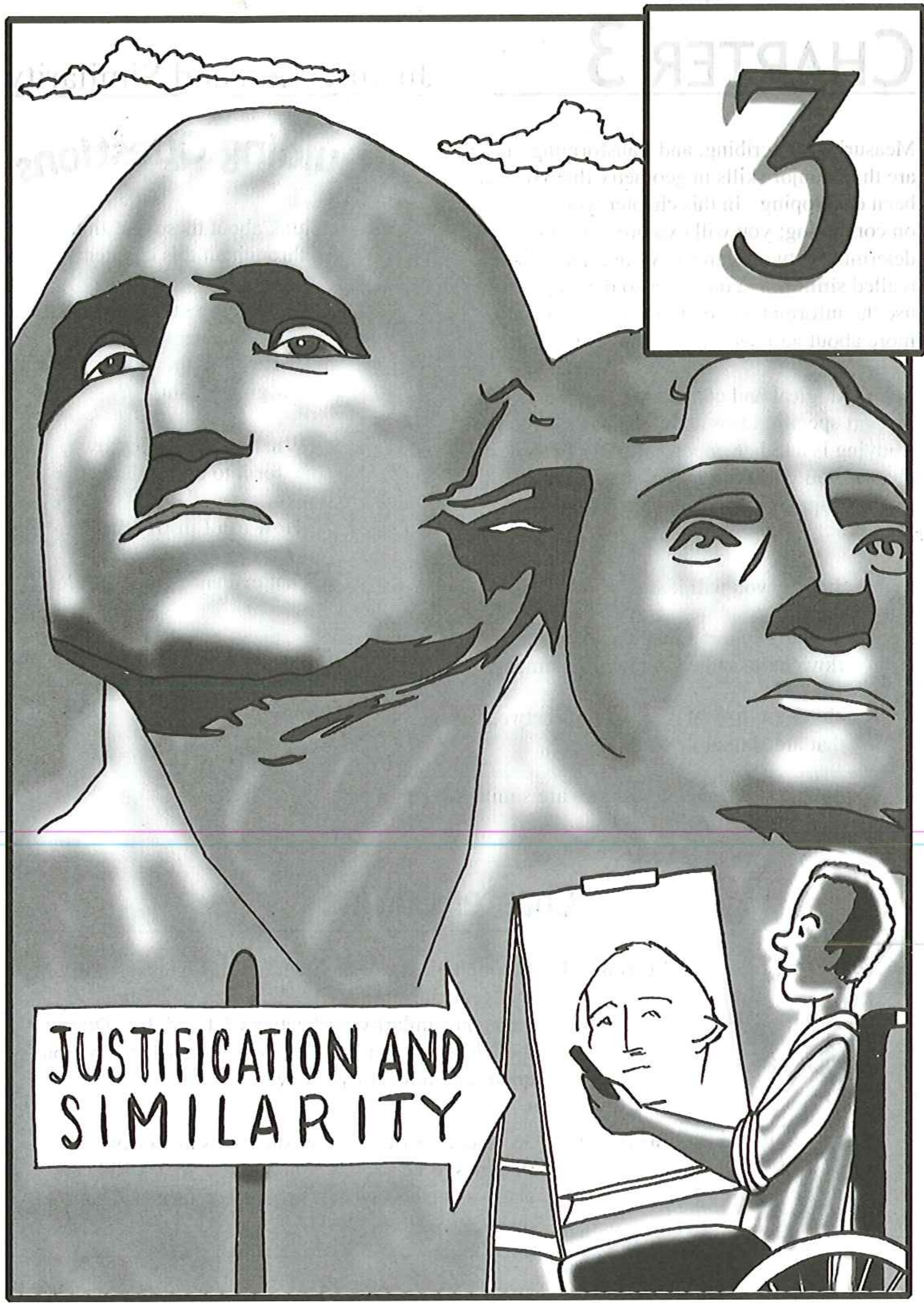

3
JUSTIFICATION AND SIMILARITY

CHAPTER 3 Justification and Similarity

Measuring, describing, and transforming: these are three major skills in geometry that you have been developing. In this chapter, you will focus on comparing; you will explore ways to determine if two figures have the same shape (called **similar**). You will also develop ways to use the information about one figure to learn more about another that has the same shape.

Making logical and convincing arguments that support specific ideas about the shapes you are studying is another important skill. In this chapter you will learn how you can document facts to support a conclusion in a flowchart.

In this chapter, you will learn:

- how to support a mathematical statement using flowcharts and conditional statements.
- about the special relationships between shapes that are similar or congruent.
- how to determine if triangles are similar or congruent.

Guiding Questions

Think about these questions throughout this chapter:

Is it still the same shape?

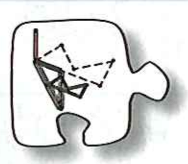
How did the shape grow or shrink?

What do I need to show for it to be similar?

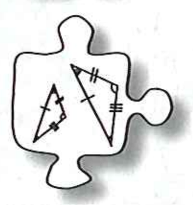
How can I justify that?

What evidence can I state?

Chapter Outline



Section 3.1 Through an exploration with rubber bands, students will generate similar figures, which will launch a focus on similarity for Sections 3.1 and 3.2. During these lessons, students will determine what common qualities similar shapes have.



Section 3.2 As students discover the conditions that cause triangles to be similar or congruent, they will learn about using a flowchart to organize facts and support their conclusions.

3.1.1 What do these shapes have in common?

Similarity

In Section 1.3, you organized shapes into groups based on their size, angles, sides, and other characteristics. You identified shapes using their characteristics and investigated relationships between different kinds of shapes, so that now you can tell if two shapes are both parallelograms or trapezoids, for example. But what makes two figures look alike?

Today you will be introduced to a new transformation that enlarges a figure while maintaining its shape, called a **dilation**. After creating new enlarged shapes, you and your team will explore the interesting relationships that exist between figures that have the same shape.

In your teams, you should keep the following questions in mind as you work together today:

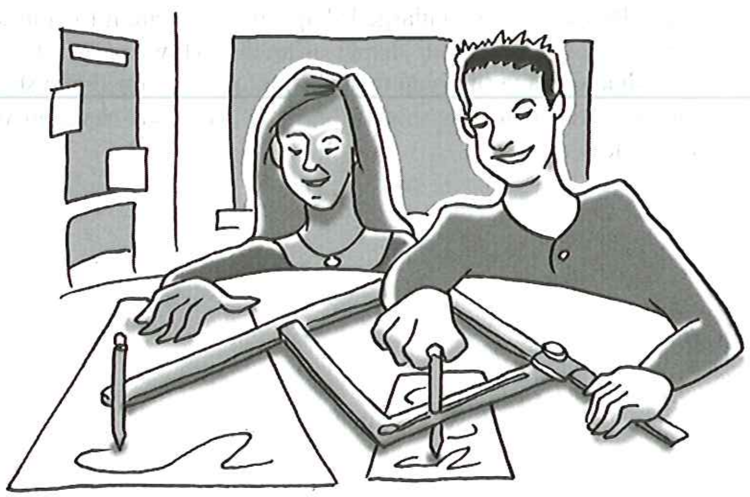
What do the shapes have in common?

How can you demonstrate that the shapes are the same?

What specifically is different about the shapes?

3-1. WARM-UP STRETCH

Before computers and copy machines existed, it sometimes took hours to enlarge documents or to shrink text on items such as jewelry. A pantograph device (like the one below) was often used to duplicate written documents and artistic drawings. You will now employ the same geometric principles by using rubber bands to draw enlarged copies of a design. Your teacher will show you how to do this.



3-2. In problem 3-1, you created designs that were **similar**, meaning that they have the same shape. But how can you determine if two figures are similar? What do similar shapes have in common? To find out, your team will need to create similar shapes that you can measure and compare.

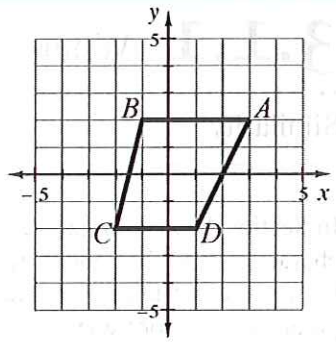


Diagram #1

a. Obtain a Lesson 3.1.1 Resource Page from your teacher (or download it from www.cpm.org). On it, find the quadrilateral shown in Diagram #1 at right.

Dilate (stretch) the quadrilateral from the origin by a factor of 2, 3, 4, or 5 to form $A'B'C'D'$. Each team member should pick a different enlargement factor. You may want to imagine that your rubber band chain is stretched from the origin so that the knot traces the perimeter of the original figure.

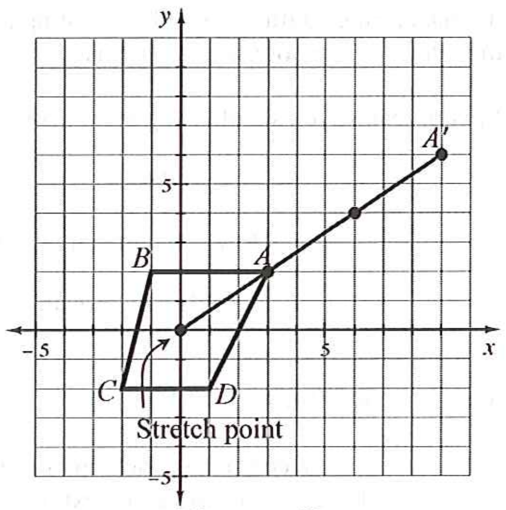
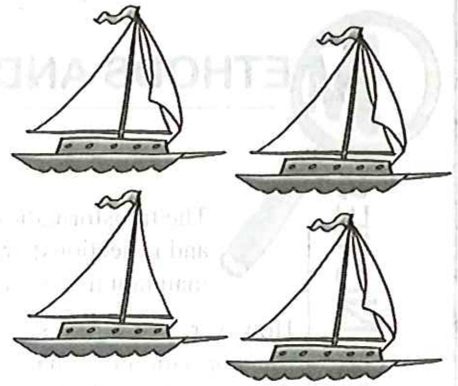


Diagram #2

For example, if your job is to stretch *ABCD* by a factor of 3, then A' would be located as shown in Diagram #2 at right.

b. Carefully cut out your enlarged shape and compare it to your teammates' shapes. How are the four shapes different? How are they the same? As you **investigate**, make sure you record what qualities make the shapes different and what qualities make the shapes the same. Be ready to report your conclusions to the class.

3-3. WHICH SHAPE IS THE EXCEPTION?



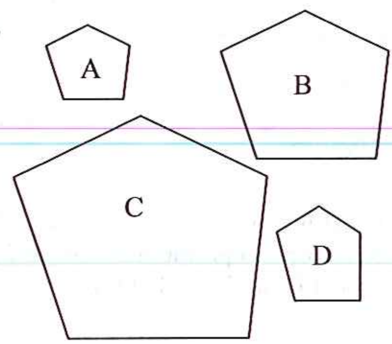
Sometimes shapes look the same and sometimes they look very different. What characteristics make figures alike so that we can say that they are the same shape? How are shapes that look the same but are different sizes related to each other? Understanding these relationships will allow us to know if shapes that appear to have the same shape actually do have the same shape.

Your task: For each set of shapes below, three shapes are similar, and one is an exception, which means that it is not like the others. Find the exception in each set of shapes. Your teacher may give you tracing paper to help you in your **investigation**.

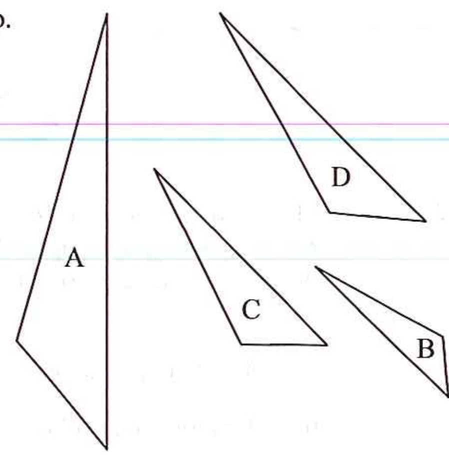
Answer each of these questions for both sets of shapes below:

- Which shape appears to be the exception? What makes that shape different from the others?
- What do the other three shapes have in common?
- Are there commonalities in the angles? Are there differences?
- Are there commonalities in the sides? Are there differences?

a.



b.



3-4. Write an entry in your Learning Log about the characteristics that figures with different sizes need to have in order to maintain the same shape. Add your own diagrams to illustrate the description. Title this entry "Same Shape, Different Size" with today's date.

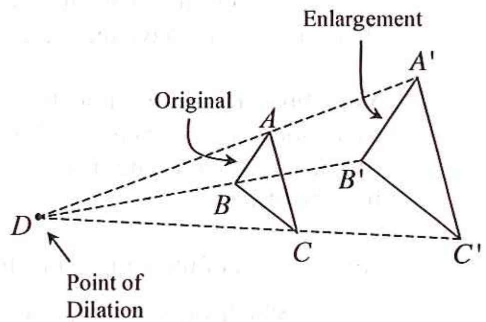
MATH NOTES

METHODS AND MEANINGS

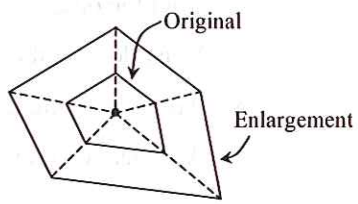
Dilations

The transformations you studied in Chapter 1 (translations, rotations, and reflections) are called rigid transformations because they all maintain the size and shape of the original figure.

However, a **dilation** is a transformation that maintains the shape of a figure but multiplies its dimensions by a chosen factor. In a dilation, a shape is stretched proportionally from a particular point, called the **point of dilation** or **stretch point**. For example, in the diagram at right, ΔABC is dilated to form $\Delta A'B'C'$. Notice that while a dilation changes the size and location of the original figure, it does not rotate or reflect the original.



Note that if the point of dilation is located inside a shape, the enlargement encloses the original, as shown at right.



Review & Preview

3-5. Plot the rectangle $ABCD$ formed with the points $A(-1, -2)$, $B(3, -2)$, $C(3, 1)$, and $D(-1, 1)$ onto graph paper. Use the method used in problem 3-2 to enlarge it from the origin by a factor of 2 (using two "rubber bands"). Label this new rectangle $A'B'C'D'$.

a. What are the dimensions of the enlarged rectangle, $A'B'C'D'$?

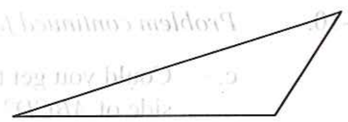
b. Find the area and the perimeter of $A'B'C'D'$.

c. Find AC (the length of $\overline{AC}$).

3-6. Solve each equation below for x. Show all work and check your answer by substituting it back into the equation and verifying that it makes the equation true.

a. $\frac{x}{3} = 6$ b. $\frac{5x+9}{2} = 12$ c. $\frac{x}{4} = \frac{9}{6}$ d. $\frac{5}{x} = \frac{20}{8}$

3-7. **Examine** the triangle at right.



a. Estimate the measure of each angle of the triangle at right.

b. Given only its shape, what is the best name for this triangle?

3-8. On graph paper, graph line $\overleftrightarrow{MU}$ if $M(-1, 1)$ and $U(4, 5)$.

a. Find the slope of $\overleftrightarrow{MU}$.

b. Find MU (the distance from M to U).

c. Are there any similarities to the calculations used in parts (a) and (b)? Any differences?

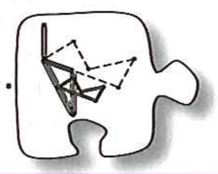
3-9. Rewrite the statements below into conditional ("If …, then …") form.

a. All equilateral triangles have 120° rotation symmetry.

b. A rectangle is a parallelogram.

c. The area of a trapezoid is half the sum of the bases multiplied by the height.

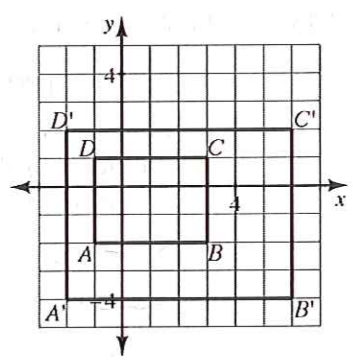
3.1.2 How can I maintain the shape?



Proportional Growth and Ratios

So far you have studied several shapes that appear to be **similar** (exactly the same shape but not necessarily the same size). But how can we know for sure that two shapes are similar? Today you will focus on the relationship between the lengths of sides of similar figures by enlarging and reducing shapes and looking for patterns.

3-10. Find your work from problem 3-5. The graph you created should resemble the graph at right.



a. In problem 3-5, you dilated (stretched) $ABCD$ to create rectangle $A'B'C'D'$, which is similar to $ABCD$. Which side of $A'B'C'D'$ corresponds to $\overline{CB}$? Which side corresponds to $\overline{AB}$?

b. Compare the lengths of each pair of corresponding sides. What do you notice? How could you get the lengths of $A'B'C'D'$ from the dimensions of $ABCD$?

Problem continues on next page →

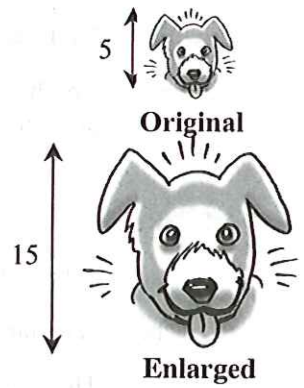
3-10. *Problem continued from previous page.*

c. Could you get the dimensions of $A'B'C'D'$ by adding the same amount to each side of $ABCD$? Try this and explain what happened.

d. Monica enlarged $ABCD$ to get a different rectangle $A''B''C''D''$ which is similar to $ABCD$. She knows that $\overline{A''B''}$ is 20 units long. How many times larger than $ABCD$ is $A''B''C''D''$? (That is, how many "rubber bands" did she use?) And how long is $\overline{B''C''}$? Show how you know.

3-11. In problem 3-10, you learned that you can create similar shapes by multiplying each side length by the same number. This number is called the **zoom factor**. You may have used a zoom factor before when using a copy machine. For example, if you set the zoom factor on a copier to 50%, the machine shrinks the image in half (that is, multiplies it by 0.5) but keeps the shape the same. In this course, the zoom factor will be used to describe the ratio of the new figure to the original figure.

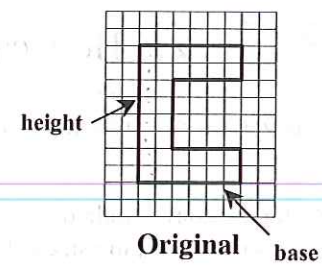
What zoom factor was used to enlarge the puppy shown at right?



3-12. Casey decided to enlarge her favorite letter: C, of course! Your team is going to help her out. Have each member of your team choose a different zoom factor below. Then on graph paper, enlarge (or reduce) the block "C" at right by your zoom factor.

a. 3   b. 2

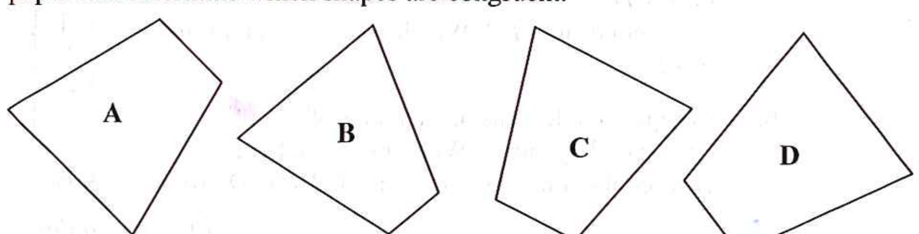
c. 1   d. $\frac{1}{2}$



3-13. Look at the different "C's" that were created in problem 3-12.

a. What happened when the zoom factor was 1?

b. When two shapes are the same shape and the same size (that is, the zoom factor is 1), they are called **congruent**. Compare the shapes below with tracing paper and determine which shapes are congruent.

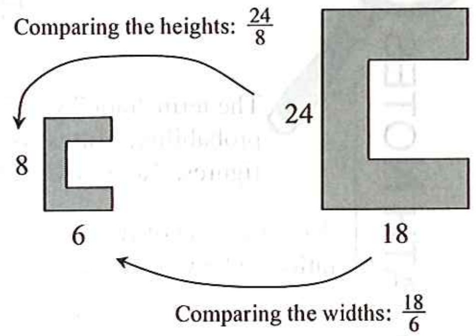


3-14. EQUAL RATIOS OF SIMILARITY

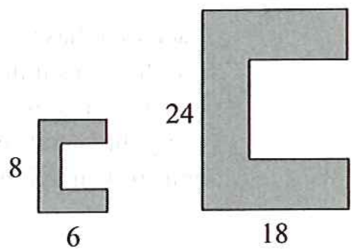
Casey wants to learn more about her enlarged "C"s. Return to your work from problem 3-12.

a. Since the zoom factor multiplies each part of the original shape, then the **ratio** of the widths must equal the ratio of the lengths.

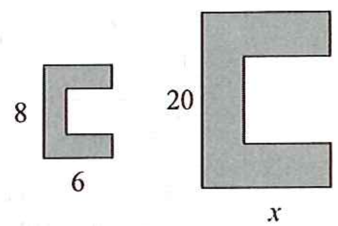
Casey decided to show these ratios in the diagram at right. Verify that her ratios are equal.



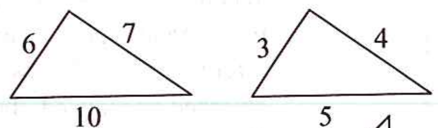
b. When looking at Casey's work, her brother wrote the equation $\frac{8}{6} = \frac{24}{18}$. Are his ratios, in fact, equal? And how could he show his work on his diagram? Copy his diagram at right and add arrows to show what sides Casey's brother compared.



c. She has decided to create an enlarged "C" for the door of her bedroom. To fit, it needs to be 20 units tall. If x is the width of this "C", write and solve an equation to find out how wide the "C" on Casey's door must be. Be ready to share your equation and solution with the class.

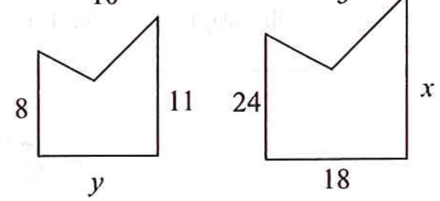


3-15. Use your observations about ratios between similar figures to answer the following questions.



a. Are the triangles above similar? How do you know?

b. If the pentagons at right are similar, what are the values of x and y?



3-16. In a new entry of your Learning Log, explain what you know about the ratio of similar figures. If you know the dimensions of one shape, how can you figure out the dimensions of another shape that is similar to it? Title this entry, "Common Ratios of Similar Figures" and include today's date.

METHODS AND MEANINGS

MATH NOTES

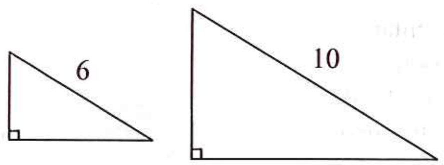
Ratio of Similarity and Zoom Factor

The term "ratio" was introduced in Chapter 1 in the context of probability. But ratios are very important when comparing two similar figures. Review what you know about ratios below.

A comparison of two quantities (numbers or measures) is called a **ratio.** A ratio can be written as:

$a:b$ or $\frac{a}{b}$ or "a to b"

Each ratio has a numeric value that can be expressed as a fraction or a decimal. For the two similar right triangles below, the ratio of the small triangle's hypotenuse to the large triangle's hypotenuse is $\frac{6}{10}$ or $\frac{3}{5}$. This means that for every three units of length in the small triangle's hypotenuse, there are five units of length in the large triangle's hypotenuse.



Original **Enlargement**

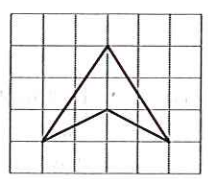
$\frac{6}{10} \longrightarrow$ means 6 units of length on one hypotenuse compared to 10 units on the other hypotenuse

The ratio between any pair of corresponding sides in similar figures is called the **ratio of similarity.**

When a figure is enlarged or reduced, each side is multiplied (or divided) by the same number. While there are many names for this number, this text will refer to this number as the **zoom factor**. To help indicate if the figure was enlarged or reduced, the zoom factor is written as the ratio of the new figure to the original figure. For the two triangles above, the zoom factor is $\frac{10}{6}$ or $\frac{5}{3}$.

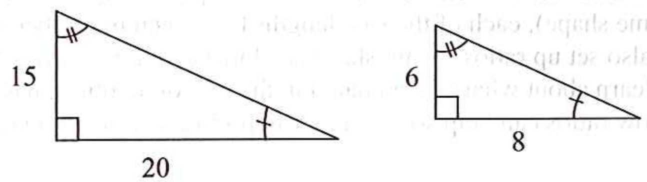
Review & Preview

3-17. Enlarge the shape at right on graph paper using a zoom factor of 4.



3-18. The ratios Casey wrote from the table in part (a) of problem 3-14 are **common ratios between corresponding sides** of the two shapes. That is, they are ratios between the matching sides of two shapes.

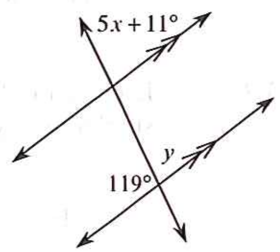
a. Look at the two similar shapes below. Which sides correspond? Write common ratios with the names of sides and lengths, just like Bernhard did.



b. Find the hypotenuse of each triangle above. Is the ratio of the hypotenuses equal the ratios you found in part (a)?

3-19. The temperature in San Antonio, Texas is currently 77°F and is increasing by 3° per hour. The current temperature in Bombay, India is 92°F and the temperature is dropping by 2° per hour. When will it be as hot in San Antonio as it is in Bombay? What will the temperature be?

3-20. **Examine** the relationships in the diagram at right. Then solve for x and y, if possible.



3-21. Read the following statements and decide if, when combined, they present a *convincing* argument. You may need to refer back to your Shape Toolkit as you consider the following statements and decide if the conclusion is correct. Be sure to **justify** your reasoning.

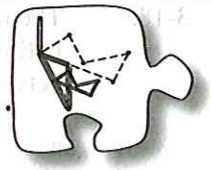
Fact #1: A square has four sides of equal length.

Fact #2: A square is a rectangle because it has four right angles.

Fact #3: A rhombus also has four sides of equal length

Conclusion: Therefore, a rhombus is a rectangle.

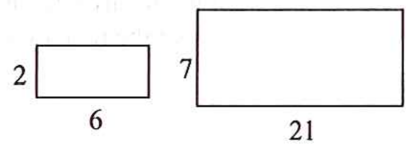
3.1.3 How are the shapes related?



Using Ratios of Similarity

You have learned that when we enlarge or reduce a shape so that it remains similar (that is, it maintains the same shape), each of the side lengths have been multiplied by a common zoom factor. We can also set up ratios within shapes and make comparisons to other similar shapes. Today you will learn about what effect changing the size of an object has on its perimeter. You will also learn how ratios can help solve similarity problems when drawing the figures is impractical.

3-22. **Examine** the rectangles at right.

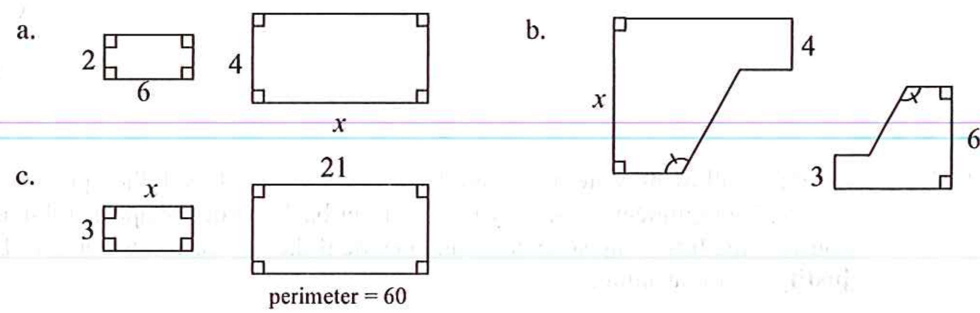


a. Use ratios to show that these shapes are similar (figures that have the same shape, but not necessarily the same size).

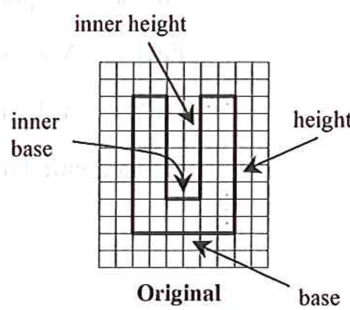
b. What other ratios could you use?

c. Linh claims that these shapes are not similar. When she compared the heights, she wrote $\frac{2}{7}$. Then she compared the bases and got $\frac{21}{6}$. Why is Linh having trouble? Explain completely.

3-23. Each pair of figures below is similar. Review what you have learned so far about similarity as you solve for x.



3-24. Casey's back at it! Now she wants you to enlarge the block "U" for her spirit flag.



a. Copy her "U" onto graph paper.

b. Now draw a larger "U" with a zoom factor of $\frac{3}{2} = 1.5$. What is the height of the new "U"?

c. Find the ratio of the perimeters. That is, find $\frac{\text{Perimeter New}}{\text{Perimeter Original}}$. What do you notice?

d. Casey enlarged "U" proportionally so that it has a height of 10. What was her zoom factor? What is the base of this new "U"? **Justify** your conclusion.

3-25. After enlarging his "U" in problem 3-24, Al has an idea. He drew a 60° angle, as shown in Diagram #1 at right. Then, he extended the sides of the angle so that they are twice as long, as shown in Diagram #2. "Therefore, the new angle must have measure 120°," he explained. Do you agree? Discuss this with your team and write a response to Al.

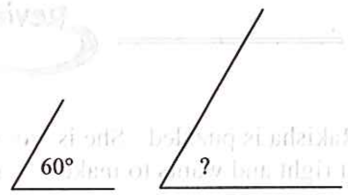
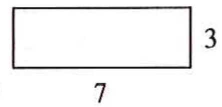


Diagram #1 Diagram #2

3-26. Al noticed that the ratio of the perimeters of two similar figures is equal to the ratio of the side lengths. "*What about the area? Does it grow the same way?*" he wondered.



a. Find the area and perimeter of the rectangle above.

b. Test Al's question by enlarging the rectangle by a zoom factor of 2. Then find the new area and perimeter.

c. Answer Al's question: Does the perimeter double? Does the area double? Explain what happened.

METHODS AND MEANINGS

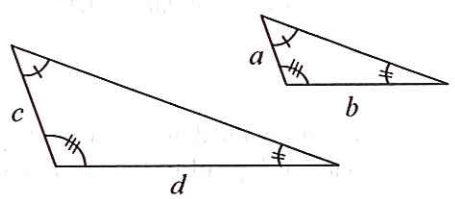
MATH NOTES

Proportional Equations

A **proportional equation** is one that compares two or more ratios. Proportional equations can express comparisons between two similar objects or compare two corresponding parts of an object.

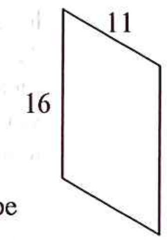
For example, the following equations can be written for the similar triangles at right:

$$\frac{a}{c} = \frac{b}{d} \quad or \quad \frac{a}{b} = \frac{c}{d}.$$



Review & Preview

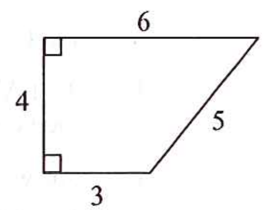
3-27. Rakisha is puzzled. She is working with the parallelogram drawn at right and wants to make it smaller instead of bigger.



a. What should she do if she wants the sides of her new shape to be *half as long* as the original sides? What zoom factor should she use? Find the dimensions of her new shape.

b. While drawing some other shapes, Rakisha ended up with a shape congruent to the original parallelogram. What is the common ratio between pairs of corresponding sides?

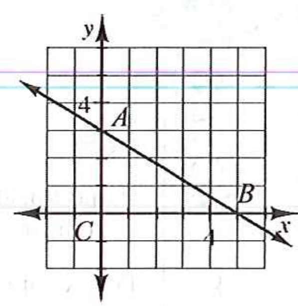
3-28. Enlarge the shape at right on graph paper using a zoom factor of 2. Then find the perimeter and area of both shapes. What do you notice when you compare the perimeters? The areas?



3-29. Solve each equation below. Show all work and check your answer.

a. $\frac{14}{5} = \frac{x}{3}$ b. $\frac{10}{m} = \frac{5}{11}$ c. $\frac{t-2}{12} = \frac{7}{8}$ d. $\frac{x+1}{5} = \frac{x}{3}$

3-30. **Examine** the graph of line $\overleftrightarrow{AB}$ at right.

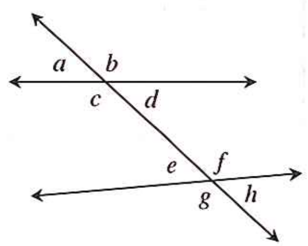


a. Find the equation of $\overleftrightarrow{AB}$.

b. Find the area and perimeter of $\triangle ABC$.

c. Write an equation of the line through A that is perpendicular to $\overleftrightarrow{AB}$.

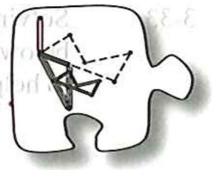
3-31. **Examine** the diagram at right. Name the geometric relationships of the angles below.



a. *d* and *e* b. *e* and *h*

c. *a* and *e* d. *c* and *d*

3.1.4 How can I use equivalent ratios?

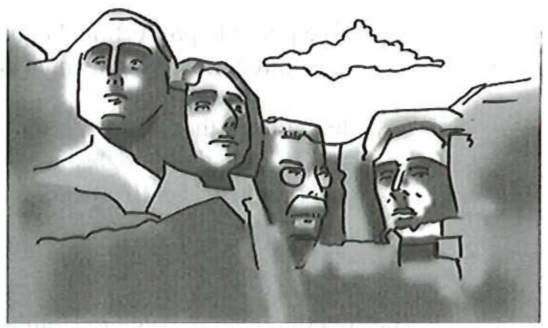


Applications and Notation

Now that you have a good understanding of how to use ratios in similar figures to solve problems, how can you extend these ideas to situations outside the classroom? You will start by considering a situation for which we want to find the length of something that would be difficult to physically measure.

3-32. GEORGE WASHINGTON'S NOSE

On her way to visit Horace Mann University, Casey stopped by Mount Rushmore in South Dakota. The park ranger gave a talk that described the history of the monument and provided some interesting facts. Casey could not believe that the carving of George Washington's face is 60 feet tall from his chin to the top of his head!



However, when a tourist asked about the length of Washington's nose, the ranger was stumped! Casey came to her rescue by measuring, calculating and getting an answer. How did Casey get her answer?

Your task: Figure out the length of George Washington's nose on the monument. Work with your study team to come up with a **strategy**. Show all measurements and calculations on your paper with clear labels so anyone could understand your work.

Discussion Points

What is this question asking you to find?

How can you use similarity to solve this problem?

Is there something in this room that you can use to compare to the monument?

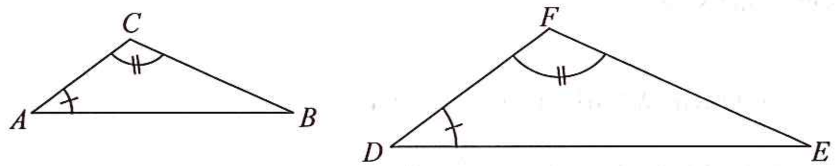
What parts do you need to compare?

Do you have any math tools that can help you gather information?

3-33. Solving problem 3-32, you may have written a proportional equation like the one below. When solving proportional situations, is very important that parts be labeled to help you follow your work.

$$\frac{\text{Length of George's Nose}}{\text{Length of George's Head}} = \frac{\text{Length of Student's Nose}}{\text{Length of Student's Head}}$$

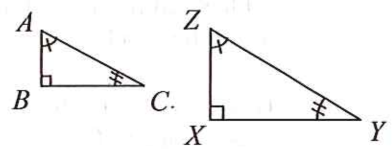
Likewise, when working with geometric shapes such as the similar triangles below, it is easier to explain which sides you are comparing by using notation that everyone understands.



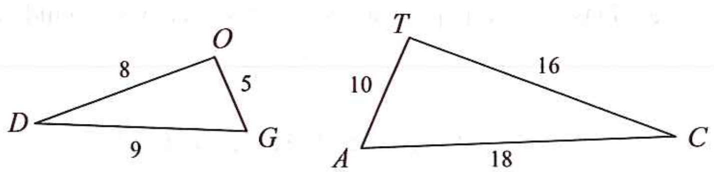
a. One possible proportional equation for these triangles is $\frac{AC}{AB} = \frac{DF}{DE}$. Write at least three more proportional equations based on the similar triangles above.

b. Jeb noticed that $m\measuredangle A = m\measuredangle D$ and $m\measuredangle C = m\measuredangle F$. But what about $m\measuredangle B$ and $m\measuredangle E$? Do these angles have the same measure? Or is there not enough information? **Justify** your conclusions.

3-34. The two triangles at right are similar. Read the Math Notes box for this lesson to learn about how to write a statement to show that two shapes are similar.



Then **examine** the two triangles below. Which of the following statements are correctly written and which are not? Note that more than one statement may be correct. Discuss your answers with your team.



a. $\Delta DOG \sim \Delta CAT$

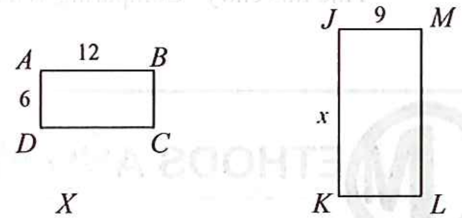
b. $\Delta DOG \sim \Delta CTA$

c. $\Delta OGD \sim \Delta ATC$

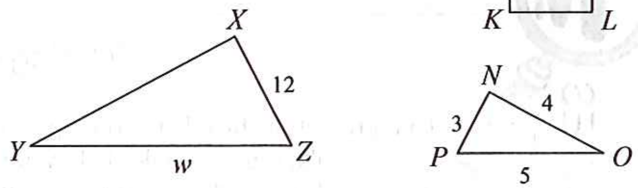
d. $\Delta DGO \sim \Delta CAT$

3-35. Find the value of the variable in each pair of similar figures below. You may want to set up tables to help you write equations.

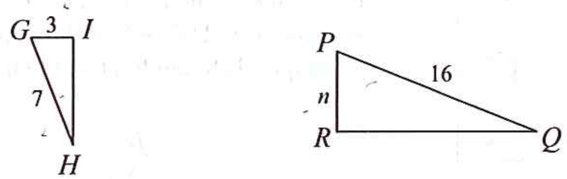
a. $ABCD \sim JKLM$



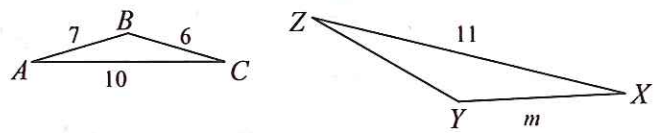
b. $\Delta NOP \sim \Delta XYZ$



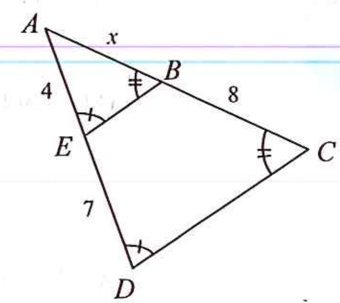
c. $\Delta GHI \sim \Delta PQR$



d. $\Delta ABC \sim \Delta XYZ$



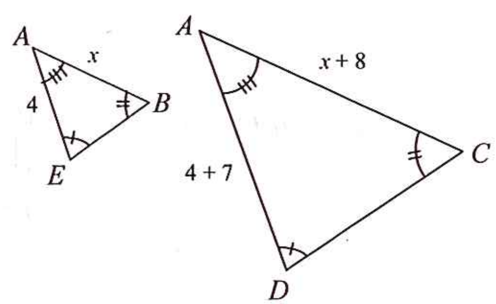
3-36. Rochida was given the diagram at right and told that the two triangles are similar.



a. Rochida knows that to be similar, all corresponding angles must be equal. Are all three sets of angles equal? How can you tell?

b. Rochida decides to redraw the shape as two separate triangles, as shown at right. Write a proportional equation using the corresponding sides.

c. Solve the equation for x. How long is $\overline{AB}$? How long is $\overline{AC}$?



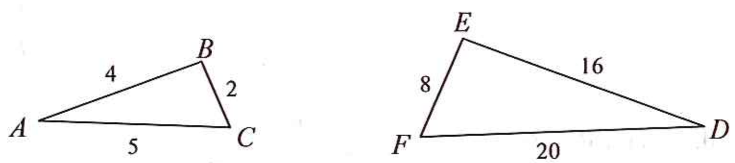
3-37. Write a Learning Log entry describing the different ways you can compare two similar objects or quantities with equivalent ratios. Title this entry "Comparing With Ratios" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Writing a Similarity Statement

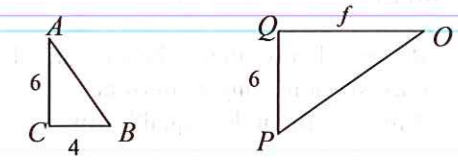
To represent the fact that two shapes are **similar**, use the symbol "~". For example, if the triangles below are similar, this can be stated as $\triangle ABC \sim \triangle DEF$. The order of the letters in the name of each triangle determines which sides and angles correspond. For example, in the statement $\triangle ABC \sim \triangle DEF$, you can determine that angle A corresponds to angle D and that side $\overline{BC}$ corresponds to side $\overline{EF}$.



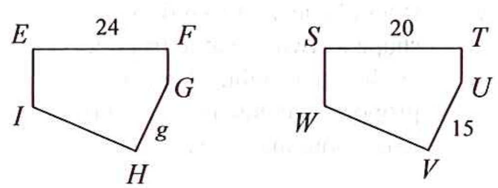
Review & Preview

3-38. Solve for the missing lengths in the sets of similar figures below.

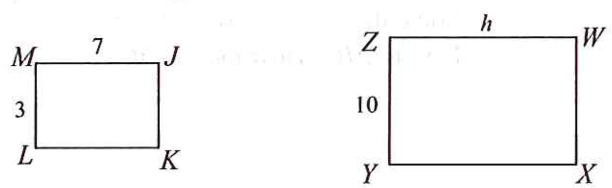
a. $\triangle ABC \sim \triangle OPQ$



b. $EFGHI \sim STUVW$



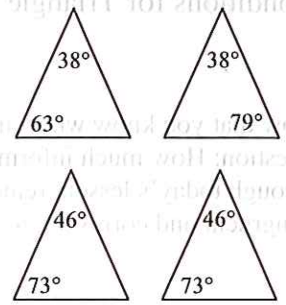
c. $JKLM \sim WXYZ$



3-39. In recent lessons, you have learned that similar triangles have equal corr angles. Is it possible to have equal corresponding angles when the triangles appear to match? What if you are not given all three angle measures? Conside two cases below.

a. Find the measure of the third angle in the first pair of triangles at right. Compare the two triangles. What do you notice?

b. **Examine** the second pair of triangles at right. Without calculating, do you know that the unmarked angles must be equal? Why or why not?



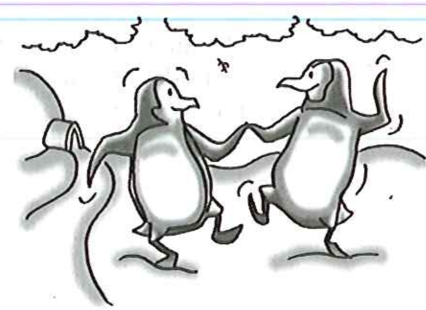
3-40. Sandy has a square, equilateral triangle, rhombus, and regular hexagon in her Shape Bucket, while Robert has a scalene triangle, kite, isosceles trapezoid, non-special quadrilateral, and obtuse isosceles triangle in his. Sandy will randomly select a shape from her Shape Bucket, while Robert will randomly select a shape from his.

a. Who has a greater probability of selecting a quadrilateral? **Justify** your conclusion.

b. Who has a greater probability of selecting an equilateral shape? **Justify** your conclusion.

c. What is more likely to happen: Sandy selecting a shape with at least two sides that are parallel or Robert selecting a shape with at least two sides that are equal?

3-41. Frank and Alice are penguins. At birth, Frank's beak was 1.95 inches long, while Alice's was 1.50 inches long. If Frank's beak grows by 0.25 inches per year and Alice's grows by 0.40 inches per year, how old will they be when their beaks are the same length?



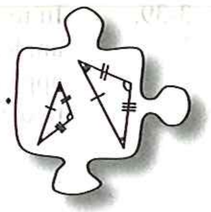
3-42. Plot $ABCDE$ formed with the points $A(-3, -2)$, $B(5, -2)$, $C(5, 3)$, $D(1, 6)$, and $E(-3, 3)$ onto graph paper.

a. Use the method from problem 3-2 to enlarge it from the origin by a factor of 2. Label this new shape $A'B'C'D'E'$.

b. Find the area and the perimeter of both figures.

What information do I need?

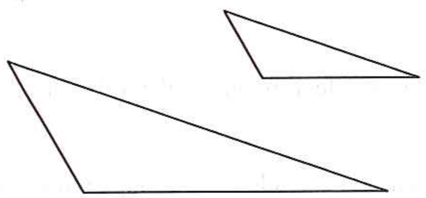
for Triangle Similarity



Now that you know what similar shapes have in common, you are ready to turn to a related question: How much information do I need to know that two triangles are similar? As you work through today's lesson, remember that similar polygons have corresponding angles that are congruent and corresponding sides that are proportional.

3-43. ARE THEY SIMILAR?

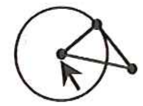
Erica thinks the triangles below might be similar. However, she knows not to trust the way figures look in a diagram, so she asks for your help.



a. If two shapes are similar, what must be true about their angles and sides?

b. Obtain the Lesson 3.2.1 Resource Page from your teacher. Measure the angles and sides of Erica's triangles and help her decide if the triangles are similar or not.

3-44. HOW MUCH IS ENOUGH?



Jovan is tired of measuring all the angles and sides to determine if two triangles are similar. "There must be an easier way," he thinks. "What if I know that all of the side lengths have a common ratio? Does that mean that the triangles are similar?"

a. Before experimenting, make a prediction. Do you think that the triangles have to be similar if the pairs of corresponding sides share a common ratio?

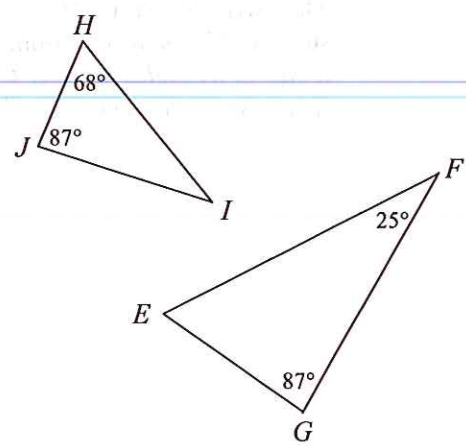
b. Experiment with Jovan's idea. To do this, use a dynamic geometry tool to test triangles with proportional side lengths. If you do not have access to a dynamic tool, cut straws or strands of linguini into the lengths below and create two triangles. Can you create two triangles that are not similar? **Investigate,** sketch your shapes, and write down your conclusion.

Triangle #1: side lengths 3, 5, and 6

Triangle #2: side lengths 6, 10, and 12

c. Jovan then asks, "Is it enough to know that each pair of corresponding angles are congruent? Does that mean the triangles are similar?" Again use a dynamic geometry tool to test triangles with equal corresponding angles or use linguini with protractors to create three angles that form a triangle. Can you create two triangles with the same three angle measures that are not similar? **Investigate**, sketch your shapes, and write down your conclusion.

3-45. Scott is looking at the set of shapes at right. He thinks that $\Delta EFG \sim \Delta HIJ$ but he is not sure that the shapes are drawn to scale.

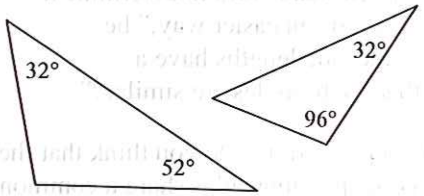


a. Are the corresponding angles equal? Convince Scott that these triangles are similar.

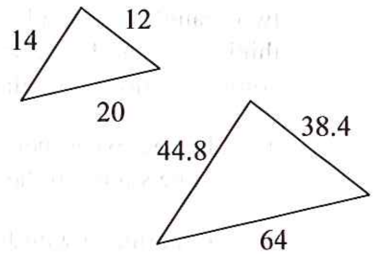
b. How many pairs of angles need to be congruent to be sure that triangles are similar?

3-46. Based on your conclusions from problems 3-44 and 3-45, decide if each pair of triangles below is similar. Explain your **reasoning**.

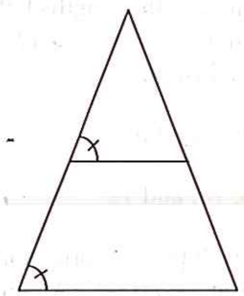
a.



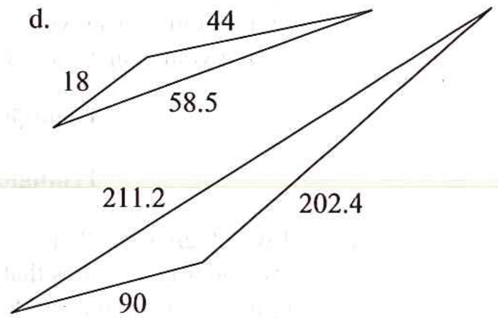
b.



c.



d.



3-47. Read the Math Notes box for this lesson, which introduces new names for the observations you made in problems 3-44 and 3-45. Then write a Learning Log entry about what you learned today. Be sure to address the question: *how much information do I need about a pair of triangles in order to be sure that they are similar?* Title this entry "AA ~ and SSS ~" and include today's date.

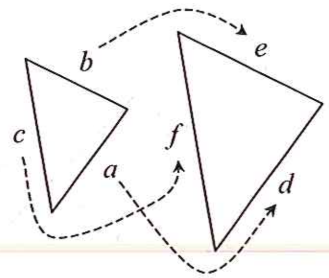
MATH NOTES

METHODS AND MEANINGS

Conditions for Triangle Similarity

For two shapes to be similar, corresponding angles must have equal measure and corresponding sides must be proportional.

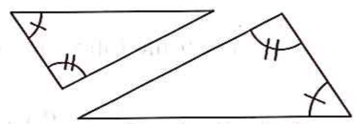
However, if you are testing for similarity between two triangles, then it is sufficient to know that all three corresponding side lengths share a common ratio.



SSS ~: If $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$, then the triangles are similar.

This guarantees similarity and is referred to as the **SSS Triangle Similarity Conjecture** (which can be abbreviated as "SSS Similarity" or "SSS ~" for short.)

Also, for two triangles it is sufficient to know that two pairs of corresponding angles have equal measures because then the third pair of angles must have equal measure.

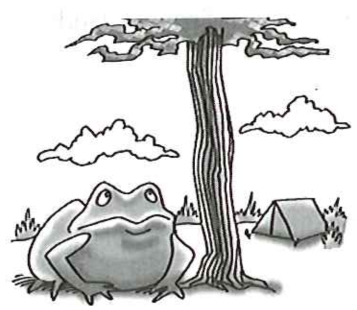


AA ~: If 2 pairs of angles have equal measure, then the triangles are similar.

This is known as the **AA Triangle Similarity Conjecture** (which can be abbreviated as "AA Similarity" or "AA ~" for short).

Review & Preview

3-48. Assume that all trees are green.



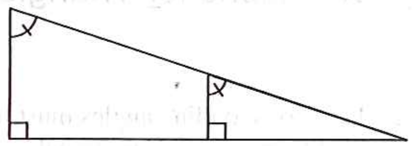
a. Does this statement mean that an oak tree must be green? Explain why or why not.

b. Does this statement mean that anything green must be a tree? Explain why or why not.

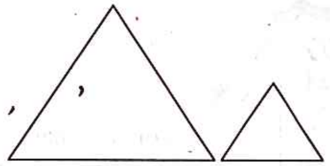
c. Are the statements "All trees are green" and "All green things are trees" saying the same thing? Explain why or why not.

3-49. Decide if each pair of triangles below is similar. If the triangles are similar, **justify** your conclusion by stating the similarity conjecture you used. If the triangles are not similar, explain how you know.

a.

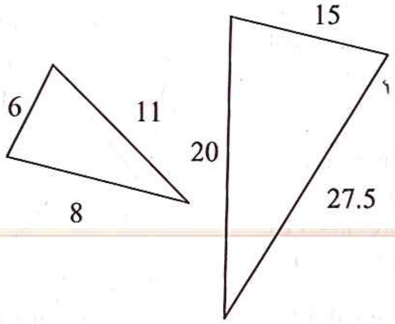


b.

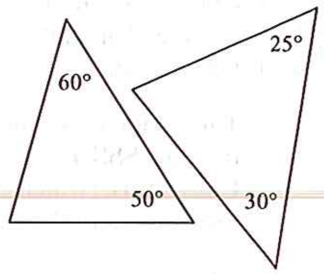


Equilateral Triangles

c.



d.



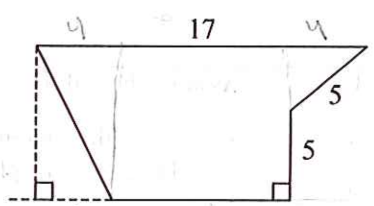
3-50. Consider the following arrow diagram:

Lines are parallel → *alternate interior angles are equal.*

a. Write this arrow diagram as a conditional ("If…, then…") statement.

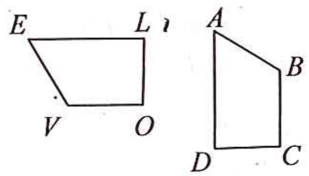
b. Write a similar conditional statement about corresponding angles, then write it as an arrow diagram.

3-51. Find the area and perimeter of the shape at right. Show all work.

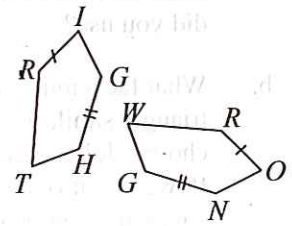


3-52. Assume that each pair of figures below is similar. Write a similarity statement to illustrate which parts of each shape correspond. Remember: letter order is important!

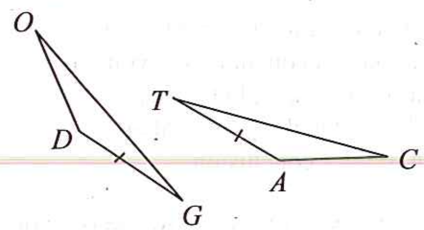
a. $ABCD \sim ?$



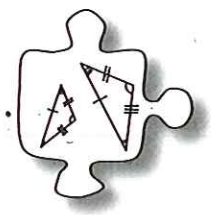
b. $RIGHT \sim ?$



c. $\triangle$____ $\sim \triangle$____



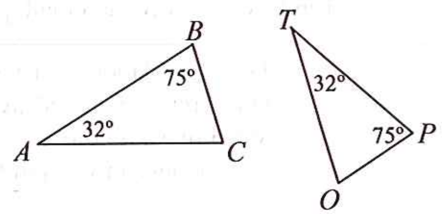
3.2.2 How can I organize my information?



Creating a Flowchart

In Lesson 3.2.1, you developed the AA ~ and SSS ~ conjectures to help confirm that triangles are similar. Today you will continue working with similarity and will learn how to use flowcharts to organize your **reasoning**.

3-53. **Examine** the triangles at right.

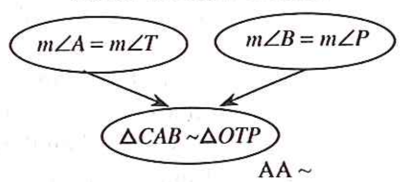


a. Are these triangles similar? Use full sentences to explain your **reasoning**.

b. Julio decided to use a diagram (called a **flowchart**) to explain his reasoning.

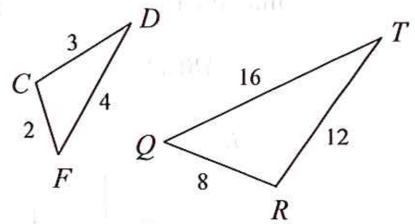
Compare your explanation to Julio's flowchart. Did Julio use the same **reasoning** you used?

JULIO'S FLOWCHART



3-54. Besides showing your **reasoning**, a flowchart can be used to organize your work as you determine whether or not triangles are similar.

a. Are these triangles similar? Which triangle similarity conjecture (see the Math Notes box from Lesson 3.2.1) did you use?



b. What facts must you know to use the triangle similarity conjecture you chose? Julio started to list the facts in a flowchart at right. Copy them on your paper and complete the third oval.

Facts:

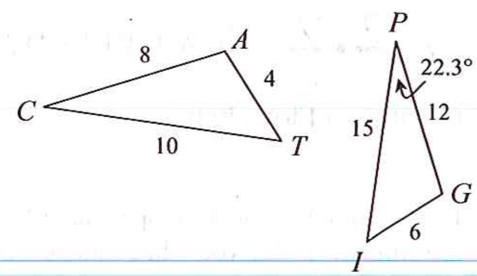
$\frac{12}{3} = 4$ $\frac{8}{2} = 4$ ( )

c. Once you have the needed facts in place, you can conclude that you have similar triangles. Add to your flowchart by making an oval and filling in your conclusion.

Conclusion: ( )

d. Finally, draw arrows to show the flow of the facts that lead to your conclusion and record the similarity conjecture you used, following Julio's example from problem 3-53.

3-55. Now **examine** the triangles at right.

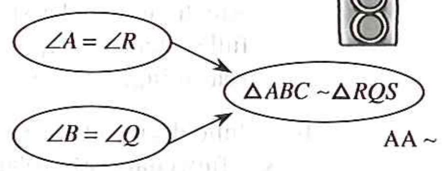


a. Are these triangles similar? **Justify** your conclusion using a flowchart.

b. What is $m\measuredangle C$? How do you know?

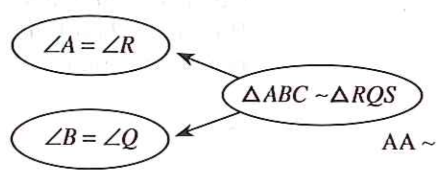
3-56. Lindsay was solving a math problem and drew the flowchart below:

a. Draw and label two triangles that could represent Lindsay's problem. What question did the problem ask her? How can you tell?

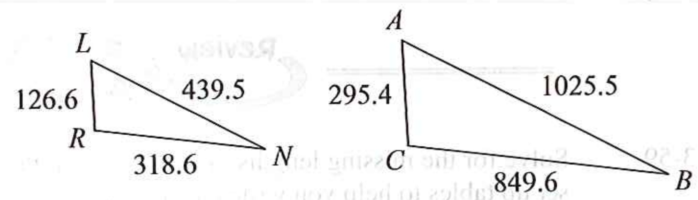


b. Lindsay's teammate was working on the same problem and made a mistake in his flowchart:

How is this flowchart different from Lindsay's? Why is this the wrong way to explain the **reasoning** in Lindsay's problem?



3-57. Ramon is **examining** the triangles at right. He suspects they may be similar by SSS ~.



a. Why is SSS ~ the best conjecture to test for these triangles?

b. Set up ovals for the facts you need to know to show that the triangles are similar. Complete any necessary calculations and fill in the ovals.

c. Are the triangles similar? If so, complete your flowchart using an appropriate similarity statement. If not, explain how you know.

3-58. In your Learning Log, explain how to set up a flowchart. For example, how do you know how many ovals you should use? How do you know what to put inside the ovals? Provide an example. Label this entry "Using Flowcharts" and include today's date.

METHODS AND MEANINGS

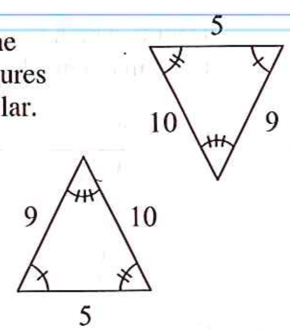
MATH NOTES

Congruent Shapes

If two figures have the same shape and are the same size, they are **congruent**. Since the figures must have the same shape, they must be similar.

Two figures are congruent if they meet both of the following conditions:

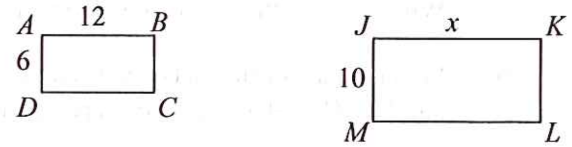
- the two figures are similar, and
- their side lengths have a common ratio of 1.



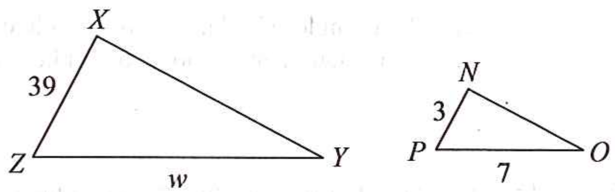
Review & Preview

3-59. Solve for the missing lengths in the sets of similar figures below. You may want to set up tables to help you write equations.

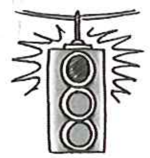
a. *ABCD* ~ *JKLM*



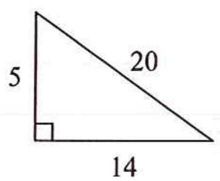
b. △*NOP* ~△*XYZ*



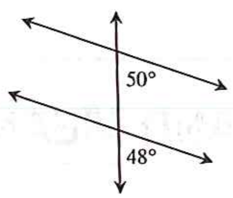
3-60. **Examine** each diagram below. Which diagrams are possible? Which are impossible? **Justify** each conclusion.



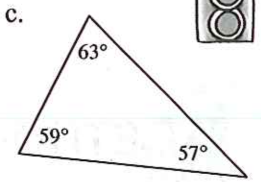
a.



b.

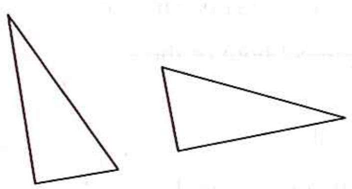


c.

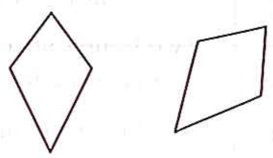


3-61. Decide which transformations were used on each pair of shapes below. Note that there may have been more than one transformation.

a.



b.



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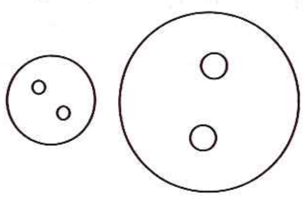

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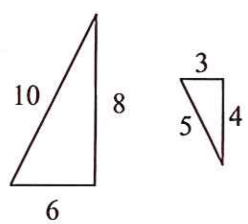
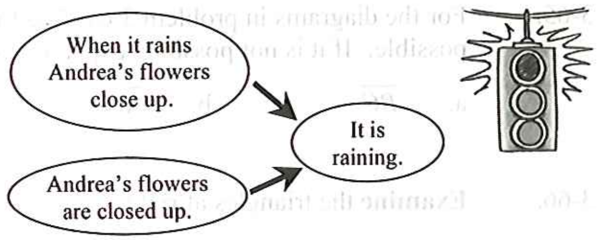


3-62. Determine whether or not the **reasoning** in the flowchart at right is correct. If it is wrong, redo the flowchart to make it correct.



3-63. Sketch each triangle if possible. If not possible, explain why not.

a. Right isosceles triangles

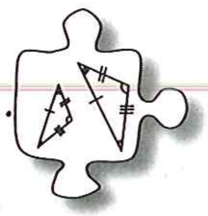
b. Right obtuse triangles

c. Scalene equilateral triangles

d. Acute scalene triangles

3.2.3 How can I use equivalent ratios?

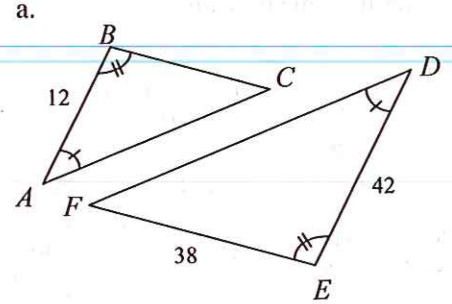
Triangle Similarity and Congruence



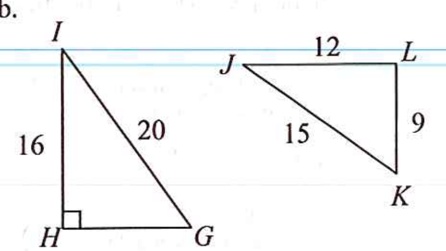
By looking at side ratios and at angles, you are now able to determine whether two figures are similar. But how can you tell if two shapes are the same shape *and* the same size? In this lesson you will **examine** properties that guarantee that shapes are exact replicas of one another.

3-64. Decide if each pair of triangles below is similar. Use a flowchart to organize your facts and conclusion for each pair of triangles.

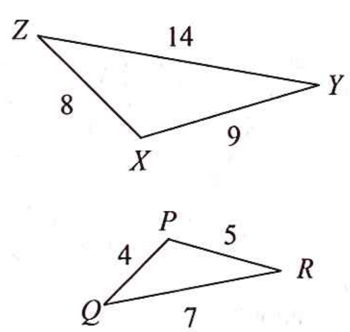
a.



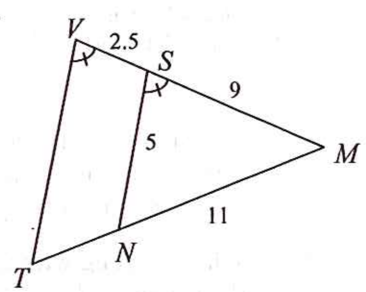
b.



c.



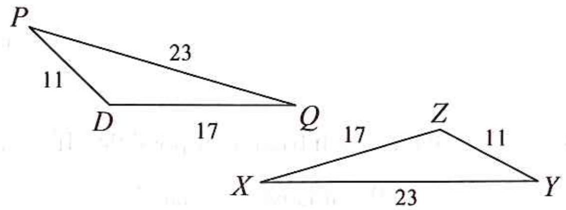
d.



3-65. For the diagrams in problem 3-64, find the lengths of the sides listed below, if possible. If it is not possible, explain why not.

a. $\overline{BC}$ b. $\overline{AC}$ c. $\overline{VT}$ d. $\overline{TN}$

3-66. **Examine** the triangles at right.



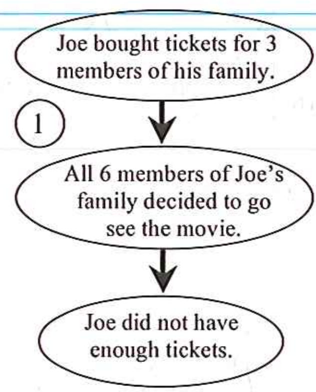
a. Are these triangles similar? How do you know? Use a flowchart to organize your explanation.

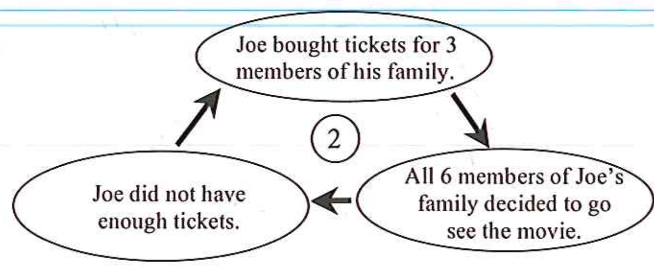
b. Kamraan says, "These triangles aren't just similar—they're congruent!" Is Kamraan correct? What special value in your flowchart indicates that the triangles are congruent?

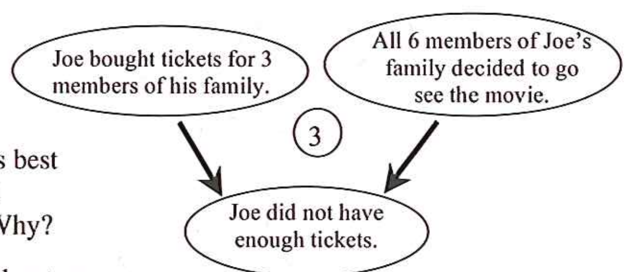
c. Write a conjecture (in "If…, then…" form) that explains how you know when two shapes are congruent.

3-67. Flowcharts can also be used to represent real-life situations. For example, yesterday Joe found out that three people in his family (including Joe) wanted to see a movie, so he went to the theater and bought three tickets. Unfortunately, while he was gone, three more family members decided to go. When everyone arrived at the theater, Joe did not have enough tickets.

Joe sat down later that night and tried to create a flowchart to describe what had happened. Here are the three possibilities he came up with:







a. Which of these flowcharts best captures the situation that happened on Saturday? Why?

b. What is wrong with the other two flowcharts as descriptions of this situation?

MATH NOTES

METHODS AND MEANINGS

Solving a Quadratic Equation

In a previous course, you learned how to solve **quadratic equations** (equations that can be written in the form $ax^2 + bx + c = 0$). Review two methods for solving quadratic equations below.

Some quadratic equations can be solved by **factoring** and using the **Zero Product Property**. For example, because $x^2 - 3x - 10 = (x-5)(x+2)$, the quadratic equation $x^2 - 3x - 10 = 0$ can be rewritten as $(x-5)(x+2) = 0$. The Zero Product Property states that if $ab = 0$, then $a = 0$ or $b = 0$. So, if $(x-5)(x+2) = 0$, then $x - 5 = 0$ or $x + 2 = 0$. Therefore, $x = 5$ or $x = -2$.

Another method for solving quadratic equations is the **Quadratic Formula.** This method is particularly helpful for solving quadratic equations that are difficult or impossible to factor. Before using the Quadratic Formula, the quadratic equation you want to solve must be in standard form, that is, written as $ax^2 + bx + c = 0$.

In this form, a is the coefficient of the x^2 term, b is the coefficient of the x term, and c is the constant term. The Quadratic Formula states:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula gives two possible answers due to the "$\pm$" symbol. This symbol (read as "plus or minus") is shorthand notation that tells us to calculate the formula twice: once using addition and once using subtraction. Therefore, every Quadratic Formula problem must be simplified twice to give:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

To solve $x^2 - 3x - 10 = 0$ using the Quadratic Formula, substitute $a = 1$, $b = -3$, and $c = -10$ into the formula, as shown below.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)} \rightarrow \frac{3 \pm \sqrt{49}}{2} \rightarrow \frac{3 \pm 7}{2} \rightarrow x = 5 \text{ or } x = -2$$

Review & Preview

3-68. On graph paper, graph the parabola $y = 2x^2 - 5x - 3$.

a. What are the roots (x-intercepts) of the parabola?

b. Read the Math Notes box for this lesson. Then solve the equation $2x^2 - 5x - 3 = 0$ algebraically. Did you solutions match your roots from part (a)?

3-69. On graph paper, plot $ABCD$ if $A(0, 3)$, $B(2, 5)$, $C(6, 3)$, and $D(4, 1)$.

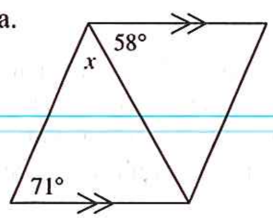
a. Rotate $ABCD$ 90° clockwise (↻) about the origin to form $A'B'C'D'$. Name the coordinates of B'.

b. Translate $A'B'C'D'$ up 8 units and left 7 units to form $A''B''C''D''$. Name the coordinates of C''.

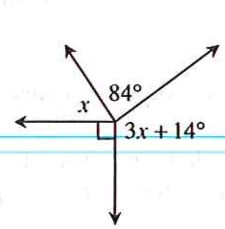
c. After rotating $ABCD$ 180° to form $A'''B'''C'''D'''$, Arah noticed that $A'''B'''C'''D'''$ position and orientation was the same as $ABCD$. What was the point of rotation? How did you find it?

3-70. Use the relationships in each diagram below to solve for x. **Justify** your solution by stating which geometry relationships you used.

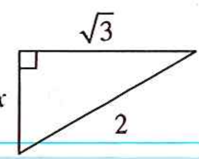
a.



b.

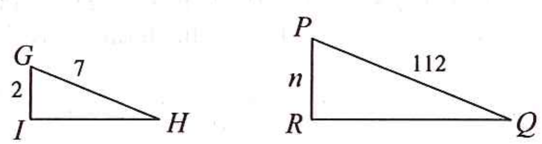


c.

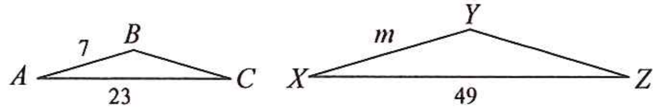


3-71. Solve for the missing lengths in the sets of similar figures below. Show all work.

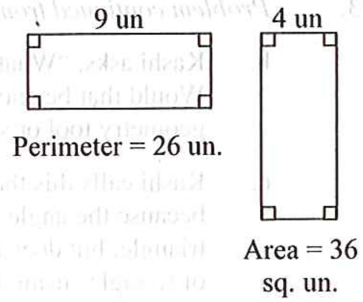
a. $\Delta GHI \sim \Delta PQR$



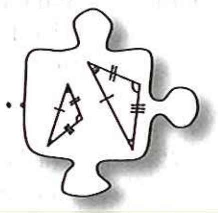
b. $\Delta ABC \sim \Delta XYZ$



3-72. Explain how you know that the shapes at right are similar.



3.2.4 What information do I need?



More Conditions for Triangle Similarity

So far, you have worked with two methods for determining that triangles are similar: the AA ~ and the SSS ~ Conjectures. Are these the only ways to determine if two triangles are similar? Today you will **investigate** similar triangles and complete your list of triangle similarity conjectures.

Keep the following questions in mind as you work together today:

How much information do you need?

Are the triangles similar? How can you tell?

Can I find a triangle with this information that is not similar?

3-73. Robel's team is using the SSS ~ Conjecture to show that two triangles are similar. "This is too much work," Robel says. "When we're using the AA ~ Conjecture, we only need to look at two angles. Let's just calculate the ratios for *two* pairs of corresponding sides to determine that triangles are similar."

Is SS ~ a valid similarity conjecture for triangles? That is, if two pairs of corresponding side lengths share a common ratio, must the triangles be similar?

In this problem, you will **investigate** this question using a dynamic geometry tool or another manipulative (such as linguini) provided by your teacher.

a. Robel has a triangle with side lengths 4 cm and 5 cm. If your triangle has two sides that share a common ratio with Robel's, does your triangle have to be similar to his? Use a dynamic geometry tool or straws or linguini to **investigate** this question.

Problem continues on next page →

3-73. *Problem continued from previous page.*

b. Kashi asks, "What if the angles between the two sides have the same measure? Would that be enough to know the triangles are similar?" Use the dynamic geometry tool or straws and linguini to answer Kashi's question.

c. Kashi calls this the "SAS ~ Conjecture," placing the "A" between the two "S"s because the angle is *between* the two sides. He knows it works for Robel's triangle, but does it work on all other triangles? Test this method on a variety of triangles using the dynamic geometry tool or straws and linguini.

3-74. What other triangle similarity conjectures involving sides and angles might there be? List the names of every other possible triangle similarity conjecture you can think of that involves sides and angles.

3-75. Cori's team put "SSA ~" on their list of possible triangle similarity conjectures. Use a dynamic geometry tool or straws or linguini to **investigate** whether SSA ~ is a valid triangle similarity conjecture. If a triangle has two sides sharing a common ratio with Robel's, and has the same angle "outside" these sides as Robel's, must it be similar to Robel's triangle? If you determine SSA ~ is not a valid similarity conjecture, cross it off your list!

3-76. Betsy's team came up with a similarity conjecture they call "AAS ~," but Betsy thinks they should cross it off their list. Betsy says, "This similarity conjecture has extra, unnecessary information. There's no point in having it on our list."

a. What is Betsy talking about? Why does the AAS ~ method contain more information than you need?

b. Go through your list of possible triangle similarity conjectures, crossing off all the invalid ones and all the ones that contain unnecessary information.

c. How many valid triangle similarity conjectures (without extra information) are there? List them.

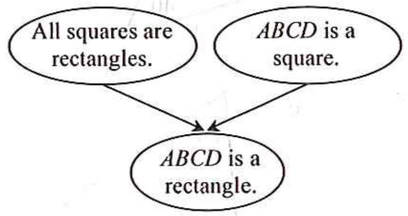
3-77. Reflect on what you have learned today. In your Learning Log, write down the triangle similarity conjectures that help to determine if triangles are similar. You can write these conjectures as conditional statements (in "If..., then..." form) or as arrow diagrams. Title this entry "Triangle Similarity Conjectures" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Writing a Flowchart

A **flowchart** helps to organize facts and indicate which facts lead to a conclusion. The bubbles contain facts, while the arrows point to a conclusion that can be made from a fact or multiple facts.



For example, in the flowchart at right, two independent (unconnected) facts are stated: *"All squares are rectangles"* and *"ABCD is a square."* However, these facts lead to the conclusion that *ABCD* must be a rectangle. Note that the arrows point toward the conclusion.

Review & Preview

3-78. If possible, draw a triangle that has exactly the following number of lines of symmetry. Then name the kind of triangle drawn.

a. 0 b. 1 c. 2 d. 3

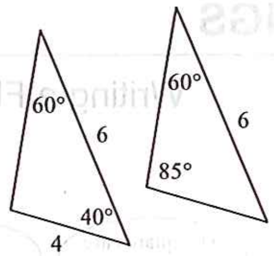
3-79. Do two lines always intersect? Consider this as you answer the questions below.

a. Write a system of linear equations that does not have a solution. Write each equation in your system in **slope-intercept form** ($y = mx + b$). Graph your system on graph paper and explain why it does not have a solution.

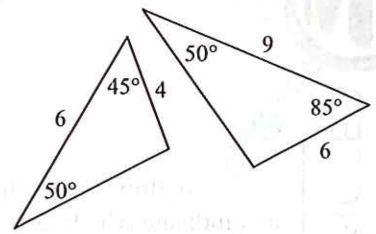
b. How can you tell algebraically that a system of linear equations has no solution? Solve your system of equations from part (a) algebraically and demonstrate how you know that the system has no solution.

3-80. Determine which of the following pairs of triangles are similar. Explain your work.

a.



b.



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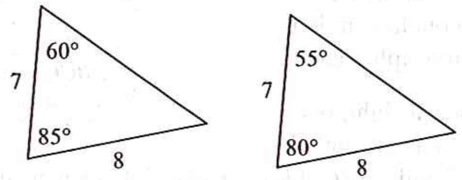
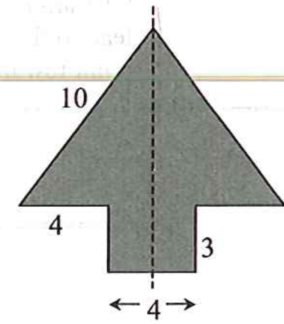
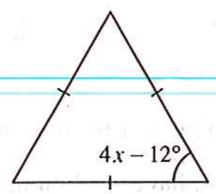


3-81. The dashed line at right represents the line of symmetry of the shaded figure. Find the area and perimeter of the shaded region. Show all work.

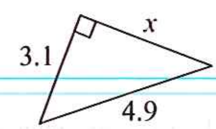


3-82. **Examine** the diagrams below. For each one, write and solve an equation to find x. Show all work.

a.



b.



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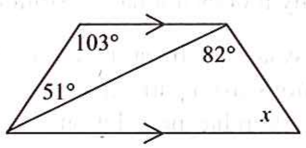

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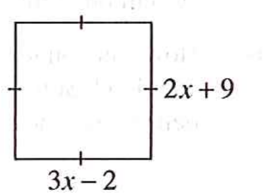
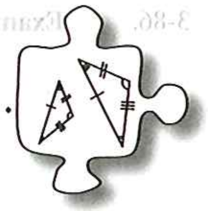


3.2.5 Are the triangles similar?

Determining Similarity

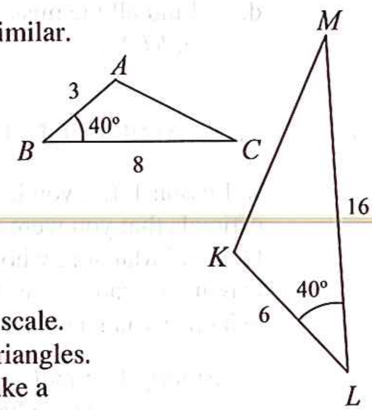


You now have a complete list of the three Triangles Similarity Conjectures (AA ~, SSS ~, and SAS ~) that can be used to verify that two triangles are similar. Today you will continue to practice applying these conjectures and using flowcharts to organize your **reasoning**.

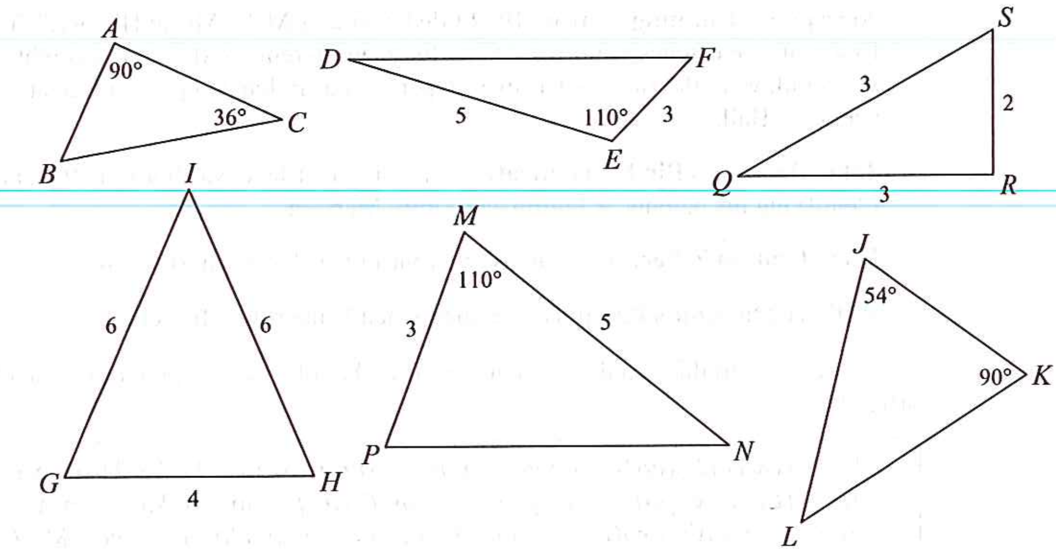
3-83. Lynn wants to show that the triangles at right are similar.

a. What similarity conjecture should Lynn use?

b. Make a flow chart showing that these triangles are similar.



3-84. Below are six triangles, none of which is drawn to scale. Among the six triangles are three pairs of similar triangles. Identify the similar triangles, then for each pair make a flowchart **justifying** the similarity.

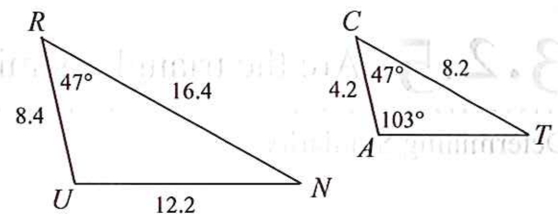


3-85. Revisit the similar triangles from problem 3-84.

a. Which pair of triangles are congruent? How do you know?

b. Suppose that in problem 3-84, $AB = 3$ cm, $AC = 4$ cm, and $KJ = 12$ cm. Find all the other side lengths in $\triangle ABC$ and $\triangle JKL$.

3-86. **Examine** the triangles at right.



a. Are these triangles similar? If so, make a flowchart **justifying** their similarity.

b. Charles has $\Delta CAT \sim \Delta RUN$ as the conclusion of his flowchart. Leesa has $\Delta NRU \sim \Delta TCA$ as her conclusion. Who is correct? Why?

c. Are ΔCAT and ΔRUN congruent? Explain how you know.

d. Find all the missing side lengths and all the angle measures of ΔCAT and ΔRUN.

3-87. THE FAMILY FORTUNE, Part Two

In Lesson 1.1.4, you had to convince city officials that you were a relative of Molly "Ol' Granny" Marston, who had just passed away leaving a sizable inheritance. Below is the evidence you had available:

Family Portrait — a photo showing three young children. On the back you see the date 1968.

Newspaper Clipping — from 1972 titled "Triplets Make Music History." The first sentence catches your eye: "Jake, Judy, and Jeremiah Marston, all eight years old, were the first triplets ever to perform a six-handed piano piece at Carnegie Hall."

Jake Marston's Birth Certificate — showing that Jake was born in 1964, and identifying his parents as Phillip and Molly Marston.

Your Learner's Permit — signed by your father, Jeremiah Marston.

Wilbert Marston's Passport — issued when Wilbert was fifteen.

As their answer to this problem, one team wrote the following argument for the city officials:

> *The birth certificate shows that Jake Marston was Molly Marston's son. The newspaper clipping shows that Jeremiah Marston was Jake Marston's brother. Therefore, Jeremiah Marston was Molly Marston's son. The learner's permit shows that Jeremiah Marston is my father. Therefore, I am Molly Marston's grandchild.*

Your task: Make a flowchart showing the **reasoning** in this team's argument. This flowchart will have more levels than the ones you have made in the past, because certain conclusions will be used as facts to support other conclusions. So plan carefully before you start to draw your chart.

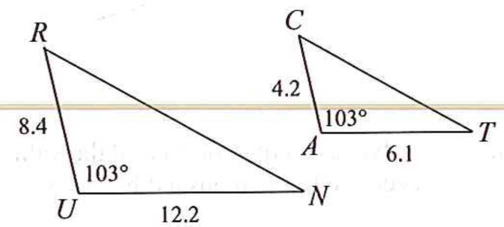
MATH NOTES

METHODS AND MEANINGS

Complete Conditions for Triangle Similarity

There are exactly three valid, non-redundant triangle similarity conjectures that use only sides and angles. (A conjecture is "redundant" if it includes more information than is necessary to establish triangle similarity.) They are abbreviated as: SSS ~, AA ~, and SAS ~. In the SAS ~ Conjecture, the "A" is placed between the two "S"s to indicate that the angle must be *between* the two sides used.

For example, ΔRUN and ΔCAT below are similar by SAS ~. $\frac{RU}{CA} = 2$ and $\frac{UN}{AT} = 2$, so two pairs of corresponding side lengths share a common ratio. The measure of the angle *between* $\overline{RU}$ and $\overline{UN}$, $\measuredangle U$, equals the measure of the angle between $\overline{CA}$ and $\overline{AT}$, $\measuredangle A$, so the conditions for SAS ~ are met.



Review & Preview

3-88. Sketch each triangle described below, if possible. If not possible, explain why it is not possible.

a. Equilateral obtuse triangle

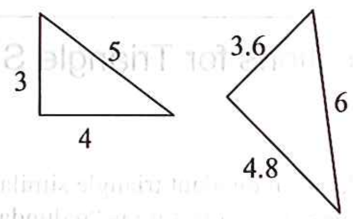
b. Right scalene triangle

c. Obtuse isosceles triangle

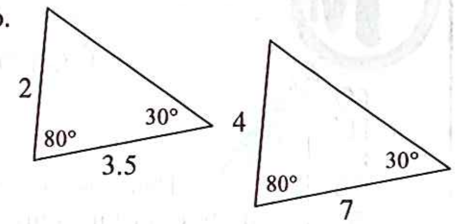
d. Acute right triangle

3-89. Determine which similarity conjectures (AA ~, SSS ~, or SAS ~) could be used to establish that the following pairs of triangles are similar. List as many as you can.

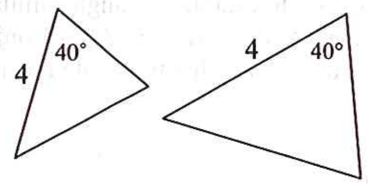
a.



b.



c.



3-90. Solve each equation to find the value of x. Leave your answers in decimal form accurate to the thousandths place.

a. $\frac{3.2}{x} = \frac{7.5}{x^2}$

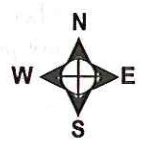
b. $4(x-2)+3(-x+4)=-2(x-3)$

c. $2x^2+7x-15=0$

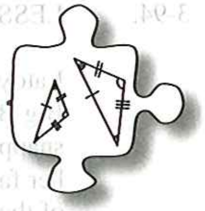
d. $3x^2-2x=+1$

3-91. On graph paper, sketch a rectangle with side lengths of 15 units and 9 units. Shrink the rectangle by a zoom factor of $\frac{1}{3}$. Make a table showing the area and perimeter of both rectangles.

3-92. Susan lives 20 miles northeast of Matt. Simone lives 15 miles dues south of Susan. If Matt lives due west of Simone, approximately how many miles does he live from Simone? Draw a diagram and show all work.



3.2.6 What can I do with similar triangles?



Applying Similarity

In previous lessons, you have learned methods for finding similar triangles. Once you find triangles are similar, how can that help you? Today you will apply similar triangles to analyze situations and solve new applications. As you work on today's problems, ask the following questions in your team:

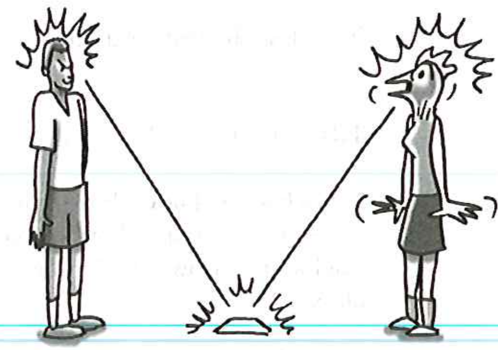
What is the relationship?

Are any triangles similar? What similarity conjecture can we use?

3-93. YOU ARE GETTING SLEEPY…

Legend has it that if you stare into a person's eyes in a special way, you can hypnotize them into squawking like a chicken. Here's how it works.

Place a mirror on the floor. Your victim has to stand exactly 200 cm away from the mirror and stare into it. The only tricky part is that you need to figure out where you have to stand so that when you stare into the mirror, you are also staring into your victim's eyes.



If your calculations are correct and you stand at the *exact* distance, your victim will squawk like a chicken!

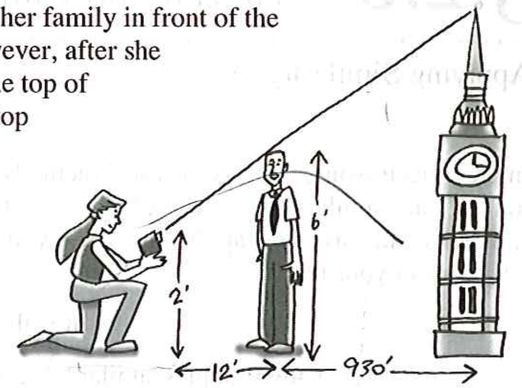
a. Choose a member of your team to hypnotize. Before heading to the mirror, first analyze this situation. Draw a diagram showing you and your victim standing on opposite sides of a mirror. Measure the heights of both yourself and your victim (heights to the eyes, of course), and label all the lengths you can on the diagram. (Remember: your victim will need to stand 200 cm from the mirror.)

b. Are there similar triangles in your diagram? **Justify** your conclusion. (Hint: Remember what you know about how light reflects off mirrors.) Then calculate how far you will need to stand from the mirror to hypnotize your victim.

c. Now for the moment of truth! Have your teammate stand 200 cm away from the mirror, while you stand at your calculated distance from the mirror. Do you make eye contact? If not, check your measurements and calculations and try again.

3-94. LESSONS FROM ABROAD

Latoya was trying to take a picture of her family in front of the Big Ben clock tower in London. However, after she snapped the photo, she realized that the top of her father's head exactly blocked the top of the clock tower!



While disappointed with the picture, Latoya thought she might be able to estimate the height of the tower using her math knowledge. Since Latoya took the picture while kneeling, the camera was 2 feet above the ground. The camera was also 12 feet from her 6-foot tall father, and he was standing about 930 feet from the base of the tower.

a. Sketch the diagram above on your paper and locate as many triangles as you can. Can you find any triangles that must be similar? If so, explain how you know they are similar.

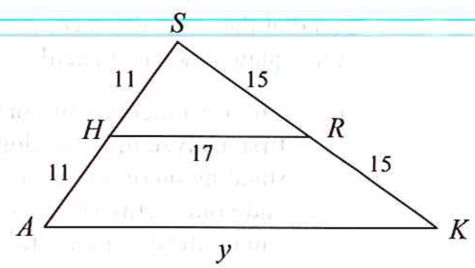
b. Use the similar triangles to determine the height of the Big Ben clock tower.

3-95. TRIANGLE CHALLENGE

Use what you know about triangles and angle relationships to answer these questions about the diagram below. As you work, make a careful record of your **reasoning** (including a flowchart for any similarity arguments) and be ready to share it with the class.

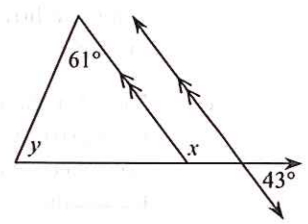
a. First challenge: Find y.

b. Second challenge: Is $\overline{HR}$ parallel to $\overline{AK}$? How do you know?

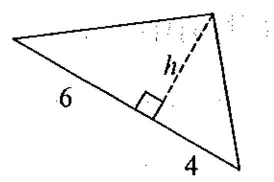


Review & Preview

3-96. Use the relationships in the diagram at right to solve for x and y. **Justify** your solutions.



3-97. The area of the triangle at right is 25 square units. Find the value of h. Then find the perimeter of the entire triangle. Show all work.

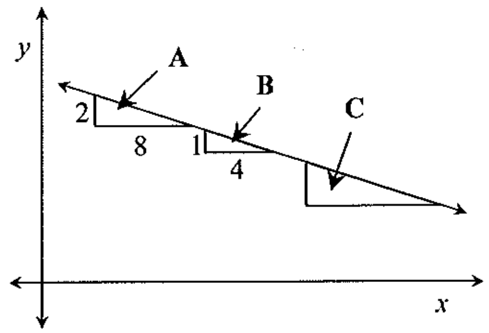


3-98. **Examine** the graph of the line at right.

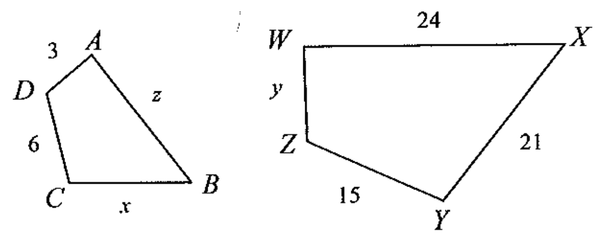
a. Find the slope of this line using slope triangle A.

b. Find the slope using slope triangle B.

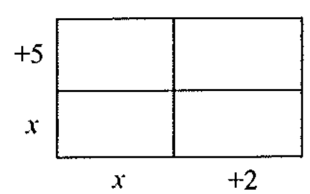
c. Without calculating, what does the slope ratio for slope triangle C have to be?



3-99. If $ABCD \sim WXYZ$, find x, y, and z.



3-100. The area of the rectangle shown at right is 40 square units. Write and solve an equation to find x. Then find the dimensions of the rectangle.



Chapter 3 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned in this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following two subjects. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Connections: How are the topics, ideas, and words that you learned in previous courses connected to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

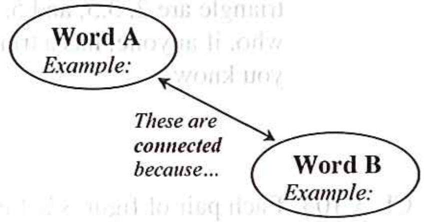
The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

AA ~	angle	conditional
congruent	conjecture	**corresponding sides**
dilation	**enlarge**	**flowchart**
hypotenuse	**logical argument**	original
perimeter	**proportional equation**	ratio
relationship	**SAS ~**	**sides**
similarity	**SSS ~**	**statement**
translate	vertex	**zoom factor**

Problem continues on next page →

② *Problem continued from previous page.*

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the example below. A word can be connected to any other word as long as there is a justified connection. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

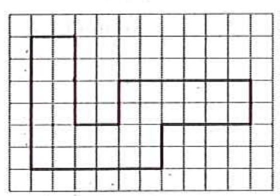
This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will direct you how to do this. Your teacher may give you a "GO" page to work on. "GO" stands for "Graphic Organizer," a tool you can use to organize your thoughts and communicate your ideas clearly.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you feel comfortable with and which types you need more help with. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 3-101. **Examine** the shape at right.



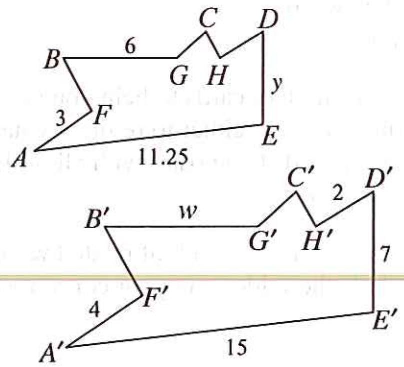
a. On graph paper, enlarge this shape by a factor of 3.

b. Now redraw the enlarged shape from part (a) using a zoom factor of $\frac{1}{2}$.

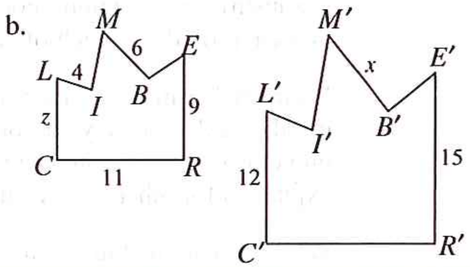
CL 3-102. Jermaine has a triangle with sides 8, 14, and 20. Sadie and Aisha both think that they have triangles that are similar to Jermaine's triangle. The sides of Sadie's triangle are 2, 3.5, and 5. The sides of Aisha's triangle are 4, 10, and 16. Decide who, if anyone, has a triangle similar to Jermaine's triangle. Be sure to explain how you know.

CL 3-103. Each pair of figures below is similar. Find the lengths of all unknown sides.

a.



b.



CL 3-104. Draw an example for each geometric term below. Diagrams should be clearly marked with all necessary information. You should also include a brief description of the qualities of each term.

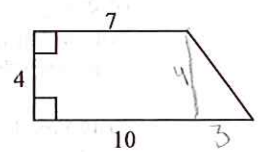
a. supplementary angles

b. alternate interior angles

c. parallel lines

d. complementary angles

CL 3-105. Find the perimeter and area of the shape at right.

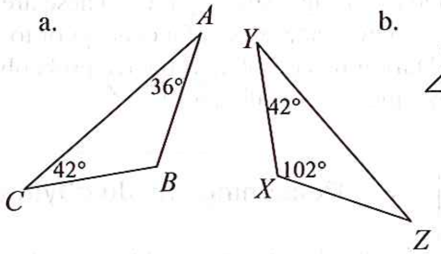


CL 3-106. Create a flowchart that represents the story:

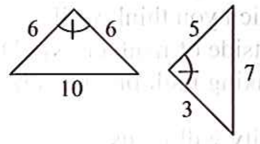
Marcelle and Harpo live at Apt. I, 8 Logic St. Marcelle took his guitar to band practice across town and isn't back yet. Harpo hears guitar music in the hallway. He decides that someone else in the building also plays guitar.

CL 3-107. For each pair of triangles below, determine whether or not the triangles are similar. If they are similar, show your **reasoning** in a flowchart. If they are not similar, explain how you know.

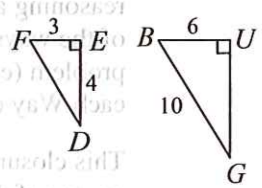
a.



b.

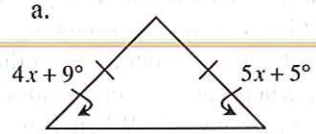


c.

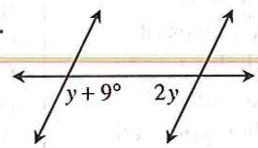


CL 3-108. Use the relationships in the diagrams below to find the values of the variables, if possible. The diagrams are not drawn to scale.

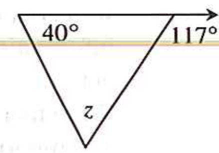
a.



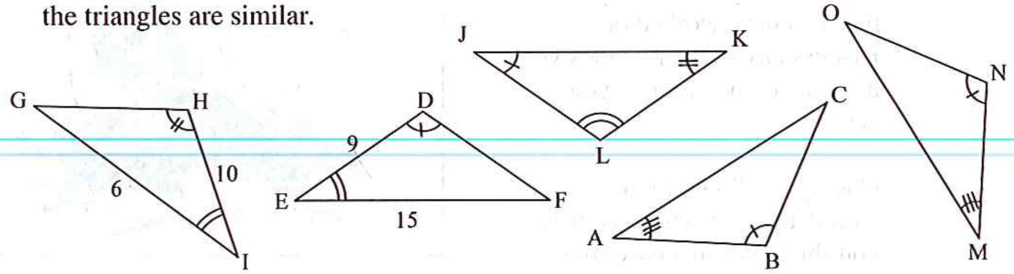
b.



c.



CL 3-109. Among the triangles below are pairs of similar triangles. Find the pairs of similar triangles and state the triangle similarity conjecture that you used to determine that the triangles are similar.



CL 3-110. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: investigating, examining, reasoning and justifying, visualizing, and choosing a strategy/tool. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **reasoning and justifying**. Read the description of this Way of Thinking at right.

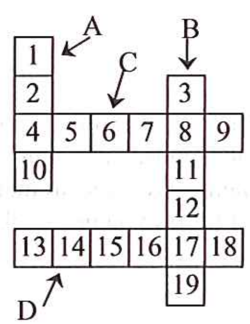
Think about the problems you have worked on in this chapter. When did you need to make sense out of multiple pieces of information? What helps you to determine what makes sense? Were there times when you made assumptions instead of relying on facts? How have you used **reasoning** in a previous math class? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any of the methods you have developed to help you **reason**.

Reasoning and Justifying

To use logical **reasoning** means to organize facts with the purpose of making and communicating a convincing argument. As you develop this way of thinking, you will learn what pieces of information are facts and how you can combine facts to make convincing (uncontestable) arguments. You think this way when you answer questions like "*Is that always true?*" When you catch yourself defending a statement or idea, you are using logical reasoning.

Once your discussion is complete, think about the way you think as you answer the questions below.

a. At right is a crossword puzzle and a list of words that fit within the puzzle. Do you know where all of the words MUST go or is there more than one possible solution? Write an argument that will convince your teacher of your answer.

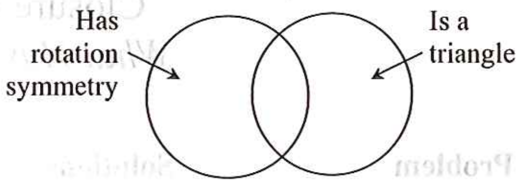


IRON
SODIUM
HELIUM
OXYGEN

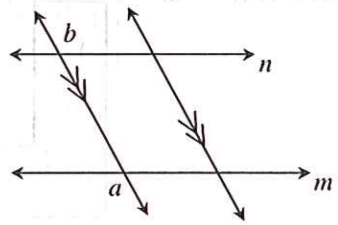
Problem continues on next page →

⑤ *Problem continued from previous page.*

b. **Examine** the Venn diagram at right. What shape(s) can go in the intersection? **Justify** your statements so that they are convincing.



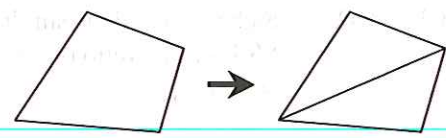
c. Being able to **justify** your thinking is especially important when you are working in a study team. While you may have a correct idea, if you cannot convince your team that your ideas are valid, your teammates may not agree.



Consider the diagram above. Assume that your teammates think angles a and b must be congruent. Do you agree? How can you explain your **reasoning** so that your team is convinced? Use a diagram to support your argument.

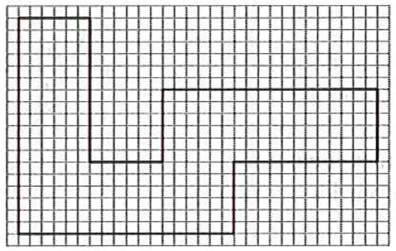
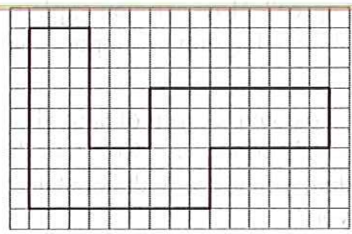
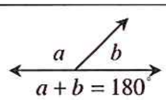
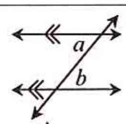
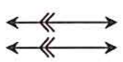
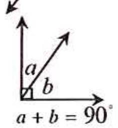
d. What if someone else is trying to convince you of something? How do you think as you follow someone else's argument? Consider this as you read Lila's **reasoning** below. Then decide if you agree with her statement or not. If you agree, what helped convince you?

I know that the sum of the angles of a triangle is 180°, but I don't think that is true for a quadrilateral. If I draw a diagonal, I split my quadrilateral into two triangles. I know that the angles of each of these triangles add up to 180°. Therefore, I think the angles of the quadrilateral must add up to 360°.



Here is my quadrilateral. *I divided it into 2 triangles.*

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solutions	Need Help?	More Practice
CL 3-101.	a.  b. 	Lessons 3.1.1 and 3.1.2 Math Notes boxes, problem 3-11	Problems 3-12, 3-24, 3-26, 3-27, 3-28, 3-91
CL 3-102.	Sadie's triangle is similar to Jermaine by SSS ~; The ratio of the corresponding sides is $\frac{1}{4}$.	Lessons 3.2.1 and 3.2.5 Math Notes boxes	Problems 3-46, 3-24, 3-54, 3-55, 3-57, 3-66, 3-80, 3-83, 3-84, 3-86, 3-89
CL 3-103.	a. $w = 8$, $y = \frac{21}{4} = 5.25$ b. $x = 10$, $z = \frac{36}{5} = 7.2$	Lessons 3.1.2 and 3.1.3 Math Notes boxes	Problems 3-23, 3-35, 3-36, 3-38, 3-59, 3-65, 3-71, 3-99
CL 3-104.	a.  b.  c.  d. 	Lessons 2.1.1 and 2.1.4 Math Notes boxes, Problem 2-3	Problems 2-13, 2-16, 2-17, 2-18, 2-23, 2-24, 2-25, 2-62, 2-72, 3-20, 3-31, 3-60

Problem	Solutions	Need Help?	More Practice
CL 3-105.	Perimeter = 26 un., Area = 34 un.^{2}	Lessons 1.1.3, 2.2.4, and 2.3.3 Math Notes boxes	Problems 2-66, 2-75, 2-79, 2-90, 2-120, 3-5, 3-42, 3-51, 3-81
CL 3-106.	*Harpo hears guitar music in the hallway.* + *Marcelle took his guitar to band practice across town and isn't back yet.* → *Harpo decides that someone else in the building also plays guitar.*	Lesson 3.2.4 Math Notes box	Problems 3-62, 3-56, 3-67, 3-87
CL 3-107.	a. $\angle BAC = \angle XZY$ + $\angle ACB = \angle ZYX$ → $\Delta ABC \sim \Delta ZXY$ AA ~ b. Not similar because corresponding sides do not have the same ratio. c. $\frac{FD}{BG} = \frac{5}{10} = \frac{1}{2}$ + $\frac{DE}{GU} = \frac{4}{8} = \frac{1}{2}$ + $\frac{FE}{BU} = \frac{3}{6} = \frac{1}{2}$ → $\Delta FED \sim \Delta BUG$ SSS ~	Lessons 3.2.1 and 3.2.5 Math Notes Boxes	Problems 3-46, 3-49, 3-53, 3-54, 3-55, 3-57, 3-64, 3-66, 3-80, 3-83, 3-84, 3-89
CL 3-108.	a. $x = 4°$ b. Cannot determine because the lines are not marked parallel. c. $z = 77°$	Lessons 2.1.1 and 2.1.4 Math Notes boxes, Problem 2-3	Problems 2-13, 2-16, 2-17, 2-18, 2-23, 2-24, 2-25, 2-31, 2-32, 2-38, 2-49, 2-51, 2-62, 2-72, 2-111, 3-20, 3-31, 3-60, 3-70, 3-96
CL 3-109.	$\Delta ABC \sim \Delta MNO$, by AA ~ $\Delta EDF \sim \Delta IGH$, by SAS ~ $\Delta EDF \sim \Delta LJK$, by AA ~ $\Delta IGH \sim \Delta LJK$, by AA ~	Lessons 3.2.1 and 3.2.4 Math Notes boxes	Problems 3-46, 3-49, 3-53, 3-54, 3-55, 3-57, 3-64, 3-66, 3-80, 3-83, 3-84, 3-89