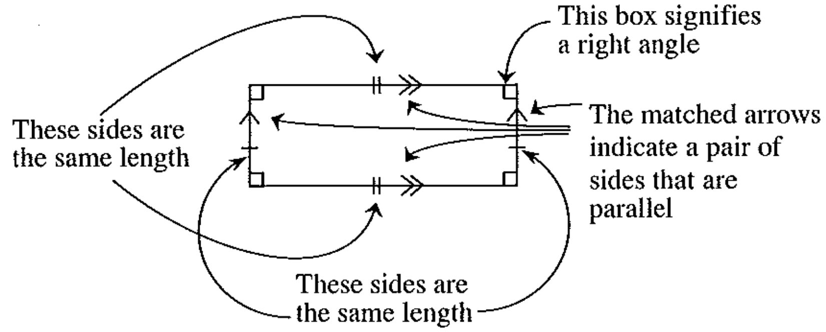
Obtain the Lesson 1.3.2A Resource Page entitled "Shapes Toolkit" from your teacher.

a. Label each shape that you have learned about so far with its geometrical name.

b. In the space provided, describe the shape based on the descriptions generated from problem 1-106. Leave space so that later observations can be added for each shape. Note that the description for **rectangle** has been provided as an example.

c. On the diagram for each shape, mark sides that must have equal length or that must be parallel. Also mark any angles that measure 90°. See the descriptions below.

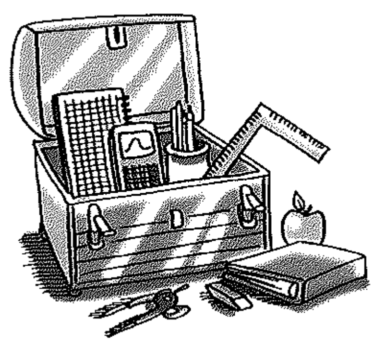
- To show that two sides have the same length, use "tick marks" on the sides. However, to show that one pair of equal sides may not be the same length as the other pair of equal sides, you should use one tick mark on two of the opposite, equal sides and two tick marks on the other two opposite, equal sides, as shown below.



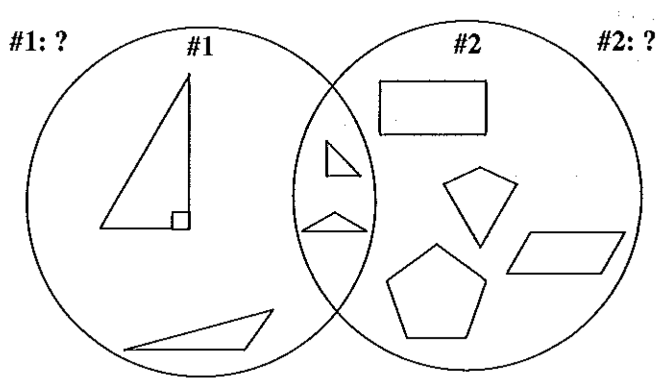
- To show that the rectangle has two pairs of parallel sides, use one ">" mark on one pair of parallel sides and two ">>" marks on the other two parallel sides, as shown above.

- Also mark any right angles by placing a small square at the right angle vertex (the corner). See the example above.

d. The Shapes Toolkit Resource Page is the first page of a special information organizer, called your Geometry Toolkit, which you will be using for this course. It is a reference tool that you can use when you need to remember the name or description of a shape. Find a safe place in your Geometry binder to keep your Toolkit.



1-108. **Examine** the Venn diagram below.

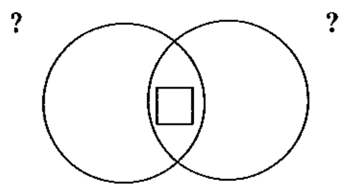


a. What attribute does each circle represent? How can you tell?

b. Where would the equilateral hexagon from your Shape Bucket go in this Venn? What about the trapezoid? Justify your **reasoning**.

c. Create another shape that would belong outside both circles. Does your shape have a name that you have studied so far? If not, give it a new name.

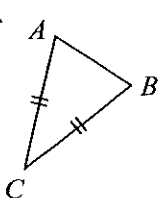
1-109. Elizabeth has a Venn diagram that she started at right. It turns out that the **only** shape in the Shape Bucket that could go in the intersection (where the two circles overlap) is a square! What are the possible attributes that her circles could represent? Discuss this with your team and be ready to share your ideas with the class.



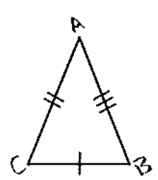
Review & Preview

1-110. If no sides of a triangle have the same length, the triangle is called **scalene** (pronounced SCALE-een). However, if the triangle has two sides that are the same length, the triangle is called **isosceles** (pronounced eye-SOS-a-lees). Use the markings in each diagram below to decide if ΔABC is isosceles or scalene. Assume the diagrams are not drawn to scale.

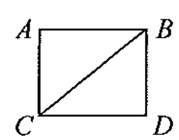
a.



b.



c. *ABDC* is a square



1-111. In Lesson 1.3.3, you will start to learn how to make predictions in mathematics using probability. Decide if the following events must happen, cannot happen, or possibly will happen.

a. You will drink water some time today.

b. An earthquake will strike in your region tomorrow.

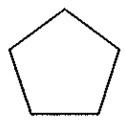
c. You will read a book.

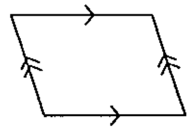
d. If a shape is a trapezoid, it will have a pair of parallel sides.

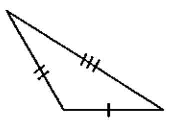
e. You will shave your head this month.

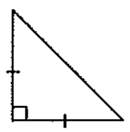
1-112. Without referring to your Shapes Toolkit, see if you can recall the names of each of the shapes below. Then check your answers with definitions from your Shapes Toolkit. How did you do?

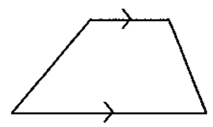
a.


b.


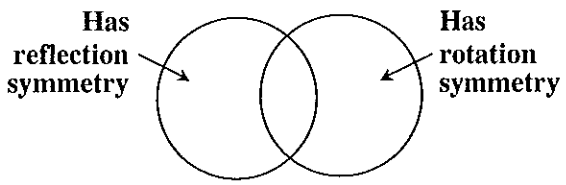
c.


d.


e.


f.


1-113. Copy the Venn diagram at right onto your paper. Then carefully place each capitalized letter of the alphabet below into your Venn diagram based on its type of symmetry.



A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z

1-114. **Multiple Choice:** Which equation below correctly represents the relationship of the sides given in the diagram at right?

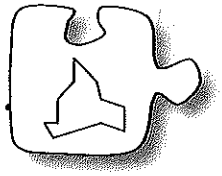
a. $3x-2+2x+17=360°$

b. $3x-2+2x+17=180°$

c. $3x-2+2x+17=90°$

d. $3x-2=2x+17$

1.3.3 What are the chances?



Introduction to Probability

You have learned a great deal about the shapes in the Shape Bucket and with that knowledge can draw conclusions and make predictions. For example, if you are told that a shape is not a triangle, you could predict that it might be a quadrilateral.

One mathematical tool that can be used to predict how likely it is that something will happen is **probability**, the formal word for chance. There are many examples of the uses of probability in our lives. One such example is the weather. If a forecast predicts that there is a 95% likelihood of rain, you would probably bring a raincoat and umbrella with you for that day. If you buy a lottery ticket, the small print on the back of your ticket usually tells you the probability of randomly selecting winning numbers, which, unfortunately, is very low.

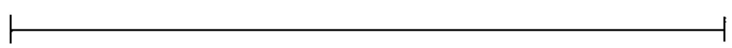
Your task today is to learn the principles of probability so that you can make predictions.

1-115. WHAT'S THE CHANCE OF THAT HAPPENING?

In your teams, discuss the following list of events and consider the likelihood of each event occurring:

1. Your school's basketball team having a winning season this year
2. The sky being cloudy tomorrow
3. Your math teacher assigning homework tonight
4. The sun setting this evening
5. Meeting a famous person on the way home
6. Snow falling in Arizona in July

a. Copy the probability line shown below on your paper. Where should these events lie? Place each event listed above at the appropriate location on the probability line. Do any events have no chance of happening? These should be placed on the left end of the number line. If you are sure the event will happen, place it on the right end.



NO CHANCE OF HAPPENING | MIGHT HAPPEN | CERTAIN TO HAPPEN

Problem continues on next page →

1-115. *Problem continued from previous page.*

b. With your team, make up three new situations and place them on the probability line. One event should be certain to happen, one event should have no chance of happening, and one should possibly occur.

c. **Probability** can be written as a ratio, as a decimal, or as a percent. When using ratios or decimals, probability is expressed using numbers from 0 to 1. An event that has no chance of happening is said to have a probability of 0. An event that is certain to happen is said to have a probability of 1. Events that "might happen" will have values somewhere between 0 and 1. See the Math Notes box at the end of this lesson for more information about probability.

Which events from part (a) have a probability of 0? A probability of 1? Which event has closest to a 25% chance? 50% chance?

1-116. Probability is used to make predictions. For example, if you were to reach into the Shape Bucket and randomly pull out a shape, you could use probability to predict the chances of the shape having a right angle. Since there are 16 total shapes and 4 that have right angles, the probability is:

$$P(\text{right angle}) = \frac{\text{4 shapes with right angles}}{\text{16 total shapes}} = \frac{4}{16} = \frac{1}{4} = 0.25 = 25\%$$

The example above shows all forms of writing probability: $\frac{4}{16}$ (read "4 out of 16") is the probability as a ratio, 0.25 is its decimal form, and 25% is its equivalent percent.

Find the probabilities of randomly selecting the following shapes from a Shape Bucket that contains all 16 basic shapes.

a. P(quadrilateral)

b. P(shape with an obtuse angle)

c. P(equilateral triangle)

d. P(shape with parallel sides)

1-117. What else can you predict about the shape randomly pulled from the Shape Bucket?

a. In your teams, describe at least two new probability conditions for the shapes in the bucket (like those in problem 1-116). One of the events should have a probability of 0 or a probability of 1. Write each P(condition) on a separate blank piece of paper and give it to your teacher.

b. Your teacher will select conditions generated by the class in part (a). For each condition, work with your team to determine the probability.

1-118. Laura's team has a shape bucket that is missing several pieces. While talking to her teammates, Laura claims, "The probability of choosing a shape with four right angles is ¼ ."

Barbara adds, "Well...there are 2 shapes that have four right angles."

Montry responds, "I think you are both right."

What does Montry mean? Explain how Laura and Barbara could both be correct.

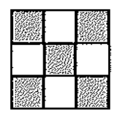
1-119. What else can probability be used to predict? Analyze each of the situations below:

a. What is the probability of randomly drawing a face card (king, queen or jack) from a deck of 52 cards, when you draw one card?

b. Eduardo has in his pocket \$1 in pennies, \$1 in nickels, and \$1 in dimes. If he randomly pulls out just one coin, what is the probability that he will pull out a dime?

c. P(rolling an 8) with one regular die if you roll the die just once.

d. P(dart hitting a shaded region) if the dart is randomly thrown and hits the target at right.



target

1-120. In your Learning Log, write an entry describing what you learned today about probability. Be sure to show an example. Title this entry "Probability" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Ratio and Probability

A comparison of two quantities is called a **ratio**. A ratio can be written as:

$$a : b \quad or \quad \frac{a}{b} \quad or \quad \text{"}a \text{ to } b\text{"}$$

Each ratio has a numeric value that can be expressed as a fraction or decimal. For example, a normal deck of 52 playing cards has 13 cards with hearts. To compare the number of cards with hearts to the total number of playing cards, you could write:

$$\frac{\text{Number of Hearts}}{\text{Total Number of Cards}} = \frac{13}{52} = \frac{1}{4} = 0.25\,.$$

Probability is a measure of the likelihood that an event will occur at random. It is expressed using numbers with values that range from 0 to 1, or from 0% to 100%. For example, an event that has no chance of happening is said to have a probability of 0 or 0%. An event that is certain to happen is said to have a probability of 1 or 100%. Events that "might happen" have values somewhere between 0 and 1 or between 0% and 100%.

The probability of an event happening is written as the ratio of the number of ways that the desired outcome can occur to the total number of possible outcomes (assuming that each possible outcome is equally likely).

$$P(\text{event}) = \frac{\text{Number of Desired Outcomes}}{\text{Total Possible Outcomes}}$$

For example, on a standard die, P(5) means the probability of rolling a 5. To calculate the probability, first determine how many possible outcomes exist. Since a die has six different numbered sides, the number of possible outcomes is 6. Of the six sides, only one of the sides has a 5 on it. Since the die has an equal chance of landing on any of its six sides, the probability is written:

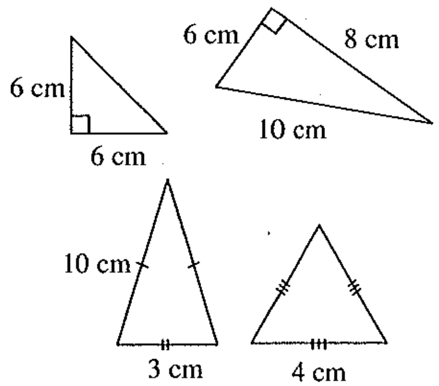
$$P(5) = \frac{\text{1 side with the number five}}{\text{6 total sides}} = \frac{1}{6} \text{ or } 0.1\overline{6} \text{ or approximately } 16.7\%$$

Review & Preview

1-121. **Examine** the shapes listed on your Shapes Toolkit. Name a shape that meets the following conditions: It has a pair of congruent sides, it has one right angle, and it has three vertices.

1-122. Augustin is in line to choose a new locker at school. The locker coordinator has each student reach into a bin and pull out a locker number. There is one locker at the school that all the kids dread! This locker, # 831, is supposed to be haunted, and anyone who has used it has had strange things happen to him or her! When it is Augustin's turn to reach into the bin and select a locker number, he is very nervous. He knows that there are 535 lockers left and locker # 831 is one of them. What is the probability that Augustin reaches in and pulls out the dreaded locker # 831? Should he be worried? Explain.

1-123. The four triangles at right are placed in a bag. If you reach into the bag without looking and pull out one triangle at random, what is the probability that:

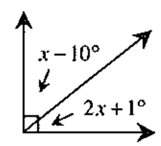


a. the triangle is scalene?

b. the triangle is isosceles?

c. at least one side of the triangle is 6 cm?

1-124. Rosalinda examined the angles at right and wrote the equation below.



$$(2x+1°)+(x-10°)=90°$$

a. Does her equation make sense? If so, explain why her equation must be true. If it is not correct, determine what is incorrect and write the equation.

b. If you have not already done so, solve her equation, clearly showing all your steps. What are the measures of the two angles?

c. Verify that your answer is correct.

1-125. Copy the table below on your paper and complete it for the rule $y=x^2+2x-3$. Then graph and connect the points on graph paper. Name the x-intercepts.

x	−4	−3	−2	−1	0	1	2
y							

Chapter 1 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned in this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following two subjects. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Connections: How are the topics, ideas, and words that you learned in previous courses connected to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

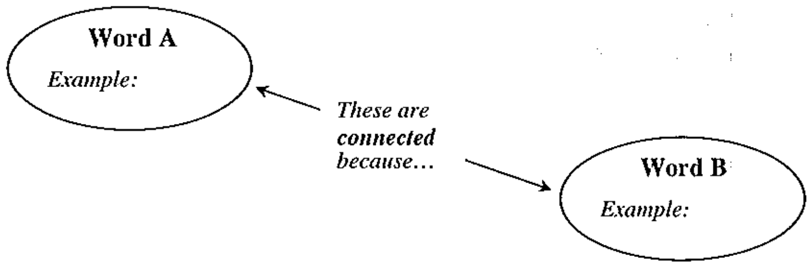
The following is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

angle	area	conjecture
equilateral	graph	image
line segment	measurement	parallel
perimeter	perpendicular	polygon
probability	protractor	random
ratio	reflection	regular hexagon
right angle	rotation	slope
solve	straight angle	symmetry
transformation	translation	triangle
Venn diagram	vertex (vertices)	

Problem continues on next page →

② *Problem continued from previous page.*

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the example below. A word can be connected to any other word as long as there is a justified connection. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will direct you how to do this. Your teacher may give you a "GO" page to work on (or you can download this from www.cpm.org). "GO" stands for "Graphic Organizer," a tool you can use to organize your thoughts and communicate your ideas clearly.

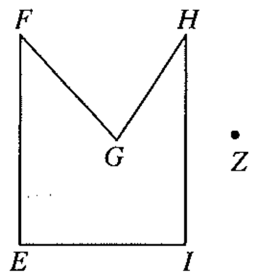
④ WHAT HAVE I LEARNED?

This section will help you recognize those types of problems you feel comfortable with and those you need more help with. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

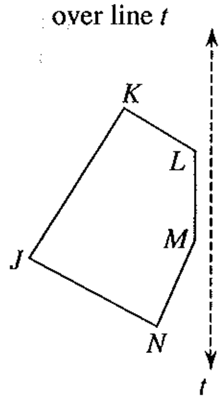
Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on similar problems.

CL 1-126. Trace the figures in parts (a) and (b) onto your paper and perform the indicated transformations. Copy the figure from part (c) onto graph paper and perform the indicated transformation. Label each image with prime notation ($A \to A'$).

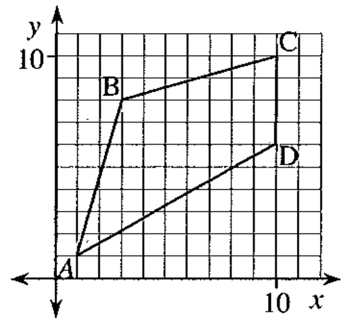
a. Rotate *EFGHI* 90° clockwise ↻ about point *Z*



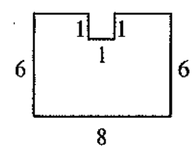
b. Reflect *JKLMN* over line *t*



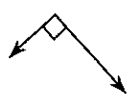
c. Translate *ABCD* down 5 units and right 3 units



CL 1-127. Assume that all angles in the diagram at right are right angles and that all the measurements are in centimeters. Find the perimeter of the figure.



CL 1-128. Estimate the measures of the angles below. Are there any that you know for sure?

a. 

b. 

c. 

d. 

CL 1-129. **Examine** the angles in problem CL 1-128. If these four angles are placed in a bag, what is the probability of randomly selecting:

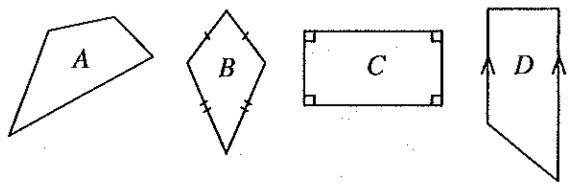
a. An acute angle

b. An angle greater than 60°

c. A 90° angle

d. An angle less than or equal to 180°

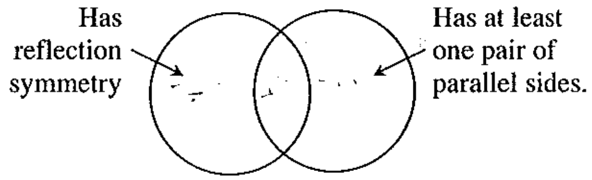
CL 1-130. **Examine** the shapes at right.



a. Describe what you know about each shape based on the information provided in the diagram.

b. Name the shape.

c. Decide where each shape would be placed in the Venn diagram at right.



CL 1-131. Solve each equation below. Check your solution.

a. $3x - 12 + 10 = 8 - 2x$

b. $\frac{x}{7} = \frac{3}{2}$

c. $5 - (x + 7) + 4x = 7(x - 1)$

d. $x^2 + 11 = 36$

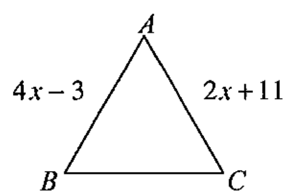
CL 1-132. Graph and connect the points in the table below. Then graph the equation in part (b) on the same set of axes. Also, find the equation for the data in the table.

a.

x	–4	–3	–2	–1	0	1	2	3	4	5	6
y	–5	–3	–1	1	3	5	7	9	11	13	15

b. $y = x^2 + x - 2$

CL 1-133. ΔABC at right is equilateral. Use what you know about an equilateral triangle to write and solve an equation for x. Then find the perimeter of ΔABC.



CL 1-134. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

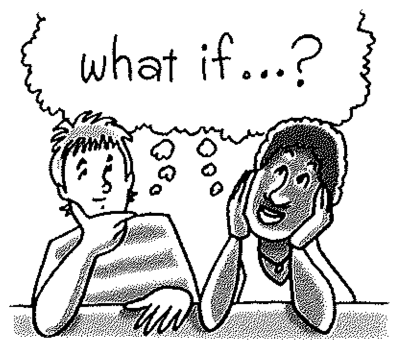
⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: investigating, examining, reasoning and justifying, visualizing, and choosing a strategy/tool. These are some of the ways in which you think while trying to make sense of a concept or attempting to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **Investigating**. Read the description of this Way of Thinking at right.

Investigating

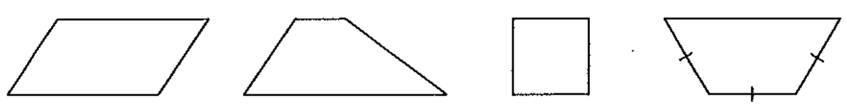
Investigating is when you create your own questions and come up with something to do to answer those questions. You are often investigating when you ask questions for which you have not learned a formula or process. Common thoughts you might have when you are investigating are: *"I wonder what would happen if I . . . "* or, *"I might find the answer to my question if I . . ." or, "That didn't quite answer my question; now I'll try . . . "*



Think about the **investigating** that you have done in this chapter. When have you tried something that you weren't specifically asked to do? What did you do when testing your idea did not work as you expected? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any of the **investigations** you have made with the rest of the class.

Once your discussion is complete, think about the way you think as you answer the following problems.

a. In the Shape Factory, you **investigated** what shapes you could create with four basic triangles. But **what if** you had started with quadrilaterals instead? **Investigate** this question as you find all shapes that could be made by rotating or reflecting the four quadrilaterals below. Remember that once rotated or reflected, the image should share a side with the original shape. Use tracing paper to help generate the new shapes. If you know the name of the new figure, state it.



PARALLELOGRAM GENERAL TRAPEZOID SQUARE SPECIAL TRAPEZOID

Problem continues on next page →

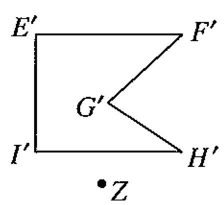
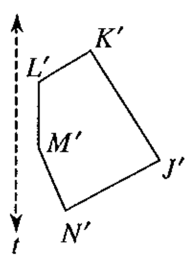
⑤ *Problem continued from previous page.*

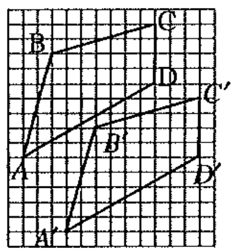
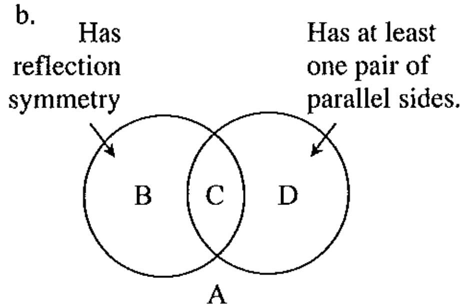
b. Now write three "What if ...?" questions that would extend your **investigation** in part (a).

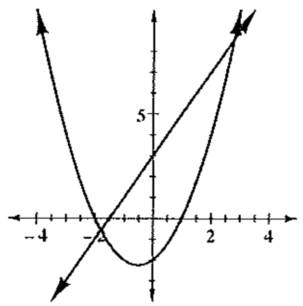
c. Now choose <u>one</u> question from part (b) to **investigate**. Decide how you will **investigate** your question. For example, will you need tracing paper? Do you need to make a table and record information?

After you have answered your question, write the results of your **investigation** clearly so that someone else can understand what question you selected, how you **investigated** your question, and any conclusions you made.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solutions	Need Help?	More Practice
CL 1-126.	a.  b. 	Lesson 1.2.2, Lessons 1.2.2 and 1.2.3 Math Notes boxes, and Problem 1-50	Problems 1-50, 1-51, 1-61, 1-64, 1-68, 1-69, 1-70, 1-71, 1-73, 1-85, 1-86, 1-94, 1-96, 1-102

Problem	Solutions	Need Help?	More Practice
CL 1-126 continued	c. 	Lesson 1.2.2, Lessons 1.2.2 and 1.2.3 Math Notes boxes, and Problem 1-50	Problems 1-50, 1-51, 1-61, 1-64, 1-68, 1-69, 1-70, 1-71, 1-73, 1-85, 1-86, 1-94, 1-96, 1-102
CL 1-127.	Perimeter = 30 centimeters	Lesson 1.1.3 Math Notes box	Problems 1-25, 1-44, 1-46, 1-65, 1-93
CL 1-128.	a. 90° b. ≈ 100° c. 180° d. ≈ 30°	Lesson 1.1.5 Math Notes box Problem 1-39	Problems 1-39, 1-42, 1-55
CL 1-129.	a. $\frac{1}{4}$ b. $\frac{3}{4}$ c. $\frac{1}{4}$ d. 1	Lesson 1.3.3 Math Notes box Problem 1-115	Problems 1-116, 1-118, 1-119, 1-122, 1-123
CL 1-130.	a. A: four sides make it a generic quadrilateral; B: Two pairs of equal sides make it a kite; C: Four right angles and a pair of parallel sides make it a rectangle; D: A quadrilateral with two parallel sides is a trapezoid. b. 	Lesson 1.3.1 Math Notes box, Shapes Toolkit, catalog from problem 1-81, problem 1-107	Problems 1-3, 1-15, 1-86, 1-87, 1-94, 1-98, 1-99, 1-101, 1-104, 1-105, 1-108, 1-109, 1-110, 1-112, 1-113, 1-121, 1-123

Problem	Solutions	Need Help?	More Practice
CL 1-131.	a. $x = 2$ b. $x = \frac{21}{2}$ c. $x = \frac{5}{4}$ d. $x + = 5$	Lesson 1.1.4 Math Notes box	Problems 1-17, 1-25, 1-32, 1-34, 1-45, 1-57, 1-74, 1-100, 1-124
CL 1-132.	a. $y = 2x + 3$ 	Lesson 1.2.1 Math Notes box	Problems 1-5, 1-16, 1-27, 1-43, 1-54, 1-62, 1-75, 1-76, 1-92, 1-103, 1-125
CL 1-133.	$4x - 3 = 2x + 11$, so $x = 7$. Therefore, each side is $4(7) - 3 = 25$ units long, and the perimeter is 75 units.	Shapes Toolkit, problems 1-15 and 1-124	Problems 1-25, 1-56, 1-65, 1-93, 1-114, 1-124