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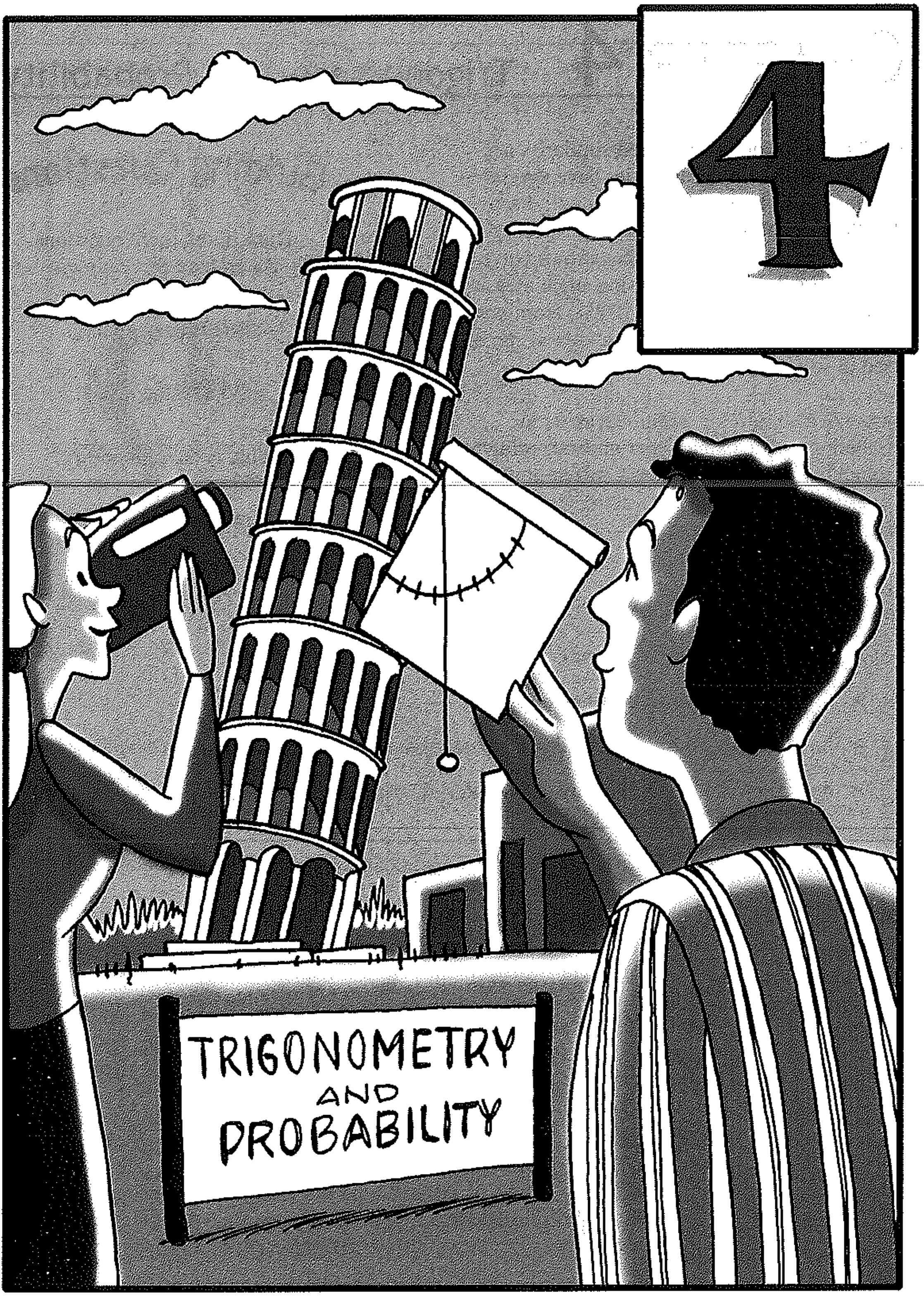


CHAPTER 4 Trigonometry and Probability

In Chapter 3, you **investigated** similarity and discovered that similar triangles have special relationships. In this chapter, you will discover that the side ratios in a right triangle can serve as a powerful mathematical tool that allows you to find missing side lengths and missing angle measures for any right triangle. You will also learn how these ratios (called trigonometric ratios) can be used in solving problems.

You will also develop additional skills in prediction as you extend your understanding of probability. You will **examine** different models to represent possibilities and to assist you in calculating probabilities.

Guiding Questions

Think about these questions throughout this chapter:

Are they similar?

What is the ratio?

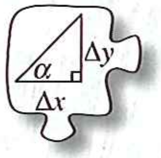
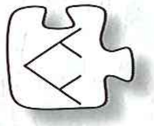
What is the probability?

How can I represent it?

In this chapter, you will learn:

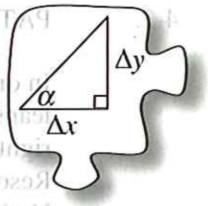
- how the tangent ratio is connected to the slope of a line.
- the trigonometric ratio of tangent.
- how to apply trigonometric ratios to find missing measurements in right triangles.
- how to model real world situations with right triangles and use trigonometric ratios to solve problems.
- several ways to model probability situations, such as tree diagrams and area models.

Chapter Outline

	**Section 4.1**	Students will **investigate** the relationship between the slope of a line and the slope angle. The slope ratio will be used to find missing measurements of a right triangle and to solve real world problems.
	**Section 4.2**	Students will continue with their study of probability by studying different models (systematic lists, tree diagrams and area models) used to represent situations involving chance.

4.1.1 What patterns can I use?

Constant Ratios in Right Triangles

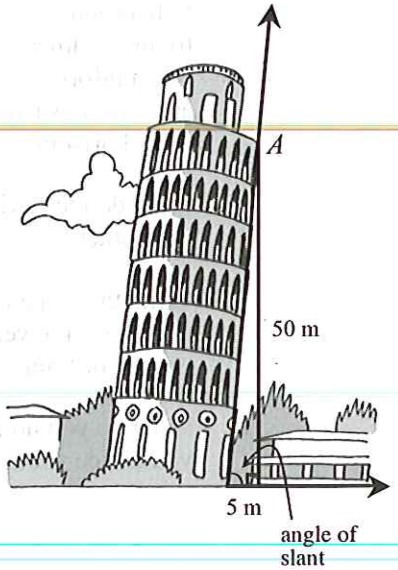


In Chapter 3, you looked for relationships and patterns among shapes such as triangles, parallelograms, and trapezoids. Now we are going to focus our attention on slope triangles, which were used in algebra to describe linear change. Are there geometric patterns within slope triangles themselves that we can use to answer other questions? In this lesson, you will look closely at slope triangles on different lines to explore their patterns.

4-1. LEANING TOWER OF PISA

For centuries, people have marveled at the Leaning Tower of Pisa due to its slant and beauty. Ever since construction of the tower started in the 1100's, the tower has slowly tilted south and has increasingly been at risk of falling over. It is feared that if the angle of slant ever falls below 83°, the tower will collapse.

Engineers closely monitor the angle at which the tower leans. With careful measuring, they know that the point labeled *A* in the diagram at right is now 50 meters off the ground. Also, they determined that when a weight is dropped from point *A*, it lands five meters from the base of the tower, as shown in the diagram.



a. With the measurements provided above, what can you determine?

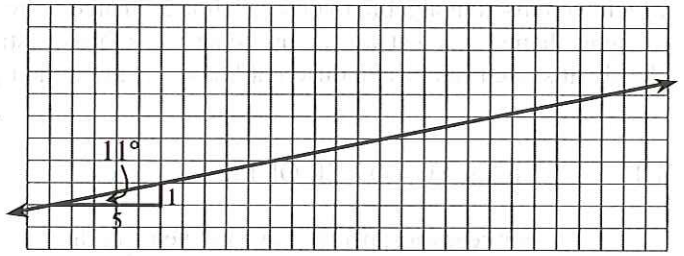
b. Can you determine the angle at which the tower leans? Why or why not?

c. At the end of Section 4.1, you will know how to find the angle for this situation and many others. However, at this point, how else can you describe the "lean" of the leaning tower?

4-2. PATTERNS IN SLOPE TRIANGLES

In order to find an angle (such as the angle at which the Leaning Tower of Pisa leans), you need to **investigate** the relationship between the angles and the sides of a right triangle. You will start by studying slope triangles. Obtain the Lesson 4.1.1 Resource Pages (two in all) from your teacher and find the graph shown below. Notice that one slope triangle has been drawn for you. **Note:** For the next several lessons angle measures will be rounded to the nearest degree.

a. Draw three new slope triangles on the line. Each should be a different size. Label each triangle with as much information as you can, such as its horizontal and vertical lengths and its angle measures.



b. What do these triangles have in common? How are these triangles related to each other?

c. Write the slope ratio for each triangle as a fraction, such as $\frac{\Delta y}{\Delta x}$. (Note: Δy represents the vertical change or "rise," while Δx represents the horizontal change or "run.") Then change the slope ratio into decimal form.

d. What do you notice about the slope ratios written in fraction form? What do you notice about the decimals?

4-3. Tara thinks she sees a pattern in these slope triangles, so she decides to make some changes in order to **investigate** whether or not the patterns remain true.

a. She asks, "What if I draw a slope triangle on this line with $\Delta y = 6$? What would be the Δx of my triangle?" Answer her question and explain how you figured it out.

b. "What if Δx is 40?" she wonders. "Then what is Δy?" Find Δy, and explain your **reasoning**.

c. Tara wonders, "What if I draw a slope triangle on a different line? Can I still use the same ratio to find a missing Δx or Δy value?" Discuss this question with your team and explain to Tara what she could expect.

4-4. CHANGING LINES

In part (c) of problem 4-3, Tara asked, "What if I draw my triangle on a different line?" With your team, **investigate** what happens to the slope ratio and slope angle when the line is different. Use the graph grids provided on your Lesson 4.1.1 Resource Pages to graph the lines described below. Use the graphs and your answers to the questions below to respond to Tara's question.

a. On graph A, graph the line $y=\frac{2}{5}x$. What is the slope ratio for this line? What does the slope angle appear to be? Does the information about this line support or change your conclusion from part (c) of problem 4-3? Explain.

b. On graph B, you are going to create $\measuredangle QPR$ so that it measures 18°. First, place your protractor so that point P is the vertex. Then find 18° and mark and label a new point, R. Draw ray $\overrightarrow{PR}$ to form $\measuredangle QPR$. Find an approximate slope ratio for this line.

c. Graph the line $y=x+4$ on graph C. Draw a slope triangle and label its horizontal and vertical lengths. What is $\frac{\Delta y}{\Delta x}$ (the slope ratio)? What is the slope angle?

4-5. TESTING CONJECTURES

The students in Ms. Coyner's class are writing conjectures based on their work today. As a team, decide if you agree or disagree with each of the conjectures below. Explain your **reasoning**.

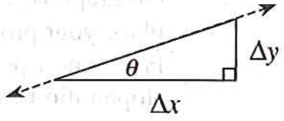
- All slope triangles have a ratio $\frac{1}{5}$.
- If the slope ratio is $\frac{1}{5}$, then the slope angle is approximately 11°.
- If the line has an 11° slope angle, then the slope ratio is approximately $\frac{1}{5}$.
- Different lines will have different slope angles and different slope ratios.

MATH NOTES

METHODS AND MEANINGS

Slope and Angle Notation

Slope is the ratio of the vertical distance to the horizontal distance in a slope triangle. The vertical part of the triangle is called Δy, (read "change in y"), while the horizontal part of the triangle is called Δx (read "change in x").



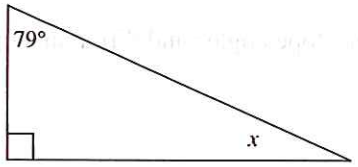
When we are missing a side length in a triangle, we often assign that length a variable from the English alphabet such as x, y, or z. However, sometimes we need to distinguish between an unknown side length and an unknown angle measure. With that in mind, mathematicians sometimes use Greek letters as variables for angle measurement. The most common variable for an angle is the Greek letter θ (*theta*), pronounced "THAY-tah." Two other Greek letters commonly used include α (*alpha*), and β (*beta*), pronounced "BAY-tah."

When a right triangle is oriented like a slope triangle, such as the one in the diagram above, the angle the line makes with the horizontal side of the triangle is called a **slope angle.**

Review & Preview

4-6. Use what you know about the angles of a triangle to find the value of x and the angles in each triangle below.

a.



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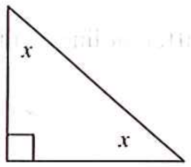

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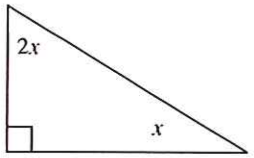

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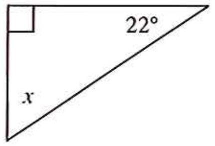
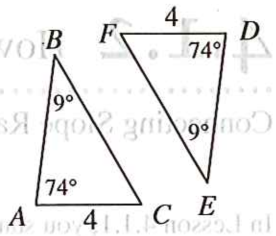


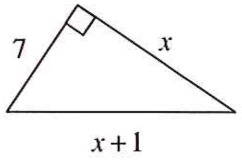
4-7. Use the triangles at right to answer the following questions.



a. Are the triangles at right similar? How do you know? Show your **reasoning** in a flowchart.

b. **Examine** your work from part (a). Are the triangles also congruent? Explain why or why not.

4-8. As Randi started to solve for x in the diagram at right, she wrote the equation $7^2 + x^2 = (x+1)^2$.



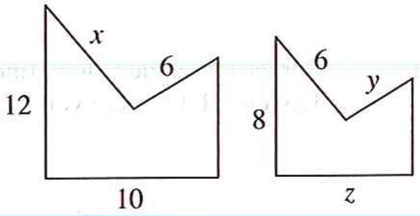
a. Is Randi's equation valid? Explain your thinking.

b. To solve her equation, first rewrite $(x+1)^2$ by multiplying $(x+1)(x+1)$. You may want to review the Math Notes box for Lesson 2.2.2.

c. Now solve your equation for x.

d. What is the perimeter of Randi's triangle?

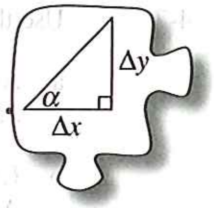
4-9. Assume that the shapes at right are similar. Find the values of x, y, and z.



4-10. Are the lines represented by the equations at right parallel? Support your **reasoning** with convincing evidence.

$$y = -\tfrac{3}{5}x + 2$$
$$y = -\tfrac{3}{5}x - 3$$

4.1.2 How important is the angle?



Connecting Slope Ratios to Specific Angles

In Lesson 4.1.1, you started **trigonometry**, the study of the measures of triangles. As you continue to **investigate** right triangles with your team today, use the following questions to guide your discussion:

What do we know about this triangle?

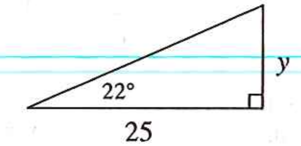
How does this triangle relate to other triangles?

Which part is Δx? Which part is Δy?

4-11. What do you know about this triangle? To what other triangles does it relate? Use any information you have to solve for x.

4-12. For each triangle below, find the missing angle or side length. Use your work from Lesson 4.1.1 to help you.

a.



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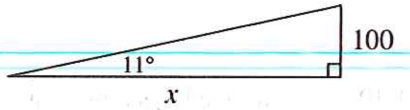

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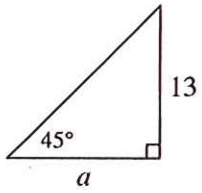

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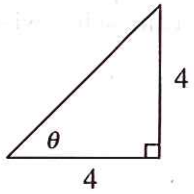

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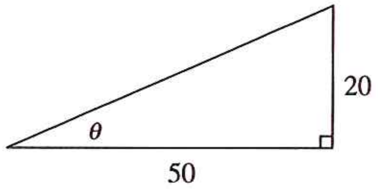

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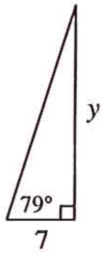
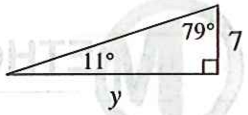


4-13. Sheila says the triangle in part (f) of problem 4-12 is the same as her drawing at right.



a. Do you agree? Use tracing paper to convince you of your conclusion.

b. Use what you know about the slope ratio of 11° to find the slope ratio for 79°.

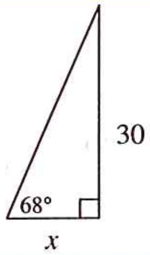
c. What is the relationship of 11° and 79°? Of their slope ratios?

4-14. For what other angles can you find the slope ratios based on the work you did in Lesson 4.1.1?

a. For example, since you know the slope ratio for 22°, what other angle do you know the slope ratio for? Use tracing paper to find a slope ratio for the complement of each slope angle you know. Use tracing paper to help re-orient the triangle if necessary.

b. Use this information to find x in the diagram at right.

c. Write a conjecture about the relationship of the slope ratios for complementary angles. You may want to start with, "*If one angle of a right triangle has the slope ratio* $\frac{a}{b}$*, then ...*"



4-15. BUILDING A TRIGONOMETRY TABLE

So far you have looked at several similar slope triangles and their corresponding slope ratios. These relationships will be very useful for finding missing side lengths or angle measures of right triangles for the rest of this chapter.

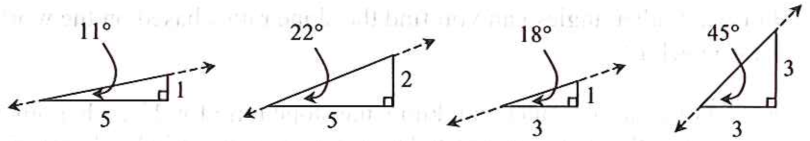
Before you forget this valuable information, organize information about the triangles and ratios you have discovered so far in the table on the Lesson 4.1.2 ("Trig Table") Resource Page provided by your teacher or download from www.cpm.org. Keep it in a safe place for future reference. Include all of the angles you have studied up to this point. An example for 11° is filled in on the table to get you started.

MATH NOTES

METHODS AND MEANINGS

Slope Ratios and Angles

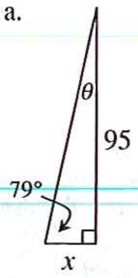
In Lesson 4.1.1, you discovered that certain slope angles produce slope triangles with special ratios. Below are the triangles you have studied so far. Note that the angles below are rounded to the nearest degree.



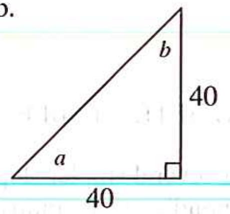
Review & Preview

4-16. Use your Trig Table from problem 4-15 to help you find the value of each variable below.

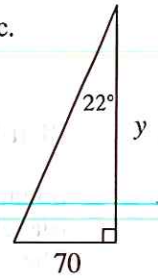
a.



b.



c.

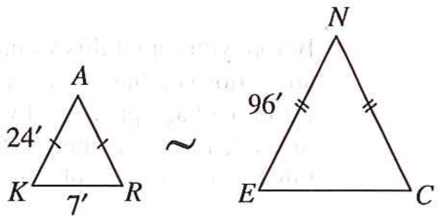


4-17. The triangles shown at right are similar.

a. What is the ratio of side length NE to side length AK?

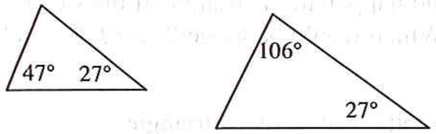
b. Use a ratio to compare the perimeters of ΔENC and ΔKAR. How is the perimeter ratio related to the side length ratio?

c. If you have not already done so, find the length of $\overline{EC}$.



4-18. **Examine** each pair of figures below. Are they similar? Explain how you know.

a.



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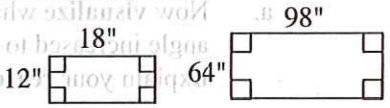

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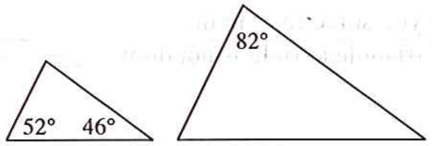
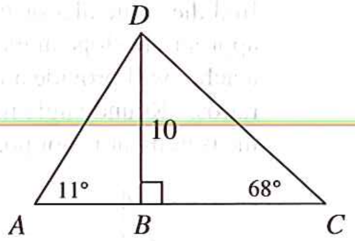
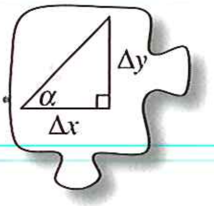


4-19. Find the equation of the line with a slope of $\frac{1}{3}$ that goes through the point (0, 9).

4-20. **Examine** the figure at right, which is not drawn to scale. Which is longer, $\overline{AB}$ or $\overline{BC}$? Explain your answer.



4.1.3 What if the angle changes?



Expanding the Trig Table

In the last few lessons, you found the slope ratios for several angles. However, so far you are limited to using the slope angles that are currently in your Trig Table. How can we find the ratios for other angles? And how are the angles related to the ratio?

Today your goal is to determine ratios for more angles and to find patterns. As you work today, keep the following questions in mind:

What happens to the slope ratio when the angle increases? Decreases?

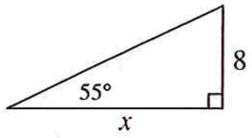
What happens to the slope ratio when the angle is 0°? 90°?

When is a slope ratio more than 1? When is it less than 1?

4-21. On your paper, draw a slope triangle with a slope angle of 45°.

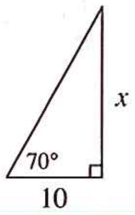
a. Now **visualize** what would happen to the triangle if the slope angle increased to 55°. Which would be longer? Δy? Or Δx? Explain your **reasoning**.

b. Using your dynamic geometry tool, create a triangle with a slope angle measuring 55°. Then use the resulting slope ratio to help you solve for x in the triangle at right. (Note: the triangle at right is not draw to scale.)



4-22. Copy each of the following triangles onto your paper. Decide whether or not the given measurements are possible. If the triangle is possible, find the value of x or θ. Use the dynamic geometry tool to find the appropriate slope angles or ratios. If technology is not available, your teacher will provide a Lesson 4.1.3 Resource Page with the needed ratios. Round angle measures to the nearest degree. If a triangle's indicated measurement is not possible, explain why.

a.



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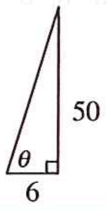

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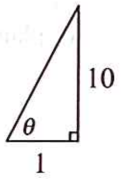

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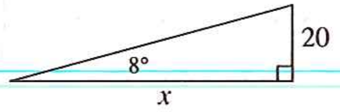

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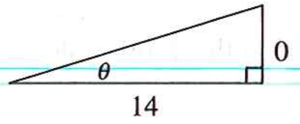

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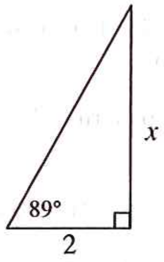
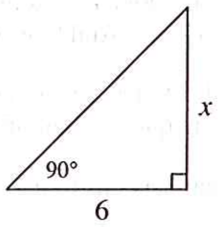


g.



4-23. If you have not already, add these new slope ratios with their corresponding angles to your Trig Table. Be sure to draw and label the triangle for each new angle. Summarize your findings—which slope triangles did not work? Do you see any patterns?

4-24. What statements can you make about the connections between slope angle and slope ratio? In your Learning Log, write down all of your observations from this lesson. Be sure to answer the questions given at the beginning of the lesson (reprinted below). Title this entry, "Slope Angles and Slope Ratios" and include today's date.

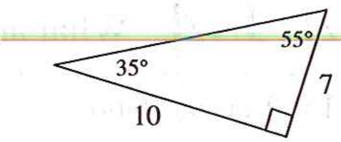
What happens to the slope ratio when the angle increases? Decreases?

What happens to the slope ratio when the angle is 0°? 90°?

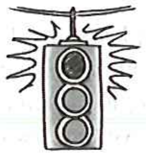
When is a slope ratio more than 1? When is it less than 1?

Review & Preview

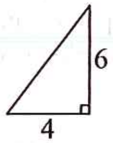
4-25. Ben thinks that the slope ratio for this triangle is $\frac{7}{10}$. Carlissa thinks the ratio is $\frac{10}{7}$. Who is correct? Explain your thinking fully.



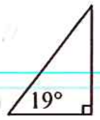
4-26. Use your observations from problem 4-24 to answer the following questions:



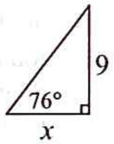
a. Thalia did not have a tool to help her find the slope angle in the triangle at right. However, she claims that the slope angle has to be more than 45°. Do you agree with Thalia? Why?



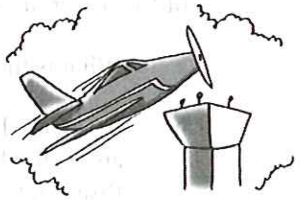
b. Lyra was trying to find the slope ratio for the triangle at right, and she says the answer is $\frac{\Delta y}{\Delta x} = 2.675$. Isiah claims that cannot be correct. Who is right? How do you know?



c. Without finding the actual value, what information do you know about x in the diagram at right?



4-27. An airplane takes off and climbs at an angle of 11°. If the plane must fly over a 120-foot tower with at least 50 feet of clearance, what is the minimum distance between the point where the plane leaves the ground and the base of the tower?



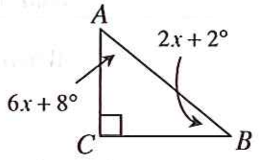
a. Draw and label a diagram for this situation.

b. What is the minimum distance between the point where the plane leaves the ground and the tower? Explain completely.

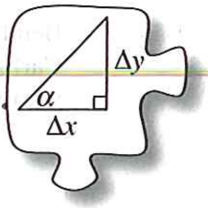
4-28. Edwina has created her own Shape Bucket and has provided the clues below about her shapes. List one possible group of shapes that could be in her bucket.

$$P(\text{equilateral}) = 1$$
$$P(\text{triangle}) = \frac{1}{3}$$

4-29. Use what you know about the sum of the angles of a triangle to find $m\measuredangle ABC$ and $m\measuredangle BAC$. Are these angles acute or obtuse? Find the sum of these two angles. How can we describe the relationship of these two angles?



4.1.4 What about other right triangles?

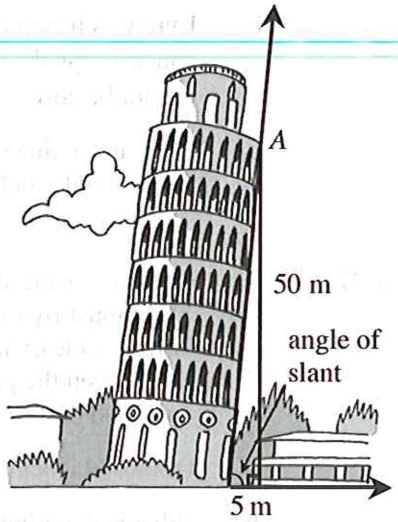


The Tangent Ratio

In Lesson 4.1.2 you started a Trig Table of angles and their related slope ratios. Unfortunately, you only have the information for a few angles. How can you quickly find the ratios for other angles when a computer is not available or when an angle is not on your Trig Table? Do you have to draw each angle to get its slope ratio? Or is there another way?

4-30. WILL IT TOPPLE?

In problem 4-1, you learned that the Leaning Tower of Pisa is expected to collapse once its angle of slant is less than 83°. Currently, the top of the seventh story (point *A* in the diagram at right) is 50 meters above the ground. In addition, when a weight is dropped from point *A*, it lands 5 meters from the base of the tower, as shown in the diagram.

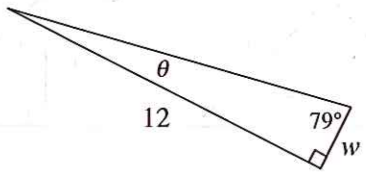


a. What is the slope ratio for the tower?

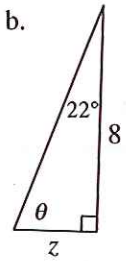
b. Use your Trig Table to determine the angle at which the Leaning Tower of Pisa slants. Is it in immediate danger of collapse?

4-31. Solve for the variables in the triangles below. It may be helpful to first orient the triangle (by rotating your paper or by using tracing paper) so that the triangle resembles a slope triangle. Use your Trig Table for reference.

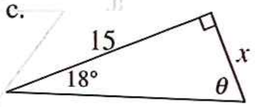
a.



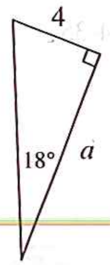
b.



c.



4-32. MULTIPLE METHODS



Tiana, Mae Lin, Eddie, and Amy are looking at the triangle at right and trying to find the missing side length.

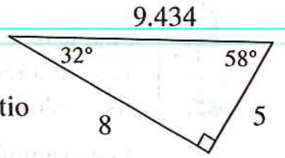
a. Tiana declares, "Hey! We can rotate the triangle so that 18° looks like a slope angle, and then $\Delta y = 4$." Will her method work? If so, use her method to solve for *a*. If not, explain why not.

b. Mae Lin says, "I see it differently. I can tell $\Delta y = 4$ without turning the triangle." How can she tell? Explain one way she could know.

c. Eddie replies, "What if we use 72° as our slope angle? Then $\Delta x = 4$." What is he talking about? Discuss with your team and explain using pictures and words.

d. Use Eddie's observation in part (c) to confirm your answer to part (a).

4-33. USING A SCIENTIFIC CALCULATOR



Examine the triangle at right.

a. According to the triangle at right, what is the slope ratio for 32°? Explain how you decided to set up the ratio. Write the ratio in both fraction and decimal form.

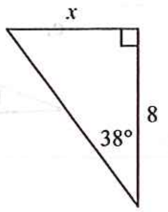
b. What is the slope ratio for the 58° angle? How do you know?

c. Scientific calculators have a button that will give the slope ratio when the slope angle is entered. In part (a), you calculated the slope ratio for 32° as 0.625. Use the "tan" button on your calculator to verify that you get ≈ 0.625 when you enter 32°. Does that button give you ≈ 1.600 when you enter 58°? Be ready to help your teammates find the button on their calculator.

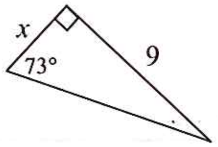
d. The ratio in a right triangle that you have been studying is referred to as the **tangent ratio**. When you want to find the slope ratio of an angle, such as 32°, it is written "tan 32°." So, an equation for this triangle can be written as $\tan 32° = \frac{5}{8}$. Read more about the tangent ratio in the Math Notes box for this lesson.

4-34. For each triangle below, trace the triangle on tracing paper. Label its legs Δy and Δx based on the given slope angle. Then write an equation (such as $\tan 14° = \frac{x}{5}$), use your scientific calculator to find a slope ratio for the given angle, and solve for x.

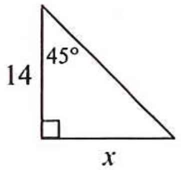
a.



b.



c.



4-35. How do you set up a tangent ratio equation? How can you know which side is Δy? How can you use your scientific calculator to find a slope ratio? Write a Learning Log entry about what you learned today. Be sure to include examples or refer to your work from today. Title this entry "The Tangent Ratio" and include today's date.

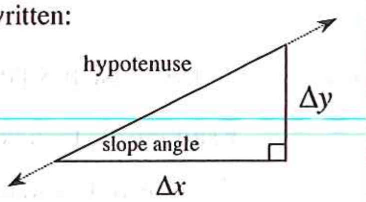
METHODS AND MEANINGS

MATH NOTES

The Tangent Ratio

For any slope angle in a slope triangle, the ratio that compares the Δy to Δx is called the **tangent ratio**. The ratio for any angle is constant, regardless of the size of the triangle. It is written:

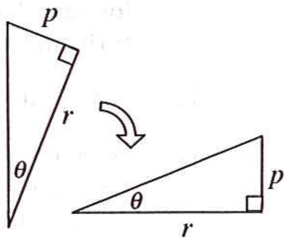
$$\tan(\text{slope angle}) = \frac{\Delta y}{\Delta x}$$



One way to identify which side is Δy and Δx is to first reorient the triangle so that it looks like a slope triangle, as shown at right.

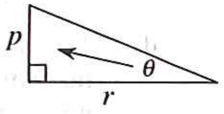
For example, when the triangle at right is rotated, the resulting slope triangle helps to show that the tangent of θ is $\frac{p}{r}$, since θ is the slope angle, p is Δy and r is Δx. This is written:

$$\tan \theta = \frac{p}{r}$$



Whether the triangle is oriented as a slope triangle or not, you can identify Δy as the leg that is always opposite (across the triangle from) the angle, while Δx is the leg closest to the angle.

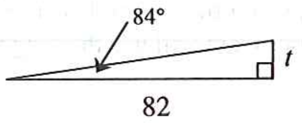
$$\tan \theta = \frac{\textit{opposite leg}}{\textit{adjacent leg}} = \frac{p}{r}$$



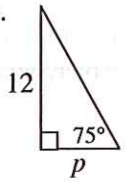
Review & Preview

4-36. Find the missing side length for each triangle. Use the tangent button on your calculator to help.

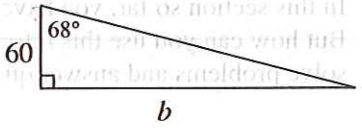
a.



b.

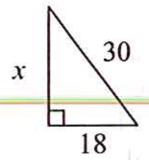


c.

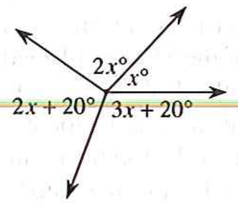


4-37. Use the relationships in the diagrams below to write an equation and solve for *x*.

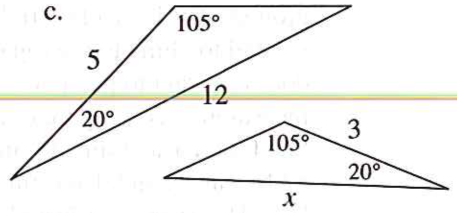
a.



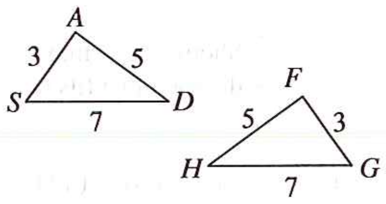
b.



c.



4-38. What is the relationship of the triangles at right? **Justify** your conclusion.



4-39. Mr. Singer made the flowchart at right about a student named Brian.

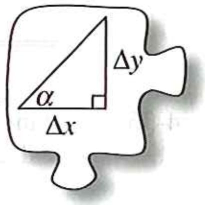
a. What is wrong with Mr. Singer's flowchart?

b. Rearrange the ovals so the flowchart makes more sense.

4-40. When she was younger, Mary had to look up at a 68° angle to see into her father's eyes whenever she was standing 15 inches away. How high above the flat ground were her father's eyes if Mary's eyes were 32″ above the ground?

4.1.5 What if I can't measure it?

Applying the Tangent Ratio



In this section so far, you have learned how to find the legs of a right triangle using an angle. But how can you use this information? Today you and your team will use the tangent ratio to solve problems and answer questions.

4-41. STATUE OF LIBERTY

Lindy gets nose-bleeds whenever she is 300 feet above ground. During a class fieldtrip, her teacher asked if she wanted to climb to the top of the Statue of Liberty. Since she does not want to get a nose-bleed, she decided to take some measurements to figure out how high the torch of the statue is. She found a spot directly under the torch and then measured 42 feet away and determined that the angle up to the torch was 82°. Her eyes are 5 feet above the ground.

Should she climb to the top or will she get a nose-bleed? Draw a diagram that fits this situation. **Justify** your conclusion.

4-42. HOW TALL IS IT?

How tall is Mount Everest? How tall is the White House? Often we want to know a measurement of something we cannot easily measure with a ruler or tape measure. Today you will work with your team to measure the height of something inside your classroom or on your school's campus in order to apply your new tangent tool.

Your Task: Get a **clinometer** (a tool that measures a slope angle) and a meter stick (or tape measure) from your teacher. As a team, decide how you will use these tools to find the height of the object selected by your teacher. Be sure to record all measurements carefully on your Lesson 4.1.5A Resource Page and include a diagram of the situation.

Discussion Points

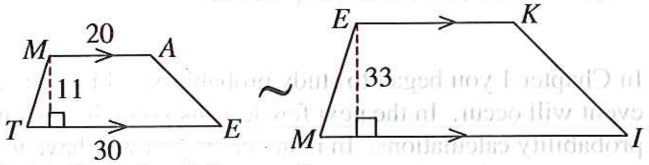
What should the diagram look like?

What measurements would be useful?

How can you use your tools effectively to get accurate measurements?

Review & Preview

4-43. The trapezoids at right are similar.

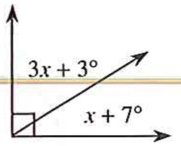


a. What is the ratio of the heights?

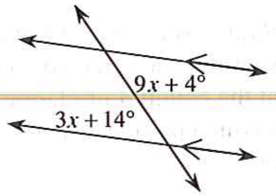
b. Compare the areas. What is the ratio of the areas?

4-44. For each diagram below, write an equation and solve for x, if possible.

a.

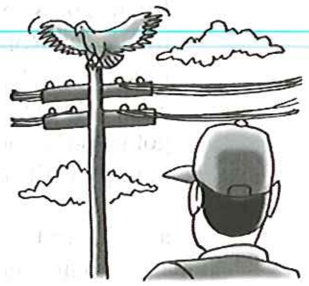


b.



4-45. Joan and Jim are planning a dinner menu including a main dish and dessert. They have 4 main dish choices (steak, tuna casserole, turkey burgers, and lasagna) and 3 dessert choices (brownies, ice cream, and chocolate chip cookies.) How many different dinner menus do they have to choose from? List all of the possible menus.

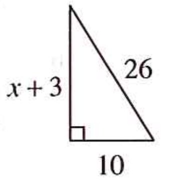
4-46. Leon is standing 60 feet from a telephone pole. As he looks up, a red-tailed hawk lands on the top of the pole. Leon's angle of sight up to the bird is 22° and his eyes are 5.2 feet above the ground.



a. Draw a detailed picture of this situation. Label with all of the given information.

b. How tall is the pole? Show all of your work.

4-47. Find the value of x in the triangle at right. Refer to problem 4-8 for help. Show all work.



4.2.1 How can I make a list?

Introduction to Probability Models

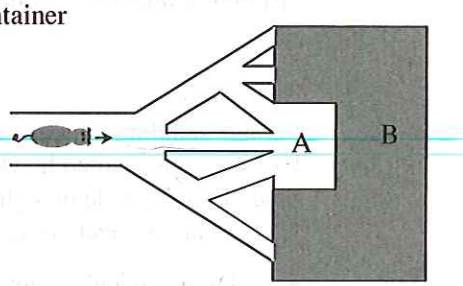
In Chapter 1 you began to study probability, which is a measure of the chance that a particular event will occur. In the next few lessons you will encounter a variety of situations that require probability calculations. In many cases you will have to develop new probability tools to help you analyze these situations. Today's lesson focuses on tools for listing *all* the possible outcomes of a probability situation.

4-48. THE RAT RACE, Part One

Ryan has a pet rat Romeo that he boasts is the smartest rat in the county. Sammy overheard Ryan at the county fair claiming that Romeo could learn to run a particular maze and find the cheese at the end.

"I don't think Romeo's that smart!" Sammy declares, "I think the rat just chooses a random path through the maze."

Ryan has built a maze with the floor plan shown at right. Ryan places the cheese in an airtight container (so Romeo can't smell the cheese!) in room A.



Ryan runs Romeo through the maze 100 times, and Romeo finds the cheese 66 times. "See how smart Romeo is?" Ryan asks, "He clearly learned something and got better at the maze as he went along." Sammy isn't so sure.

a. Does the fact that Romeo ends up in Room A mean that he has learned the route and improved his ability to return to the same room over time? Or do you think he would achieve the same results by moving randomly throughout the maze? Discuss this question with your team.

b. Predicting what will happen requires you to analyze the **probability** of possible outcomes. To accurately decide if Romeo is randomly guessing or if he is learning the route, you will need to learn more about probability. We will return to this problem at the end of Section 4.2. At this point, review the Math Notes box from Lesson 1.3.3 and then move on to problem 4-49.

4-49. **TOP OF THE CHARTS**

Renae's MP3 player can be programmed to randomly play songs from her playlist without repeating a single song. Currently, Renae's MP3 player has 5 songs loaded on it, which are listed at right. As she walks between class, she only has time to listen to one song.

PLAYLIST

a. **I Love My Mama** (country)
by the Strings of Heaven

b. **Don't Call Me Mama** (country)
Duet by Sapphire and Hank Tumbleweed

c. **Carefree and Blue** (R & B)
by Sapphire and Prism Escape

d. **Go Back To Mama** (Rock)
Duet by Bjorn Free and Sapphire

e. **Smashing Lollipops** (Rock)
by Sapphire

a. What is the probability that her MP3 player will select a country song?

b. What is the probability that Renae will listen to a song with "Mama" in the title?

c. What is the probability she listens to a duet with Hank Tumbleweed?

d. What is the probability she listens to a song that is not R & B?

4-50. While waiting for a bus after school, Renae programmed her MP3 player to randomly play two songs. Assume that the MP3 player will not play the same song twice.

a. List all the combinations of two songs that Renae could select. The order that she hears the songs does not matter for your list. How can you be sure that you listed all of the song combinations?

b. Find the probability that Renae will listen to two songs with the name "Mama" in the title.

c. What is the probability that at least one of the songs will have the name "Mama" in the title?

d. Why does it make sense that the probability in part c (d) is higher than the probability in part (c)? b

4-51. When a list is created by following a system (an orderly process), it is called a **systematic list**. Using a systematic list to answer questions involving probability can help you determine **all** of the possible outcomes. There are different **strategies** that may help you make a systematic list, but what is most important is that you methodically follow your system until it is complete. For the problem below, create a systematic list. Be prepared to share your **strategy**.

To get home, Renae can take one of four busses: #41, #28, #55, or #81. Once she is on a bus, she will randomly select one of the following equally likely activities: listening to her MP3 player, writing a letter, or reading a book.

a. List all the possible ways Renae can get home. Use a systematic list to make sure you find all the combinations of a bus and an activity.

b. Use your list to find the following probabilities:

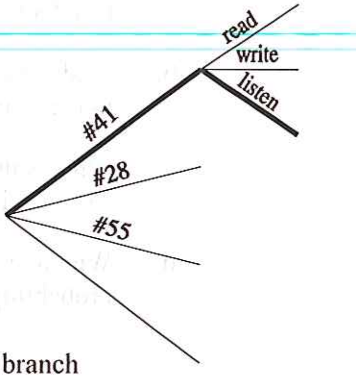
(1) P(Renae takes an odd-numbered bus)

(2) P(Renae does not write a letter)

(3) P(Renae catches the #28 bus and then reads a book)

c. Does her activity depend on which bus she takes? Explain why or why not.

4-52. Sometimes writing an entire systematic list seems unnecessarily repetitive. However, creating a **tree diagram**, like the one started at right, is one way to avoid the repetition. This structure organizes the list by connecting each bus with each activity.



In this tree, the first set of branches represents the bus options. At the end of each of these branches are branches representing the activities. This system reduces some of the writing repetition.

Renae can then read the possible scenarios by following a branch across the diagram. For example, if you follow the bold branches, Renae will take the #41 bus and will listen to her MP3 player.

On your paper, complete this tree diagram to show all of the different travel options that Renae could take.

4-53. For the evening, Renae has programmed her MP3 player to play all five songs in a random order.

PLAYLIST

a. **I Love My Mama** (country)
by the Strings of Heaven

b. **Don't Call Me Mama** (country)
Duet by Sapphire and Hank Tumbleweed

c. **Carefree and Blue** (R & B)
by Sapphire and Prism Escape

d. **Go Back To Mama** (Rock)
Duet by Bjorn Free and Sapphire

e. **Smashing Lollipops** (Rock)
by Sapphire

a. What is the probability that the first song is a country song?

b. If the first song is a country song, does that affect the probability that the second song is a country song? Explain your thinking.

c. As songs are playing, the number of songs left to play decreases. Therefore, the probability of playing each of the remaining songs is dependent on which songs that have played before it. This is an example of **dependent events.** If Renae has already listened to "Don't Call Me Mama," "Carefree and Blue," and "Smashing Lollipops," what is the probability that one of the singers of the fourth song will be Sapphire? Explain your **reasoning.**

d. In problem 4-51, you considered a situation of **independent events,** when the bus that Renae took did not affect which activity she chose. For example, what is the probability that Renae writes a letter if she takes the #41 bus? What if she takes the #55 bus?

Review & Preview

4-54. When Ms. Shreve randomly selects a student in her class, she has a $\frac{1}{3}$ probability of selecting a boy.

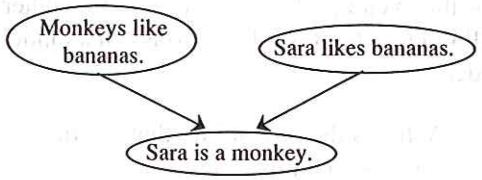
a. If her class has 36 students, how many boys are in Ms. Shreve's class?

b. If there are 11 boys in her class, how many girls are in her class?

c. What is the probability that she will select a girl?

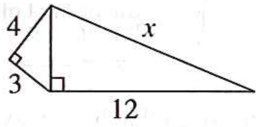
d. Assume that Ms. Shreve's class has a total of 24 students. She selected one student (who was a boy) to attend a fieldtrip and then was told she needed to select one more student to attend. What is the probability that the second randomly selected student will also be a boy?

4-55. What is wrong with the argument shown in the flow chart at right? What assumption does the argument make?

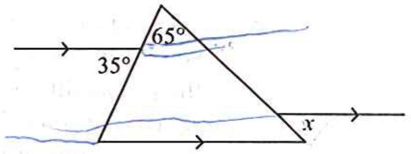


4-56. For each diagram below, solve for x. Name the relationship(s) you used. Show all work.

a.



b.



4-57. Kamillah decided to find the height of the Empire State Building. She walked 1 mile away (5280 feet) from the tower and found that she had to look up 15.5° to see the top. Assuming Manhattan is flat, if Kamillah's eyes are 5 feet above the ground how tall is the Empire State Building?

4-58. Renae wonders, "What if I program my MP3 player to randomly play 3 songs?"

a. Assuming that her MP3 player is still programmed as described in problem 4-53, how many combinations of 3 songs could she listen to? Make a list of all the combinations. Remember that the order of the songs is not important.

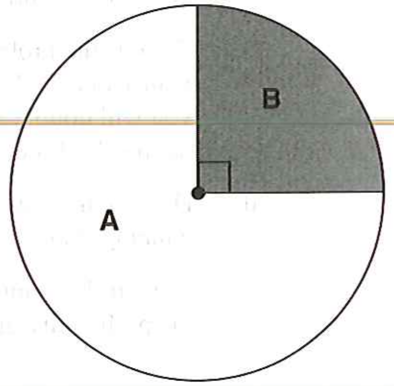
b. Compare your answer to part (a) with the number of combinations that she could have with 2 songs in problem 4-50. Why does it make sense that these are the same?

4.2.2 How can I test my prediction?

Theoretical and Experimental Probability

In Lesson 4.2.1, you organized situations using systematic lists and tree diagrams in order to calculate the probability that an event would occur. How can you verify that your calculated probabilities are accurate? Today you will analyze a couple of games. For each game, you will calculate the chance of winning. Then you will play the game to decide if your probability analysis is accurate.

4-59. Luis is going to spin the spinner at right.

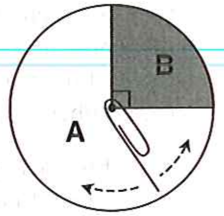


a. What are the possible results of his spin?

b. What do you predict Luis' result will be after one spin? **Justify** your prediction.

c. If Luis spun this spinner four times, what do you predict his results would be?

d. A paperclip with one end extended can be spun around the center of a spinner to collect data, as shown at right. With your team, spin a paperclip four times about the spinner above and record the results. Did your **experimental results** match your prediction in part (c)? Why or why not?



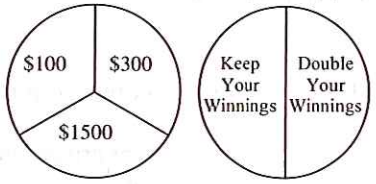
e. What if the spinner is spun 40 times? 400 times? Would you predict the result to be closer to the expected results? Why or why not?

f. What if Luis has spun the spinner 400 times and landed on region A every time? What would be the probability that the next spin would land on region B? Explain.

4-60. Congratulations! You are going to be a contestant on a new game show with a chance to win some money. You will spin two spinners to see how much money you will win. The first spinner has an equal chance of coming out \$100, \$300, or \$1500. The second spinner is equally divided between "Keep your winnings," and "Double your winnings."

a. Make a systematic list and tree diagram of all the possible outcomes.

b. What is the probability that you will take home \$200? What is the probability that you will take home more than \$500?



c. What is the probability that you will double your winnings? Does the probability that you will double your winnings depend on the result of the first spinner?

d. Does your answer for part (c) mean that you are guaranteed to double your winnings half the time you play the game?

e. What if the amounts on the first spinner were \$100, \$200, and \$1500? What is the probability that you would take home \$200? Justify your conclusion.

4-61. ROLL AND WIN

Now consider a game that is played with two regular dice, each numbered 1 through 6. First, each player chooses a number. The two dice are rolled and the numbers that come up are **added** together. If the sum is the number you chose, you win a point. For example, if a 2 and a 5 are rolled, the sum is 7, so the person who chose the number 7 would get a point.

a. First, use your intuition. Which number do you think will be the most common sum? In other words, which number would you pick to play?

b. What are the possible results when two dice are rolled and their numbers added?

c. Are the results from the separate dice dependent or independent? Does the result from one die affect the other? Explain.

d. What about the separate rolls? When the two dice are rolled, does the sum depend on the previous rolls? Why or why not?

e. Now play Roll and Win to see which sum occurs the most often. Have each member of your team choose a different number. Roll the dice 36 times and record the results. Which sum occurred most often?

4-62. One way to analyze this situation is to **make a table** like the one at right. Copy and complete this table of sums on your paper.

Dice #1

Dice #2	1	2	3	4	5	6
1				5		
2		4				
3						
4			7			
5						
6						

a. What is the probability the outcome will be odd? P(even)?

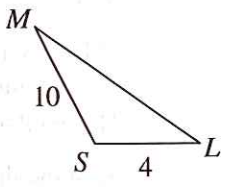
b. Which sum is the most likely result? What is the probability of rolling that sum?

c. What is the probability of rolling the sum of 2? 10? 15?

d. Compare your data from the table with that from problem 4-61. Did your experimental results match your theoretical probability? If not, explain what you could do to get results that are closer to the predicted results.

Review & Preview

4-63. What are the possible lengths for side $\overline{ML}$ in the triangle at right? Show how you know.

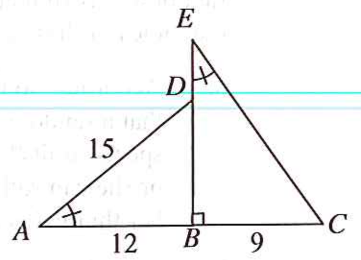


4-64. Alexis, Bart, Chuck, and Dariah all called in to a radio show to get free tickets to a concert. List all the possible orders in which their calls could have been received.

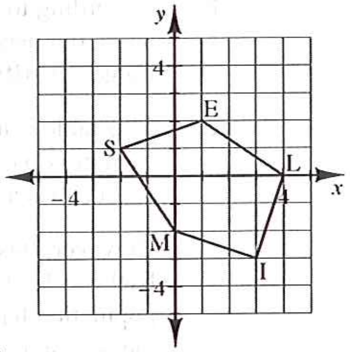
4-65. **Examine** the diagram at right. If $\overline{AC}$ passes through point B, then answer the questions below.

a. Are the triangles similar? If so, make a flowchart **justifying** your answer.

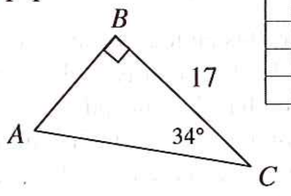
b. Are the triangles congruent? Explain how you know.



4-66. **Examine** pentagon *SMILE* at right. Do any of its sides have equal length? How do you know? Be sure to provide convincing evidence. You might want to copy the figure onto graph paper.



4-67. Find the area of the triangle at right. Show all work.



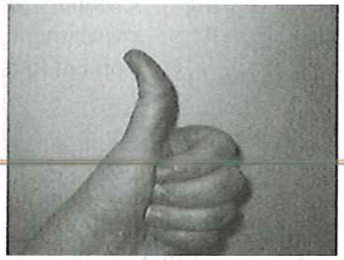
4.2.3 How can I represent it?

Using an Area Model

So far in this chapter, you have studied several different ways to represent situations involving chance. You have analyzed games using systematic lists, tree diagrams, and tables. In these games, each outcome you listed had an equal probability of occurring. But what if a game is biased so that some outcomes are more likely than others? How can you represent biased games? Today you will learn a new tool to analyze more complicated situations of chance, called an area model.

4-68. IT'S IN THE GENES

Can you bend your thumb backwards at the middle joint to make an angle, like the example at right? Or does your thumb remain straight? The ability to bend your thumb back is thought to rely on a single gene.



Example of a thumb that can bend backwards at the joint.

What about your tongue? If you can roll your tongue into a "U" shape, you probably have a special gene that enables you to do this.

Assume that half of the U.S. population can bend their thumbs backwards and that half can roll their tongues. Also assume that these genes are independent (in other words, having one gene does not affect whether or not you have the other) and randomly distributed (spread out) throughout the population. Then the possible outcomes of these genetic traits can be organized in a table like the one below.

a. According to this table, what is the probability that a random person from the U.S. has both special traits? That is, what is the chance that he or she can roll his or her tongue <u>and</u> bend his or her thumb back?

b. According to this table, what is the probability that a random person has only one of these special traits? **Justify** your conclusion.

		Bend Thumb?	
		Yes $\frac{1}{2}$	No $\frac{1}{2}$
Roll Tongue?	Yes $\frac{1}{2}$		
	No $\frac{1}{2}$		

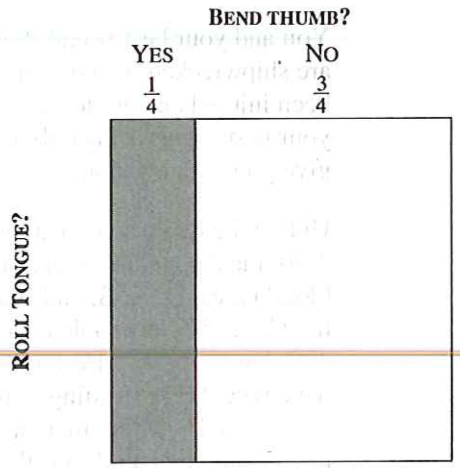
c. This table is useful because every cell in the table is equally likely. Therefore, each possible outcome, such being able to bend your thumb but not roll your tongue, has a $\frac{1}{4}$ probability.

However, this table assumes that half the population can bend their thumbs backwards. In actuality, only a small percentage (about $\frac{1}{4}$) of the U.S. population has this special trait. It also turns out that a majority (about $\frac{7}{10}$) of the population can roll their tongues. How can this table be adjusted to represent these percentages? Discuss this with your team and be prepared to share your ideas with the class.

4-69. USING AN AREA MODEL

One way to represent a probability situation that has outcomes with unequal probabilities is by using an **area model**. An area model uses a square with an area of 1 to represent all possible outcomes. The square is subdivided to represent the different possible outcomes. The area of each outcome is the probability that the outcome will occur.

For example, if $\frac{1}{4}$ of the U.S. population can bend their thumbs back, then the column representing this ability should take only one-fourth of the square's width, as shown at right.



a How should the diagram be altered to that show that $\frac{7}{10}$ of the U.S. can roll their tongues? Copy this diagram on your paper and add two rows to represent this probability.

b. The relative probabilities of the different outcomes are represented by the areas of the regions. For example, the portion of the area model representing people with both special traits is a rectangle with a width of $\frac{1}{4}$ and a height of $\frac{7}{10}$. What is the area of this rectangle? This area tells you the probability that a random person in the U.S. has both traits.

c. What is the probability that a randomly selected person can roll his or her tongue but not bend his or her thumb back? Show how you got this probability.

4-70. PROBABILITIES IN VEIN

You and your best friend may not only look different, you may also have different types of blood! For instance, members of the American Navajo population can be classified into two groups: 73% percent (73 out of 100) of the Navajo population has type "O" blood, while 27% (27 out of 100) has type "A" blood. (Blood types describe certain chemicals, called "antigens," that are found in a person's blood.)

Navajo Person #1

		O $\frac{73}{100}$	A $\frac{27}{100}$
Navajo Person #2	O $\frac{73}{100}$		
	A $\frac{27}{100}$		

a. Suppose you select two Navajo individuals at random. What is the probability that both individuals have type "A" blood? This time, drawing an area model that is exactly to scale would be challenging. Therefore, a **generic area model** (like the one above) is useful because it will still allow you to calculate the individual areas. Copy and complete this generic area model.

Problem continues on next page →

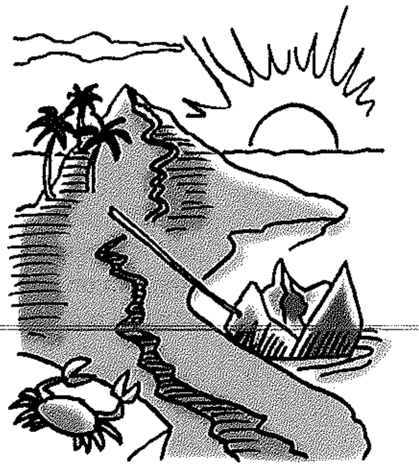
4-70. *Problem continued from previous page.*

b. What is the probability that two Navajo individuals selected at random have the same blood type?

4-71. SHIPWRECKED!

You and your best friend (both from the U.S.) are shipwrecked on a desert island! You have been injured and are losing blood rapidly, and your best friend is the only person around to give you a transfusion.

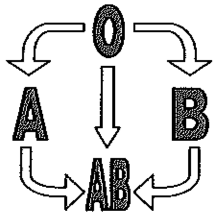
Unlike the Navajo you learned of in problem 4-70, most populations are classified into four blood types: O, A, B, and AB. For example, in the U.S., 45% of people have type O blood, 40% have type A, 11% have type B, and 4% have type AB (according to the American Red Cross, 2004). While there are other ways people's blood can differ, this problem will only take into account these four blood types.



a. Make a generic area model representing the blood types in this problem. List your friend's possible blood types along the top of the model and your possible blood types along the side.

b. What is the probability that you and your best friend have the same blood type?

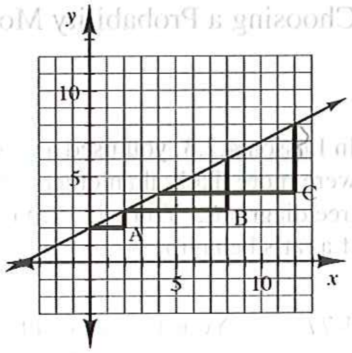
c. Luckily, two people do not have to have the same blood type for the receiver of blood to survive a transfusion. Other combinations will also work, as shown in the diagram at right. Assuming that their blood is compatible in other ways, a donor with type O blood can donate to receivers with type O, A, B, or AB, while a donor with type A blood can donate to a receiver with A or AB. A donor with type B blood can donate to a receiver with B or AB, and a donor with type AB blood can donate only to AB receivers.



Assuming that your best friend's blood is compatible with yours in other ways, determine the probability that he or she has a type of blood that can save your life!

Review & Preview

4-72. **Examine** the graph at right with slope triangles A, B, and C.



- a. Find the slope of the line using slope triangle A, slope triangle B, and then slope triangle C.
- b. Hernisha's slope triangle has a slope of $\frac{1}{2}$. What do you know about her line?

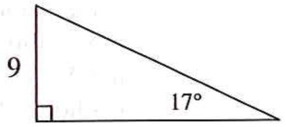
4-73. Francis and John are racing. Francis is 2 meters in front of the starting line at time $t = 0$ and he runs at a constant rate of 1 meters per second. John is 5 meters in front of the starting line and he runs at a constant rate of 0.75 meters per second. After how long will Francis catch up to John?

4-74. Can a triangle be made with sides of length 7, 10 and 20 units? **Justify** your answer.

4-75. Solve each equation below for the given variable. Show all work and check your answer.

- a. $\sqrt{x}-5=2$
- b. $-4(-2-x)=5x+6$
- c. $\frac{5}{x-2}=\frac{3}{2}$
- d. $x^2+4x-5=0$

4-76. Find the perimeter of the shape at right. Clearly show all your steps.

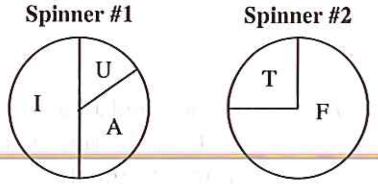


4.2.4 How can I represent it?

Choosing a Probability Model

In Lesson 4.2.3, you used area models to represent probability situations where some outcomes were more likely than others. Today you will consider how to represent these situations using tree diagrams. You will then return to a problem that was introduced in Lesson 4.2.1 to decide if a rat's behavior is random or not.

4-77. Your teacher challenges you to a spinner game. You spin the two spinners with the probabilities listed at right. The first letter should come from Spinner #1 and the second letter from Spinner #2. If the letters can form a two-letter English word, you win. Otherwise, your teacher wins.



$P(I) = \frac{1}{2}$ $P(T) = \frac{1}{4}$

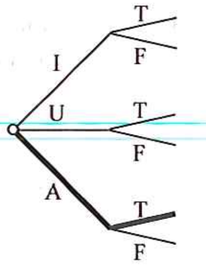
$P(U) = \frac{1}{6}$ $P(F) = \frac{3}{4}$

$P(A) = \frac{1}{3}$

a. Make a generic area model and find the probability that you will win this game.

b. Is this game fair? If you played the game 100 times, who do you think would win more often, you or your teacher? Can you be sure this will happen?

4-78. Sinclair wonders how to model the spinner game in problem 4-77 using a tree diagram. He draws the tree diagram at right.



a. Sabrina says, "That can't be right. This diagram makes it look like all the words are equally likely." What is Sabrina talking about? Why is this tree diagram misleading?

b. To make the tree diagram reflect the true probabilities in this game, Sabrina writes numbers next to each letter showing the probability that the letter will occur. So she writes a "$\frac{1}{3}$" next to "A," a "$\frac{1}{4}$" next to each "T," etc. Following Sabrina's method, label the tree diagram with numbers next to each letter.

c. According to your area model from problem 4-77, what is the probability that you will spin the word "AT"? Now **examine** the bolded branch on the tree diagram shown above. How could the numbers you have written on the tree diagram be used to find the probability of spinning "AT"?

Problem continues on next page →

4-78. *Problem continued from previous page.*

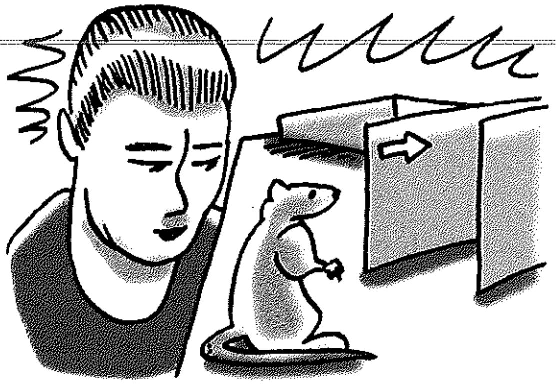
d. Does this method work for the other combinations of letters? Similarly calculate the probabilities for each of the paths of the tree diagram. At the end of each branch, write its probability. (For example, write "$\frac{1}{12}$" at the end of the "AT" branch.) Do your answers match those from problem 4-77?

e. Find all the branches with letter combinations that make words. Use the numbers written at the end of each branch to compute the total probability that you will spin a word. Does this probability match the probability you found with your area model?

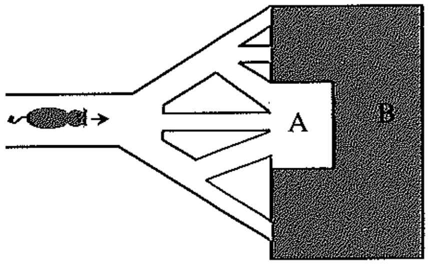
4-79. THE RAT RACE, Part Two

In problem 4-48, you read about Romeo, an amazing pet rat. As you previously learned, Sammy overheard Ryan at the county fair claiming that Romeo could learn to run a particular maze and find the cheese at the end.

"I don't think Romeo is that smart!" Sammy declares, "I think the rat just chooses a random path through the maze."



Ryan has built a maze with the floorplan shown at right. In addition, he has placed some cheese in an airtight container (so Romeo can't smell the cheese!) in room A.



a. Suppose that every time Romeo reaches a split in the maze, he is equally likely to choose any of the paths in front of him. Choose a method and find the probability that Romeo will end up in each room. In a sentence or two, explain why you chose the method you did.

b. If the rat moves through the maze randomly, how many out of 100 attempts would you expect Romeo to end up in room A? How many times would you expect him to end up in room B? Explain.

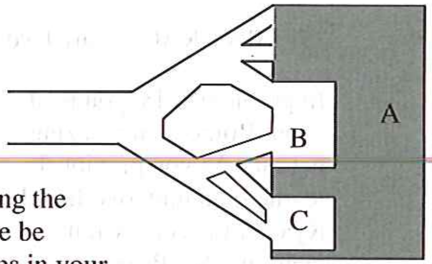
Problem continues on next page →

4-79. *Problem continued from previous page.*

c. After 100 attempts, and Romeo finds the cheese 66 times. "See how smart Romeo is?" Ryan asks, "He clearly learned something and got better at the maze as he went along." Looking at your calculations, Sammy isn't so sure.

Do you think Romeo learned and improved his ability to return to the same room over time? Or could he just have been moving randomly? Discuss this question with your team. Then, write an argument that would convince Ryan or Sammy.

4-80. Always skeptical, Sammy says, "If Romeo really can learn, he ought to be able to figure out how to run this new maze I've designed." **Examine** Sammy's maze at right.



a. To give Romeo the best chance of finding the cheese, in which room should the cheese be placed? Choose a method, show all steps in your solution process, and **justify** your answer.

b. If the cheese is in room C and Romeo finds the cheese 6 times out of every 10 tries, does he seem to be learning? Explain your conclusion.

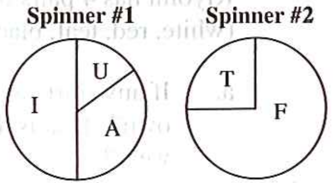
4-81. Make an entry in your Learning Log describing the various ways of representing probabilities you have learned in this chapter. Which seems easiest to use so far? Which is most versatile? Are there any conditions under which certain ones cannot be used? Label this entry "Probability Methods" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Probability Models

When all the possible outcomes of a probabilistic event have the same probability, probabilities can be calculated by listing the possible outcomes in a **systematic list**. However, when some outcomes are more probable than others, a more sophisticated model is required to calculate probabilities.



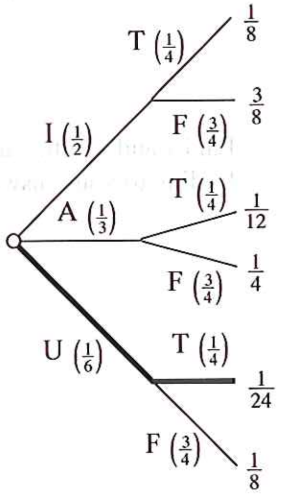
$P(I)=\frac{1}{2}$
$P(U)=\frac{1}{6}$
$P(A)=\frac{1}{3}$

$P(T)=\frac{1}{4}$
$P(F)=\frac{3}{4}$

One such model is an **area model**. In this type of model, the situation is represented by a square with area of 1 so that the areas of the parts are the probabilities of the different events that occur. For example, suppose you spin the two spinners shown above. The possible outcomes are represented in the area model at right. Notice that column "U" takes up $\frac{1}{6}$ of the width of the table since the "U" region is $\frac{1}{6}$ of Spinner #1. Similarly, the "T" row takes up $\frac{1}{4}$ of the height of the table, since the "T" region is $\frac{1}{4}$ of Spinner #2. Then the probability that the spinners come out "U" and "T" is equal to the area of the "UT" rectangle in the table: $\frac{1}{6}\cdot\frac{1}{4}=\frac{1}{24}$.

		Spinner #1		
		I $(\frac{1}{2})$	**A** $(\frac{1}{3})$	**U** $(\frac{1}{6})$
Spinner #2	**T** $(\frac{1}{4})$	IT	AT	UT
	F $(\frac{3}{4})$	IF	AF	UF

The situation can also be represented using a **tree diagram**. In this model, the branching points indicate probabilistic events, and the branches stemming from each event indicate the possible outcomes for the event. For example, in the tree diagram at right the first branching point represents spinning the first spinner. The first spinner can come out "I" "A" or "U", so each of those options has a branch. The numbers next to each letter represent the probability that that letter occurs.



The numbers at the far right of the table represent the probabilities of various outcomes. For example, the probability of spinning "U" and "T" can be found at the end of the bold branch of the tree. This probability, $\frac{1}{24}$, can be found by multiplying the fractions that appear on the bold branches.

Review & Preview

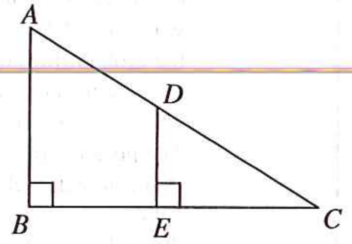
4-82. Choose a method to represent the following problem and use it to answer the questions below.

Kiyomi has 4 pairs of pants (black, green, blue jeans, and plaid) and she has 5 shirts (white, red, teal, black, and brown).

a. If any shirt can be worn with any pair of pants, how many outfits does she own? That is, how many different combinations of pants and shirts can she wear?

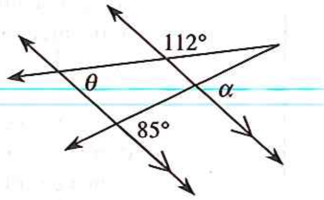
b. What is the probability that she is wearing black?

4-83. Are the triangles at right similar? If so, write a flowchart that **justifies** your conclusion. If not, explain how you know.

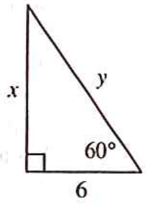


4-84. You roll a die and it comes up a "6" three times in a row. What is the probability of rolling a "6" on the next toss?

4-85. Find the values of θ and α in the diagram at right. State the relationships you used.



4-86. Find x and y in the diagram at right. Show all of the steps leading to your answer.



4.2.5 Which model should I use?

Optional: Applications of Probability Methods

Beginning with Lesson 4.2.1, you have been learning about various methods to help you determine probabilities of different outcomes. Today you will apply what you know to two probability problems. As you work in your teams today, keep the following questions in mind to guide your discussion:

Which probability do we need to find?

How can we find the probability?

Which probability model should we use?

4-87. **SHIFTY SHAUNA**

Shauna has a bad relationship with the truth—she doesn't usually tell it! In fact, whenever Shauna is asked a question, she rolls a die. If it comes up 6, she tells the truth. Otherwise, she lies.

a. If Shauna flips a fair coin and you ask her how it came out, what is the probability that she says "heads" and is telling the truth? Choose a method to solve this problem and carefully record your work. Be ready to share your solution method with the class.

b. Suppose Shauna flips a fair coin and you ask her whether it came up heads or tails. What is the probability that she says "heads"? (Hint: The answer is not $\frac{1}{12}$!)

4-88. **MIDNIGHT MYSTERY**

Each year, the students at Haardvarks School randomly select a student to play a prank. Late last night, Groundskeeper Millie saw a student steal the school's National Curling Championship trophy from the trophy case. All Millie can tell the headmaster about the crime is that the student who stole the trophy looked like he or she had red hair.

Problem continues on next page →

4-88. *Problem continued from previous page.*

Unfortunately, of the 100 students at Haardvarks, the only ones with red hair are your friend Don Measley and his siblings. Groundskeeper Millie insists that one of the 5 Measley children committed the crime and should be punished. Don is incensed: "We Measleys would never play such a stupid prank! Groundskeeper Millie claims to have seen someone with red hair, but it was so dark at the time and Millie's eyes are so bad, there is no way she could have identified the color of someone's hair!"

The headmaster isn't convinced, so he walks around with Millie at night and points to students one by one, asking Millie whether each one has red hair. Millie is right about the hair color 4 out of every 5 times.

This looks like bad news for Don, but Professor McMonacle agrees to take up his defense. "I still think," McMonacle says, "that the thief probably wasn't one of the Measleys."

Your task: Model this situation with a tree diagram or a generic area model. The two chance events in your model should be "The thief is/is not a redhead" and "Millie is/is not correct about the thief's hair color."

4-89. Look back at your model from problem 4-88 and circle every outcome in which Millie would report that the thief had red hair. (There should be two!) Then use your model to answer the following questions:

a. Suppose that to help make Don's case, you perform the following experiment repeatedly: you pick a Haardvarks student at random, show the student to Millie late at night, and see what Millie says about the student's hair color. If you performed this experiment with 100 students, how many times would you expect Millie to say the student had red hair?

b. If you performed this experiment with 100 students, how many times would you expect Millie to say the student had red hair and be correct about it?

c. If you performed this experiment with 100 students, what percentage of the students Millie *said* had red hair would *actually* have red hair?

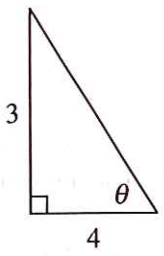
d. Can you use these calculations to defend the Measleys? Is it likely that a Measley was the thief?

4-90. Have you *proven* that none of the Measleys stole the Curling Trophy?

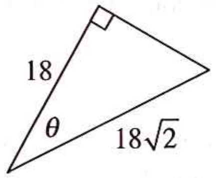
Review & Preview

4-91. Based on the measurements provided for each triangle below, decide if the angle θ must be more than, less than, or equal to 45°. Assume the diagram is not drawn to scale. Show how you know.

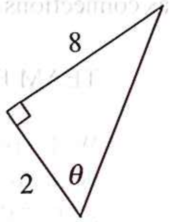
a.



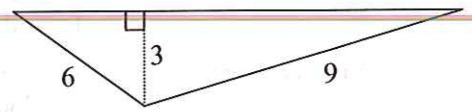
b.



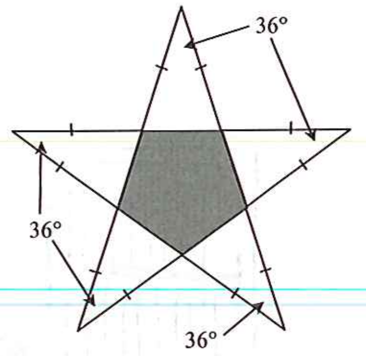
c.



4-92. Find the area and perimeter of the triangle at right.



4-93. After doing well on a test, Althea's teacher placed a gold star on her paper. When Althea **examined** the star closely, she realized that it was really a regular pentagon surrounded by 5 isosceles triangles, as shown in the diagram at right. If the star has the angle measurements shown in the diagram, find the sum of the angles inside the shaded pentagon. Show all work.



4-94. Find the slope of the line through the points (–5, 86) and (95, 16). Then find at least one more point on the line.

4-95. On graph paper, plot ΔABC if $A(-1,-1)$, $B(3,-1)$, and $C(-1,-2)$.

a. Enlarge (dilate) ΔABC from the origin so that the ratio of the side lengths is 3. Name this new triangle $\Delta A'B'C'$. List the coordinates of $\Delta A'B'C'$.

b. Rotate $\Delta A'B'C'$ 90° clockwise (↻) about the origin to find $\Delta A''B''C''$. List the coordinates of $\Delta A''B''C''$.

c. If ΔABC is translated so that the image of A is located at $(5, 3)$, where would the image of B lie?

Chapter 4 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned in this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following two subjects. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

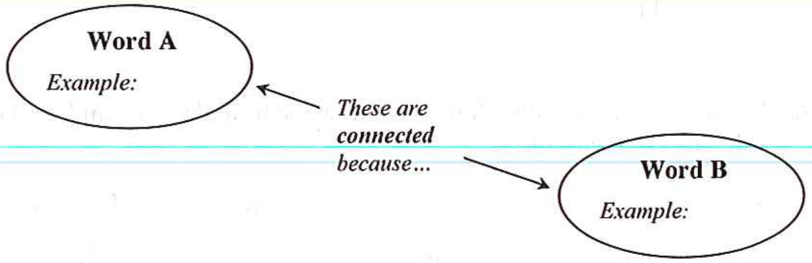
Topics:	What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.
Connections:	How are the topics, ideas, and words that you learned in previous courses connected to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

- **α (alpha)**
- angle
- **area model**
- **clinometer**
- conjecture
- **dependent events**
- hypotenuse
- **independent events**
- leg
- **orientation**
- probability
- random
- ratio
- **slope angle**
- slope ratio
- slope triangle
- **systematic list**
- **tangent**
- **θ (theta)**
- **tree diagram**
- **trigonometry**
- Δx
- Δy

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the example below. A word can be connected to any other word as long as there is a justified connection. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will direct you how to do this. Your teacher may give you a "GO" page to work on. "GO" stands for "Graphic Organizer," a tool you can use to organize your thoughts and communicate your ideas clearly.

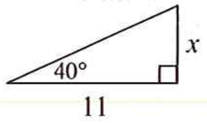
④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you feel comfortable with and which types you need more help with. This section appears at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

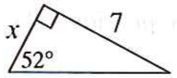
Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 4-96. Solve for the missing side length or angle below.

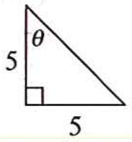
a.



b.

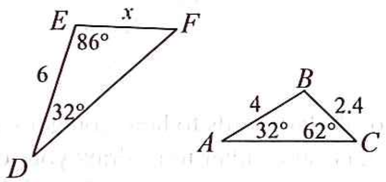


c.

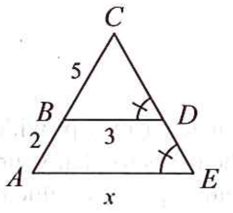


CL 4-97. Use a flowchart to show how you know the triangles are similar. Then find the value of each variable.

a.

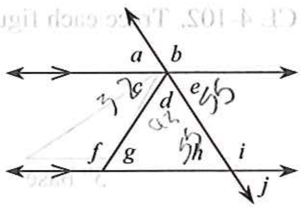


b.



CL 4-98. Salvador has a hot dog stand 58 meters from the base of the Space Needle in Seattle. He prefers to work in the shade and knows that he can calculate when his hotdog stand will be in the shade if he knows the height of the Space Needle. To measure its height, Salvador stands at the hotdog stand, gets out his clinometer, and measures the angle of sight to be 80°. Salvador's eyes are 1.5 meters above the ground. Assuming that the ground is level between the hotdog stand and the Space Needle, how tall is the Space Needle?

CL 4-99. Use the diagram at right to answer the questions below.



a. State the name of the geometric relationship between the angles below. Also describe the relationship between the angle measures, if one exists.

i. $\measuredangle a$ and $\measuredangle h$

ii. $\measuredangle b$ and $\measuredangle e$

iii. $\measuredangle c$ and $\measuredangle g$

iv. $\measuredangle g$, $\measuredangle d$, and $\measuredangle h$

b. For each problem below, **justify** your answer. If $m\measuredangle c = 32°$ and $m\measuredangle e = 55°$, then find the measure of each angle below.

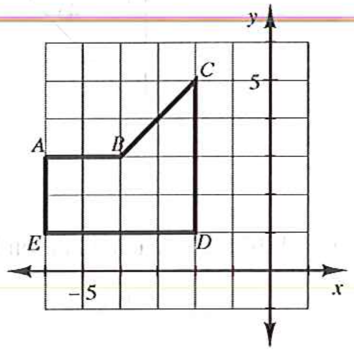
i. $m\measuredangle j$

ii. $m\measuredangle d$

iii. $m\measuredangle a$

iv. $m\measuredangle g$

CL 4-100. Draw a pair of axes in the center of a half sheet of graph paper. Then draw the figure to the right and perform the indicated transformations. For each transformation, label the resulting image $A'B'C'D'E'$.



a. Rotate $ABCDE$ 180° ↻ around the origin.

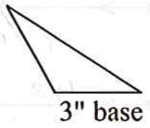
b. Rotate $ABCDE$ 90° ↺ around the origin.

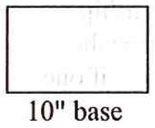
c. Reflect $ABCDE$ across the y-axis.

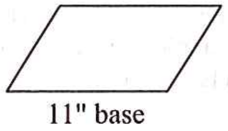
d. Translate $ABCDE$ up 5, left 7.

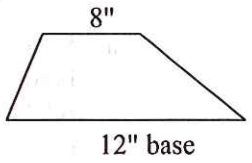
CL 4-101. In a certain town, 45% of the population has dimples and 70% has a widow's peak (a condition where the hairline above the forehead makes a "V" shape). Assuming that these physical traits are independently distributed, what is the probability that a randomly selected person has both dimples and a widow's peak? What is the probability that he or she will have neither? Use a generic area model or a tree diagram to represent this situation.

CL 4-102. Trace each figure onto your paper and label the sides with the given measurements.







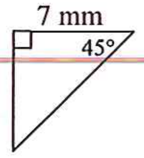


a. On your paper, draw a height that corresponds to the labeled base for each figure.

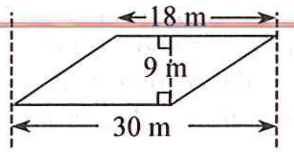
b. Assume that the height for each figure above is 7 inches. Add this information to your diagrams and find the area of each figure.

CL 4-103. Find the perimeter of each shape below. Assume the diagram in part (b) is a parallelogram.

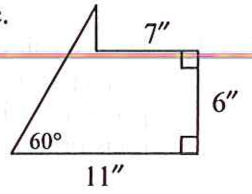
a.



b.



c.



CL 4-104. For each equation below, solve for x:

a. $\frac{x}{23} = \frac{15}{7}$

b. $(x+2)(x-5) = 6x + x^2 - 5$

c. $x^2 + 2x - 15 = 0$

d. $2x^2 - 11x = -3$

CL 4-105. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: investigating, examining, reasoning and justifying, visualizing, and choosing a strategy/tool. These are some of the ways in which you think while trying to make sense of a concept or attempting to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **visualizing**. Read the description of this Way of Thinking at right.

Think about the problems you have worked on in this chapter. When did you need to think about what something looked like to make sense of it? What helps you to create mental pictures? Were there times when a problem didn't seem to make sense without seeing it? How have you used visualizing in a previous math class? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any of the methods you have developed to help you **visualize**.

Once your discussion is complete, think about the way you think as you answer the questions below.

Visualizing

To visualize means to make a picture in your mind that represents a situation or description. As you develop this Way of Thinking, you will learn how to turn a variety of situations into mental pictures. You think this way when you ask or answer questions like, "What does it look like when . . . ?" or "How can I draw . . . ?" When you catch yourself wondering what something might look like, you are visualizing.

a. **Visualization** is required when you imagine a situation and want to draw a diagram to represent it. Read the descriptions below and **visualize** what each situation looks like. Then draw a diagram for each. Label your diagrams appropriately with any given measurements.

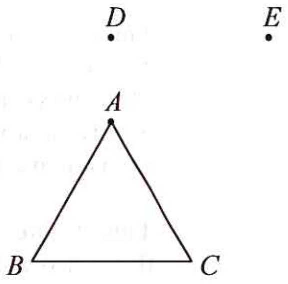
(1) *Karen is flying a kite on a windy day. Her kite is 80 feet above ground and her string is 100 feet long. Karen is holding the kite 3 feet above ground.*

(2) *The bow of a rowboat (which is 1 foot above water level) is tied to a point on a dock that is 6 feet above the water level. The length of the rope between the dock and the boat is 8 feet.*

Problem continues on next page →

⑤ *Problem continued from previous page.*

b. Sometimes, **visualization** requires you to think about how an object can move in relation to others. For example, consider equilateral $\triangle ABC$ at right.



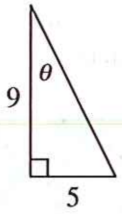
(1) **Visualize** changing $\triangle ABC$ by stretching vertex A to point D, which is right above A. What does the new triangle look like? Do you have a name for it?

(2) What happened to $m\measuredangle A$ as you stretched the triangle in part (1)? What happened to $m\measuredangle B$ and $m\measuredangle C$?

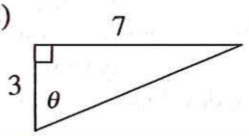
(3) Now **visualize** the result after vertex A stretched to point E. What type of triangle is the result? What happens to $m\measuredangle B$ as the triangle is stretched? What happens to $m\measuredangle C$?

c. An important use of **visualization** is to re-orient a right triangle to help you identify which leg is Δx and which leg is Δy. For each triangle below, **visualize** the triangle by rotating and/or reflecting it so that it is a slope triangle. Draw the result and label the appropriate legs Δx or Δy.

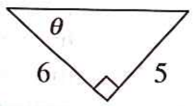
(1)



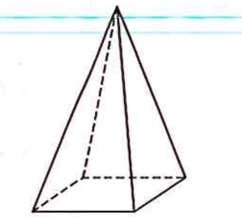
(2)



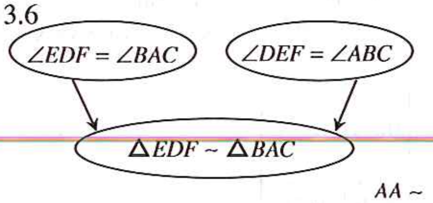
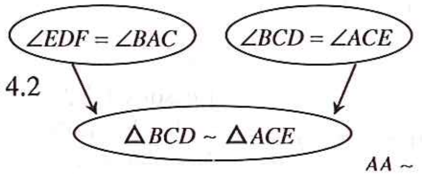
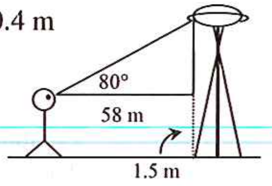
(3)

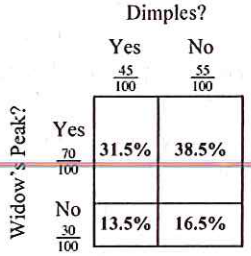


d. Finally, **visualization** can help you view an object from different perspectives. For example, consider the square-based pyramid at right. **Visualize** what you would see if you looked down at the pyramid from a point directly above the top vertex. Draw this view on your paper.



Answers and Support for
Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 4-96.	a. $x \approx 9.23$ b. $x \approx 5.47$ c. $\theta = 45°$	Lessons 4.1.2 and 4.1.4 Math Notes boxes	Problems 4-11, 4-12, 4-16, 4-22, 4-25, 4-31, 4-34, 4-36, 4-46
CL 4-97.	a. $x \approx 3.6$  b. $x = 4.2$ 	Lessons 3.1.2, 3.1.3, 3.2.1, and 3.2.5 Math Notes boxes	Problems 3-23, 3-35, 3-36, 3-38, 3-46, 3-49, 3-53, 3-54, 3-55, 3-57, 3-64, 3-66, 3-80, 3-83, 3-84, 3-89, 4-7, 4-17, 4-18, 4-38, 4-39, 4-83
CL 4-98.	Total height ≈ 330.4 m  not to scale	Lesson 4.1.4 Math Notes box, Lesson 4.1.5	Problems 4-27, 4-30, 4-40, 4-41, 4-46, 4-57
CL 4-99.	a. *i.* corresponding angles, congruent *ii.* straight angle pair, supplementary *iii.* alternate interior angles, congruent *iv.* sum of the angles in a triangle, add up to 180° b. *i.* 55°: corresponding to e *ii.* 93°: straight angle with e and c *iii.* 55°: vertical to e *iv.* 32°: alternate interior to c	Lessons 2.1.1, 2.1.4, and 2.2.1 Math Notes boxes, Problem 2-3	Problems 2-13, 2-16, 2-17, 2-18, 2-23, 2-24, 2-25, 2-31, 2-32, 2-38, 2-49, 2-51, 2-62, 2-72, 2-111, 3-20, 3-31, 3-60, 3-70, 3-96, 4-6, 4-29, 4-44, 4-85, 4-93

Problem	Solution	Need Help?	More Practice
CL 4-100.	a. $A'(6,-3)$, $B'(4,-3)$, $C'(2,-5)$, $D'(2,-1)$, $E'(6,-1)$ b. $A'(-3,-6)$, $B'(-3,-4)$, $C'(-5,-2)$, $D'(-1,-2)$, $E'(-1,-6)$ c. $A'(6,3)$, $B'(4,3)$, $C'(2,5)$, $D'(2,1)$, $E'(6,1)$ d. $A'(-13,8)$, $B'(-11,8)$, $C'(-9,10)$, $D'(-9,6)$, $E'(-13,6)$	Lessons 1.2.2 and 1.2.3 Math Notes boxes	Problems 1-50, 1-51, 1-59, 1-60, 1-61, 1-64, 1-69, 1-73, 1-85, 1-96, 2-11, 2-19, 2-20, 2-33, 2-64, 2-113, 3-96
CL 4-101.	P(both) = 31.5% P(neither) = 16.5%  	Lesson 4.2.4 Math Notes box	Problems 4-52, 4-60, 4-62, 4-68, 4-69, 4-70, 4-71, 4-77, 4-78, 4-87
CL 4-102.	a. All heights should be 7". b. Areas in square inches: 10.5, 70, 77, 70	Lessons 1.1.3 and 2.2.4 Math Notes boxes	Problems 2-66, 2-68, 2-71, 2-75, 2-76, 2-78, 2-79, 2-82, 2-83, 2-90, 2-120, 3-51, 3-81, 3-97, 4-43, 4-67, 4-92
CL 4-103.	a. ≈ 23.899 mm b. = 66 m c. ≈ 32.93"	Lessons 1.1.3, 4.1.2, and 4.1.4 Math Notes boxes	Problems 4-16, 4-20, 4-22, 4-36, 4-76, 4-92
CL 4-104.	a. $x = 49.286$ b. $x = \frac{-5}{9}$ c. $x = 3$ or $x = -5$ d. $x = \frac{11 \pm \sqrt{97}}{4} \approx 5.21$ or 0.29	Lessons 1.1.4 and 3.2.3 Math Notes box	Problems 1-17, 1-25, 1-32, 1-34, 1-45, 1-57, 1-74, 1-100, 1-124, 3-68, 3-90, 3-100, 4-8, 4-47, 4-75