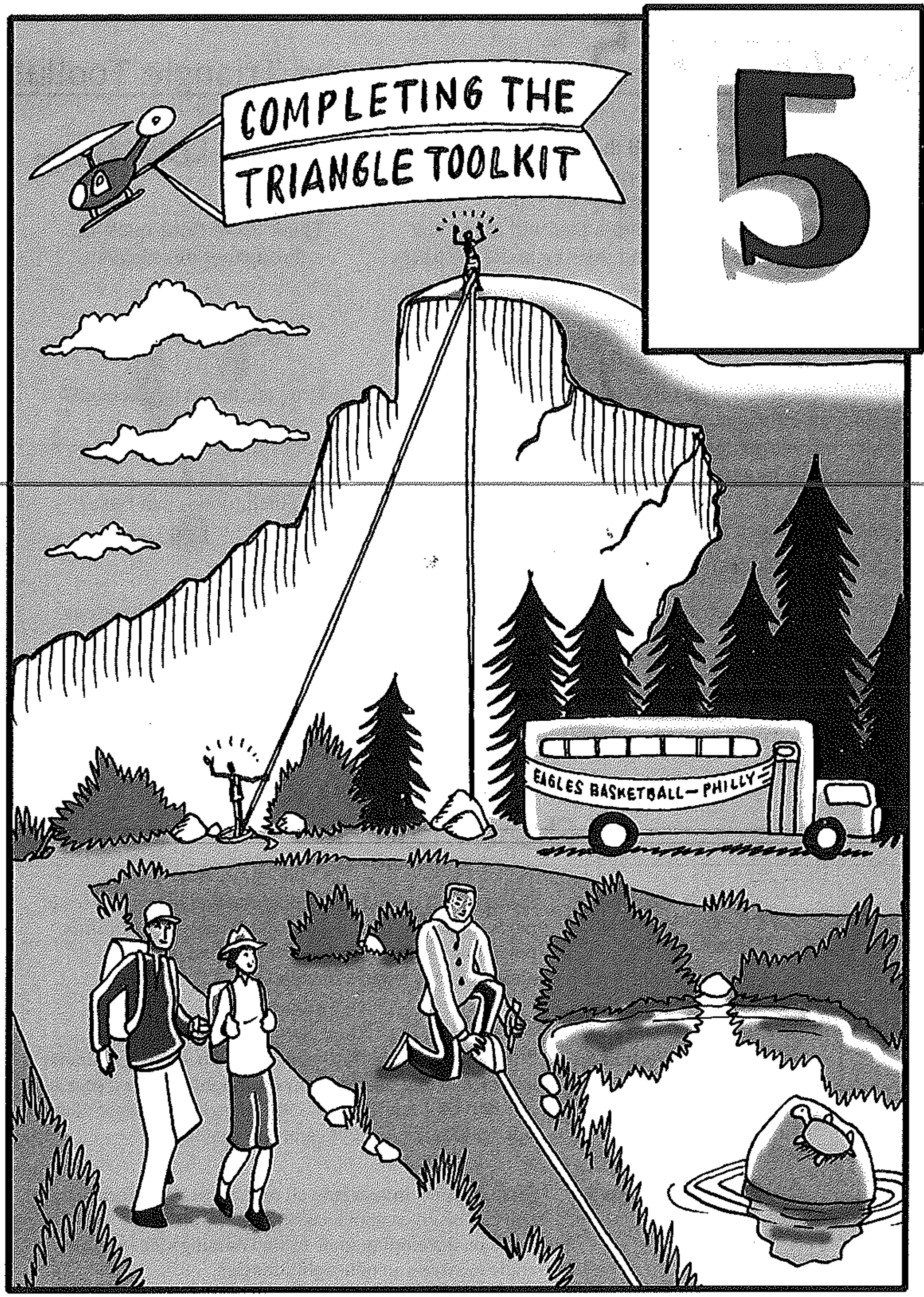

5
COMPLETING THE TRIANGLE TOOLKIT
EAGLES BASKETBALL—PHILLY

CHAPTER 5 Completing the Triangle Toolkit

In Chapter 4, you **investigated** the powerful similarity and side ratio relationships in right triangles. In this chapter, you will learn about other side ratio relationships using the hypotenuse that will allow you to find missing side lengths and missing angle measures for any right triangles.

In addition, you will develop tools to complete your triangle toolkit so that you can find the missing angle measures and side lengths for any triangle, provided that enough information is given. You will then explore ways to **choose an appropriate tool** to solve new problems in unfamiliar contexts.

Guiding Questions

Think about these questions throughout this chapter:

Is there a shortcut?

How are the shapes related?

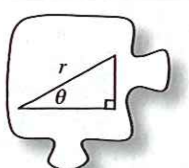
What information do I know?

Which tool(s) can I use?

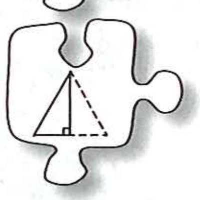
In this chapter, you will learn:

- how to recognize and use special right triangles.
- the trigonometric ratios of sine and cosine as well as the inverses of these functions.
- how to apply trigonometric ratios to find missing measurements in right triangles.
- new triangle tools called the Law of Sines and the Law of Cosines.
- how to recognize when the information provided is not enough to determine a unique triangle.

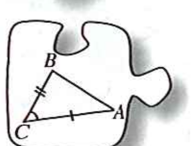
Chapter Outline



Section 5.1 Students will extend their understanding of trigonometric ratios to include sine, cosine, and inverse trigonometric functions and will use these tools to find missing measurements in right triangles.

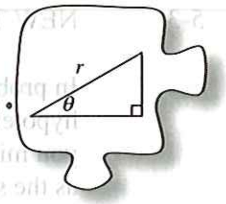


Section 5.2 Students will apply the Pythagorean Theorem and similar triangles to find patterns in special right triangles, such as 30°- 60°- 90° and 45°- 45°- 90° triangles and those with side lengths that are Pythagorean Triples.



Section 5.3 Once students **investigate** all of the types of information that can be given about a triangle (Lesson 5.3.1), they will focus on developing tools to find missing side lengths and angle measures in non-right triangles.

5.1.1 What if I know the hypotenuse?



Sine and Cosine Ratios

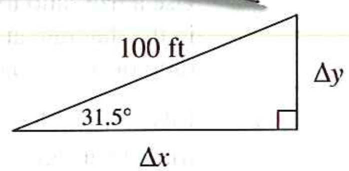
In the previous chapter, you used the idea of similarity in right triangles to find a relationship between the acute angles and the lengths of the legs of a right triangle. However, we do not always work just with the legs of a right triangle—sometimes we only know the length of the hypotenuse. By the end of today's lesson, you will be able to use two new trigonometric ratios that involve the hypotenuse of right triangles.

5-1. THE STREETS OF SAN FRANCISCO

While traveling around the beautiful city of San Francisco, Juanisha climbed several steep streets. One of the steepest, Filbert Street, has a slope angle of 31.5° according to her guidebook.

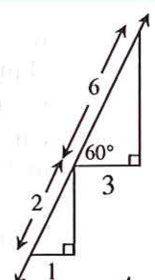
Once Juanisha finished walking 100 feet up the hill, she decided to figure out how high she had climbed. Juanisha drew the diagram below to represent this situation.

Can a tangent ratio be used to find Δy? Why or why not? Be prepared to share your thinking with the rest of the class.



Juanisha's Drawing

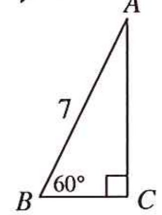
5-2. In order to find out how high Juanisha climbed in problem 5-1, you need to know more about the relationship between the ratios of the sides of a right triangle and the slope angle.



a. Use **two different strategies** to find Δy for the slope triangles shown in the diagram at right.

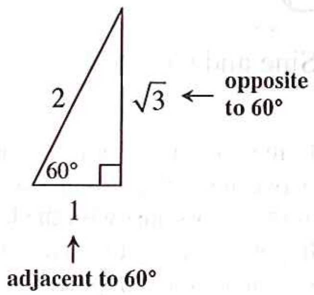
b. Find the ratio $\frac{\Delta x}{\text{hypotenuse}}$ for each triangle. Why must these ratios be equal?

c. Find BC and AC in the triangle at right. Show all work.



5-3. NEW TRIG RATIOS

In problem 5-2, you used a ratio that included the hypotenuse of ΔABC. There are several ratios that you might have used. One of those ratios is known as the **sine ratio** (pronounced "sign"). This is the ratio of the length of the side **opposite** the acute angle to the length of the **hypotenuse**.



For the triangle shown at right, the sine of 60° is $\frac{\sqrt{3}}{2} \approx 0.866$. This is written:

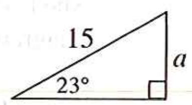
$$\sin 60° = \frac{\sqrt{3}}{2}$$

Another ratio comparing the length of the side **adjacent** to (which means "next to") the angle to the length of the **hypotenuse**, is called the **cosine ratio** (pronounced "co-sign"). For the triangle above, the cosine of 60° is $\frac{1}{2} = 0.5$. This is written:

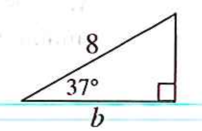
$$\cos 60° = \frac{1}{2}$$

a. Like the tangent, your calculator can give you both the sine and cosine ratios for any angle. Locate the "sin" and "cos" buttons on your calculator and use them to find the sine and cosine of 60°. Does your calculator give you the correct ratios?

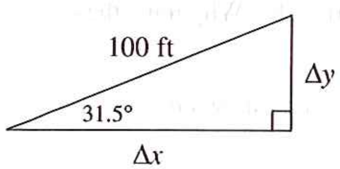
b. Use a trig ratio to write an equation and solve for a in the diagram at right. Does this require the sine ratio or the cosine ratio?



c. Likewise, write an equation and solve for b for the triangle at right.

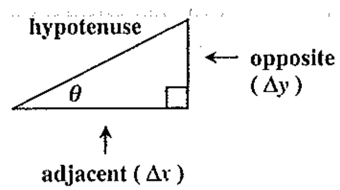


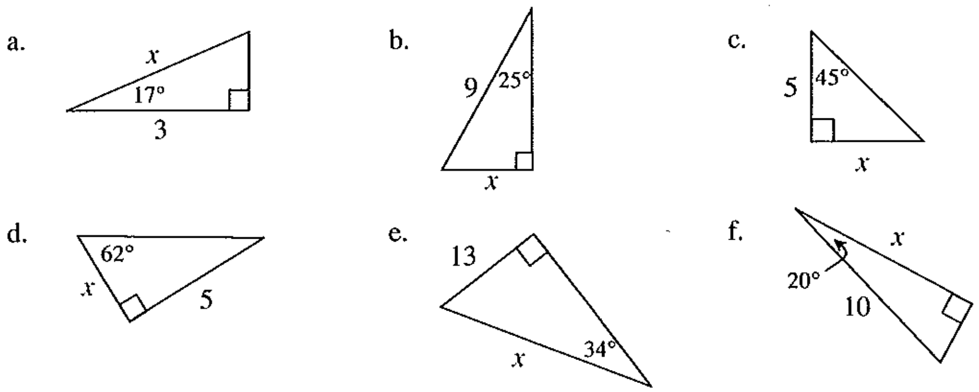
5-4. Return to the diagram from Juanisha's climb in problem 5-1. Juanisha still wants to know how many feet she climbed vertically when she walked up Filbert Street. Use one of your new trig ratios to find how high she climbed.



Juanisha's Drawing

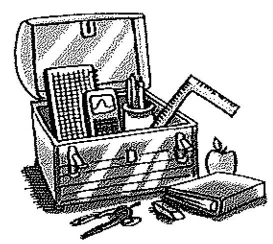
5-5. For each triangle below, decide which side is <u>opposite</u> and which is <u>adjacent</u> to the given acute angle. Then determine which of the three trig ratios will help you find x. Write and solve an equation.





5-6. TRIANGLE TOOLKIT

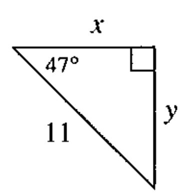
Obtain a Lesson 5.1.1 Resource Page ("Triangle Toolkit") from your teacher. This will be a continuation of the Geometry Toolkit you started in Chapter 1. Think about the tools you have developed so far to solve for the measure of sides and angles of a triangle. Then, in the space provided, add a diagram and a description of each tool you know. In later lessons, you will continue to add new triangle tools to this toolkit, so be sure to keep this resource page in a safe place. At this point, your toolkit should include:



- Pythagorean Theorem
- Tangent
- Sine
- Cosine

Review & Preview

5-7. You now have multiple trig tools to find missing side lengths of triangles. For the triangle at right, find the values of x and y. Your Triangle Toolkit might help. Which tools did you use?



5-8. Lori has written the conjectures below. For each one, decide if it is true or not. If you believe it is not true, find a **counterexample** (an example that proves that the statement is false).

a. If a shape has four equal sides, it cannot be a parallelogram.

b. If $\tan\theta$ is more than 1, then θ must be more than 45°.

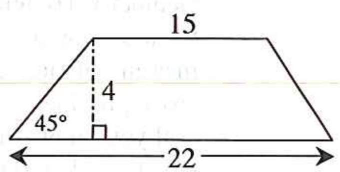
c. If two angles formed when two lines are cut by a transversal are corresponding, then the angles are congruent.

5-9. Earl hates to take out the garbage and to wash the dishes, so he decided to make a deal with his parents: He will flip a coin once for each chore and will perform the chore if the coin lands on heads. What he doesn't know is that his parents are going to use a weighted coin that lands on heads 80% of the time!

a. What is the probability that Earl will have to do both chores?

b. What is the probability that Earl will have to take out the garbage, but will not need to wash the dishes?

5-10. Copy the trapezoid at right on your paper. Then find its area and perimeter. Keep your work organized so that you can later explain how you solved it. (Note: The diagram is not drawn to scale.)



5-11. Solve each of the equations below for the given variable. Be sure to check your answers.

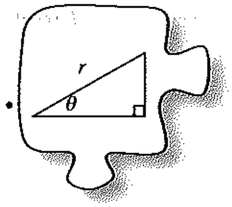
a. $4(2x+5)-11=4x-3$

b. $\frac{2m-1}{19}=\frac{m}{10}$

c. $3p^2+10p-8=0$

d. $\sqrt{x+2}=5$

5.1.2 Which tool should I use?



Selecting a Trig Tool

You now have several tools that will help you find the length of a side of a right triangle when given any acute angle and any side length. But how do you know which tool to use? And how can you identify the relationships between the sides and the given angle?

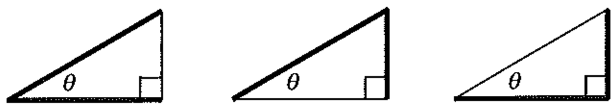
Today you will work with your team to develop **strategies** that will help you identify if cosine, sine, or tangent can be used to solve for a side of a right triangle. As you work, be sure to share any shortcuts you find that can help others identify which tool to use. As you work, keep the focus questions below in mind.

Is this triangle familiar? Is there something special about this triangle?

Which side is opposite the given angle? Which is adjacent?

Which tool should I use?

5-12. Obtain the Lesson 5.1.2 Resource Page from your teacher. On it, find the triangles shown below. Note: the diagrams are not drawn to scale.

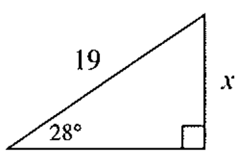


$\cos\theta = \frac{adj}{hyp}$ $\sin\theta = \frac{opp}{hyp}$ $\tan\theta = \frac{opp}{adj}$

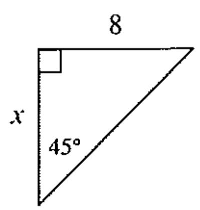
With your study team:

- Look through all the triangles first and see if any look familiar or are ones that you know how to answer right away without using a trig tool.
- Then, for all the other triangles, identify which tool you should use based on where the reference angle (the given acute angle) is located and which side lengths are involved.
- Write and solve an equation to find the missing side length.

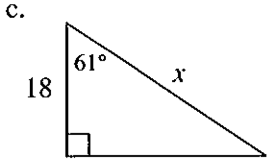
a.



b.



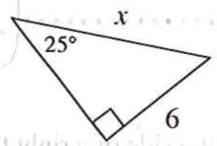
c.



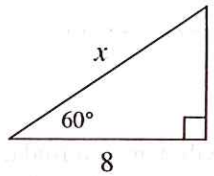
Problem continues on next page →

5-12. *Problem continued from previous page.*

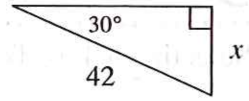
d.



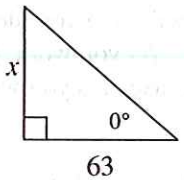
e.



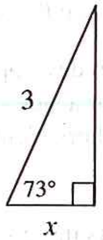
f.



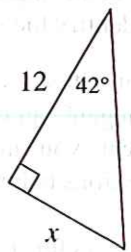
g.



h.

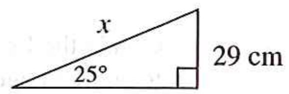


i.



5-13. Marta arrived for her geometry test only to find that she forgot her calculator. She decided to complete as much of each problem as possible.

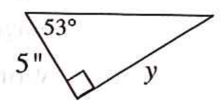
a. In the first problem on the test, Marta was asked to find the length x in the triangle shown at right. Using her algebra skills, she wrote and solved an equation. Her work is shown below. Explain what she did in each step.



$$\sin 25° = \frac{29}{x}$$
$$x(\sin 25°) = 29$$
$$x = \frac{29}{\sin 25°}$$

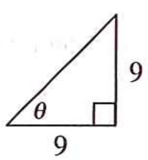
b. Marta's answer in part (a) is called an **exact answer**. Now use your calculator to help Marta find the **approximate** length of x.

c. In the next problem, Marta was asked to find y in the triangle shown at right. Find an exact answer for y without using a calculator. Then use a calculator to find an approximate value for y.

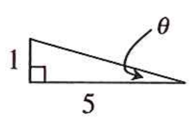


5-14. In problem 5-12, you used trig tools to find a side length. But do you have a way to find an angle? **Examine** the triangles below. Do any of them look familiar? How can you use information about the side lengths to help you figure out the reference angle (θ)? Your Trig Table from Chapter 4 may be useful.

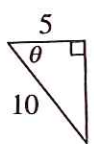
a.



b.



c.



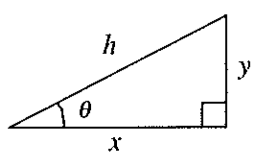
5-15. Write a Learning Log entry explaining how you know which trig tool to use. Be sure to include examples with diagrams and anything else that would be useful to refer to later. Title this entry, "Choosing a Trig Tool" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Trigonometric Ratios

You now have three **trigonometric ratios** you can use to solve for the missing side lengths and angle measurements in any right triangle. In the triangle below, when the sides are described relative to the angle θ, the opposite leg is y and the adjacent leg is x. The hypotenuse is h regardless of which acute angle is used.

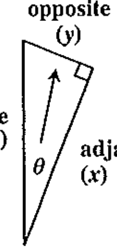


$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{y}{x}$$

$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{y}{h}$$

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{x}{h}$$

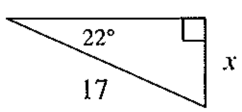
In some cases, you may want to rotate the triangle so that it looks like a slope triangle in order to easily identify the reference angle θ, the opposite leg y, the adjacent leg x, and the hypotenuse h. Instead of rotating the triangle, some people identify the opposite leg as the leg that is always opposite (not touching) the angle. For example, in the diagram at right, y is the leg opposite angle θ.



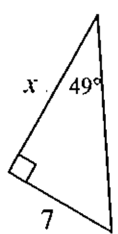
Review & Preview

5-16. For each triangle below, write an equation relating the **reference angle** (the given acute angle) with the two side lengths of the right triangle. Then solve your equation for x.

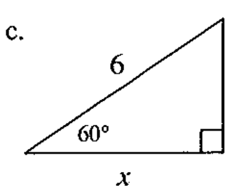
a.



b.



c.



5-17. While shopping at his local home improvement store, Chen noticed that the directions for an extension ladder state, "*This ladder is most stable when used at a 75° angle with the ground.*" He wants to buy a ladder to paint a two-story house that is 26 feet high. How long does his ladder need to be? Draw a diagram and set up an equation for this situation. Show all work.

5-18. Kendra has programmed her cell phone to randomly show one of six photos when she turns it on. Two of the photos are of her parents, one is of her niece, and three are of her boyfriend, Bruce. Today, she will need to turn her phone on twice: once before school and again after school.

a. **Choose a model** (such as a tree diagram or generic area model) to represent this situation.

b. What is the probability that both photos will be of her boyfriend?

c. What is the probability that neither photo will be of her niece?

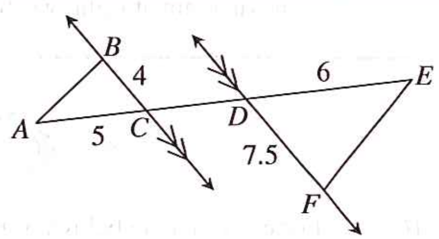
5-19. Lori has written the conjectures below. For each one, decide if it is true or not. If you believe it is not true, find a **counterexample** (an example that proves that the statement is false).

a. If a triangle has a 60° angle, it must be an equilateral triangle.

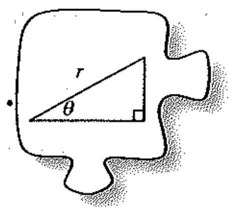
b. To find the area of a shape, you always multiply the length of the base by the height.

c. All shapes have 360° rotation symmetry.

5-20. **Examine** the triangles at right. Are the triangles similar? If so, show how you know with a flowchart. If not, explain how you know they cannot be similar.



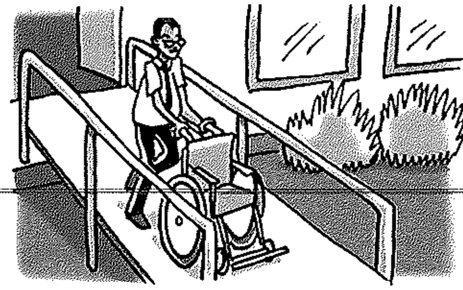
5.1.3 How can I find the angle?



Inverse Trigonometry

You now know how to find the missing side lengths in a right triangle given an acute angle and the length of any side. But what if you want to find the measure of an angle? If you are given the lengths of two sides of a right triangle, can you work backwards to find the measurements of the unknown angles? Today you will work on "undoing" the different trigonometric ratios to find the angles that correspond to those ratios.

5-21. Mr. Gow needs to build a wheelchair access ramp for the school's auditorium. The ramp must rise a total of 3 feet to get from the ground to the entrance of the building. In order to follow the state building code, the angle formed by the ramp and the ground cannot exceed 4.76°.



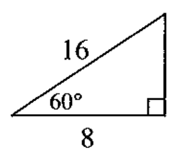
Mr. Gow has plans from the planning department that call for the ramp to start 25 feet away from the building. Will this ramp meet the state building code?

a. Draw an appropriate diagram. Add all the measurements you can. What does Mr. Gow need to find?

b. To find an angle from a trig ratio you need to "undo" the trig ratio, just like you can undo addition with subtraction, multiplication with division, or squaring by finding the square root. These examples are all pairs of **inverse** operations.

When you use a calculator to do this, you use inverse trig functions which look like "**sin**$^{-1}$", "**cos**$^{-1}$", and "**tan**$^{-1}$". These are pronounced, "inverse sine," "inverse cosine," and "inverse tangent." On many calculators, you must press the "inv" or "2nd" key first, then the "sin", "cos", or "tan" key.

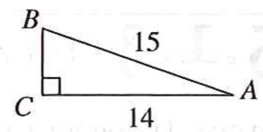
Verify that your calculator can find an inverse trig value using the triangle at right from Lesson 5.1.2. When you find $\cos^{-1}\frac{8}{16}$, do you get 60°?



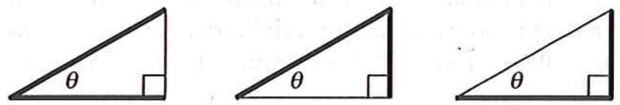
c. Return to your diagram from part (a). According to the plan, what angle will the ramp make with the ground? Will the ramp be to code?

d. At least how far from the building must the ramp start in order to meet the building code? If Mr. Gow builds the ramp exactly to code, how long will the ramp be? Show all work.

5-22. For the triangle at right, find the measures of $\measuredangle A$ and $\measuredangle B$. Once you have found the measure of the first acute angle (either $\measuredangle A$ or $\measuredangle B$), what knowledge about the angles in triangles could help you find the second acute angle?



5-23. **Examine** the triangles below. Note: the diagrams are not drawn to scale.

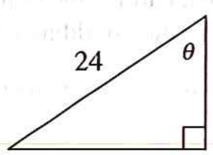


$\cos\theta = \frac{adj}{hyp}$ $\sin\theta = \frac{opp}{hyp}$ $\tan\theta = \frac{opp}{adj}$

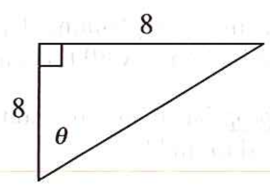
With your study team:

- Look through all the triangles first and see if any look familiar or are ones that you know how to answer right away without using a trig tool.
- Then, for all the other triangles, identify which tool to use based on where the reference angle (θ) is located and which side lengths are involved.
- Write and solve an equation to find the missing side length or angle.

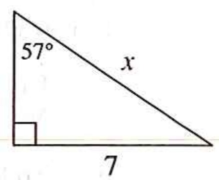
a.



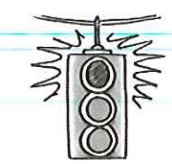
b.



c.



5-24. Peter cannot figure out what he did wrong. He wrote the equation below to find the missing angle of a triangle. However, his calculator gives him an error message each time he tries to calculate the angle.



Peter's work: $\cos\theta = \frac{8}{2.736}$

Jeri, his teammate, looked at his work and exclaimed, "Of course your calculator gives you an error! Your cosine ratio is impossible!" What is Jeri talking about? And how could she tell without seeing the triangle or checking on her calculator?

5-25. Write a Learning Log entry describing what you know about inverse trig functions. Be sure to include an example and a description of how to solve it. Title this entry, "Inverse Trig Functions" and include today's date.

Review & Preview

5-26. Solve the following equations for the given variable, if possible. Remember to check your answers.

a. $6x^2 = 150$

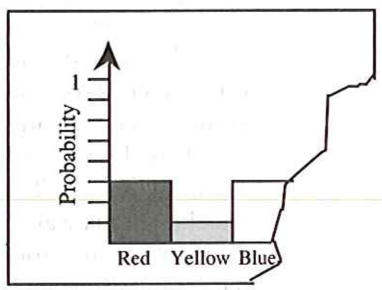
b. $4m + 3 - m = 3(m+1)$

c. $\sqrt{5x-1} = 3$

d. $(k-4)^2 = -3$

5-27. Mervin and Leela are in bumper cars. They are at opposite ends of a 100 meter track heading toward each other. If Mervin moves at a rate of 5.5 meters per second and Leela moves at a rate of 3.2 meters per second, how long does it take for them to collide?

5-28. Donnell has a bar graph which shows the probability of a colored section coming up on a spinner, but part of the graph has been ripped off.



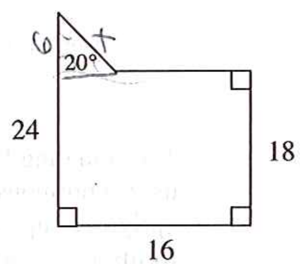
a. What is the probability of spinning red?

b. What is the probability of spinning yellow?

c. What is the probability of spinning blue?

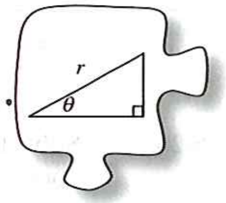
d. If there is only one color missing from the graph, namely green, what is the probability of spinning green? Why?

5-29. Find the area and the perimeter of the figure at right. Be sure to organize your work so you can explain your method later.



5-30. While playing a board game, Mimi noticed that she could roll the dice 8 times in 30 seconds. How many minutes should it take her to roll the dice 150 times?

5.1.4 How can I use trig ratios?



Trigonometric Applications

Throughout this chapter, you have developed new tools to help you determine the length of any side or the measurement of any angle of a right triangle. Trig ratios, coupled with the Pythagorean Theorem, give you the powerful ability to solve problems involving right triangles. Today you will apply this knowledge to solve some real world problems.

As you are working with your team on the problems below, be sure to draw and label a diagram and determine which trig ratio to use before you start solving.

5-31. CLIMBING IN YOSEMITE

David and Emily are climbing El Capitan, a big wall cliff in Yosemite National Park. David is on the ground holding the rope attached to Emily as she climbs. When Emily stops to rest, David wonders how high she has climbed. The rope is attached to his waist, about 3 feet off of the ground, and he has let out 40 feet of rope. This rope makes a 35° angle with the cliff wall.

a. Assuming that the rope is taut (i.e., pulled tight), approximately how high up the wall has Emily climbed?

b. How far away from the wall is David standing? Describe your method.

5-32. The Bungling Brothers Circus is in town and you are part of the crew that is setting up its enormous tent. The center pole that holds up the tent is 70 feet tall. To keep it upright, a support cable needs to be attached to the top of the pole so that the cable forms a 60° angle with the ground.

a. How long is the cable?

b. How far from the pole should the cable be attached to the ground?

5-33. Nathan is standing in a meadow, exactly 185 feet from the base of El Capitan. At 11:00 a.m., he observes Emily climbing up the wall, and determines that his angle of sight up to Emily is about 10°.

a. If Nathan's eyes are about 6 feet above the ground, about how high is Emily at 11:00 a.m.?

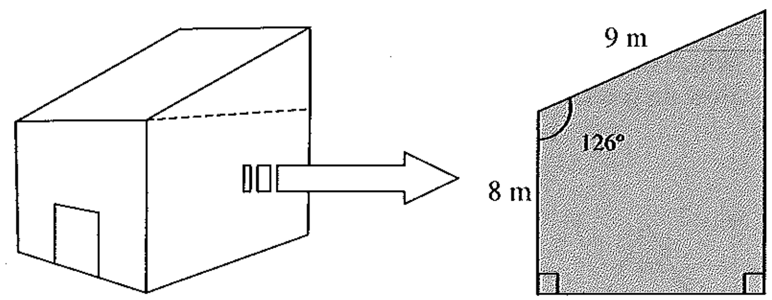
b. At 11:30 a.m., Emily has climbed some more, and Nathan's angle of sight to her is now 25°. How far has Emily climbed in the past 30 minutes?

c. If Emily climbs 32 feet higher in ten more minutes, at what angle will Nathan have to look in order to see Emily?

5-34. Forest needs to repaint the right side of his house because sunlight and rain have caused the paint to peel. Each can of paint states that it will cover 150 sq. feet. Help Forest decide how many cans of paint he should buy.

a. Copy the shaded diagram below onto your paper. Work with your team to find the area that will be painted green.

b. Assuming that Forest can only buy whole cans of paint, how many cans of paint should he buy? (Note: 1 square meter ≈ 10.764 square feet)



5-35. TEAM CHALLENGE

It is 11:55 a.m., and Emily has climbed even higher. The rope now makes an angle of 6° with the cliff wall. If David is 18 feet away from the base of El Capitan, at what angle should Nathan (who is 185 feet from the base) look up to see Emily?

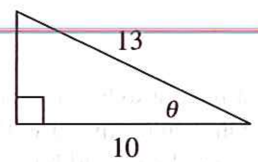
METHODS AND MEANINGS

MATH NOTES

Inverse Trigonometry

Just as subtraction "undoes" addition and multiplication "undoes" division, the inverse trigonometric functions "undo" the trig functions tangent, sine, and cosine. Specifically, **inverse trigonometric functions** are used to find the measure of an acute angle in a right triangle when a ratio of two sides is known. This is the **inverse**, or opposite, of finding the trig ratio from a known angle.

The inverse trigonometric functions that will be used in this course are $\mathbf{sin^{-1}}$, $\mathbf{cos^{-1}}$, and $\mathbf{tan^{-1}}$ (pronounced "inverse sine," "inverse cosine," and "inverse tangent"). Below is an example that shows how $\cos^{-1}$ may be used to find a missing angle, θ.

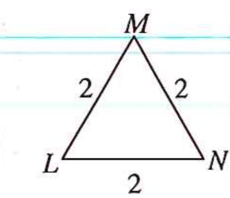


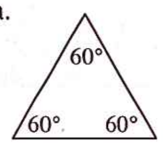
$$\cos\theta = \tfrac{10}{13}$$
$$\theta = \cos^{-1}\left(\tfrac{10}{13}\right)$$
$$\theta \approx 39.7°$$

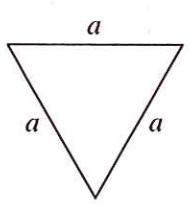
To evaluate $\cos^{-1}\left(\tfrac{10}{13}\right)$ on a scientific calculator, most calculators require the "2nd" or "INV" button to be pressed before the "cos" button.

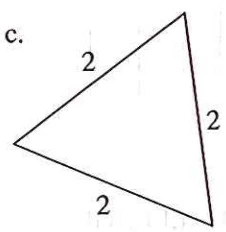
Review & Preview

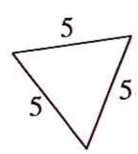
5-36. Which of the triangles below are similar to ΔLMN at right? How do you know? Explain.



a.


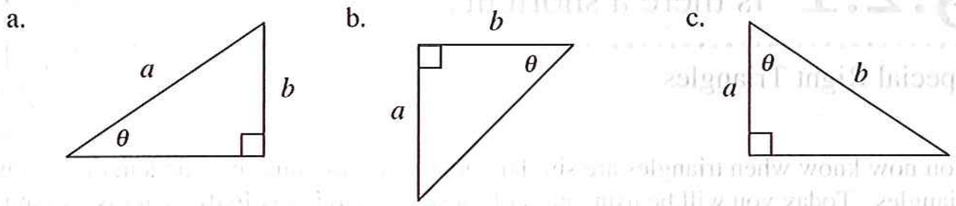
b.


c.


d.


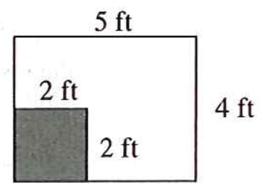
5-37. Find the equation of the line that has a 33.7° slope angle and a y-intercept at (0, 7). Assume the line has a positive slope.

5-38. For each triangle below, write a trigonometric equation relating a, b, and θ.



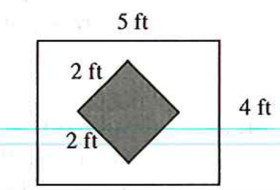
5-39. Kendrick is frantic. He remembers that several years ago he buried his Amazing Electron Ring in his little sister's sandbox, but he cannot remember where. A few minutes ago he heard that someone is willing to pay $1000 for it. He has his shovel and is ready to dig.

a. The sandbox is rectangular, measuring 4 feet by 5 feet, as shown at right. If Kendrick only has time to search in the 2 foot-square shaded region, what is the probability that he will find the ring?

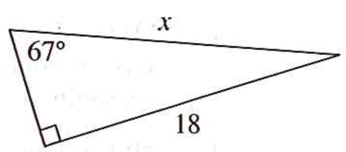


b. What is the probability that he will not find the ring? Explain how you found your answer.

c. Kendrick decides instead to dig in the square region shaded at right. Does this improve his chances for finding the ring? Why or why not?



5-40. Estelle is trying to find x in the triangle at right. She lost her scientific calculator, but luckily her teacher told her that $\sin 23° \approx 0.391$, $\cos 23° \approx 0.921$, and $\tan 23° \approx 0.424$.



a. Write an equation that Estelle could use to solve for x.

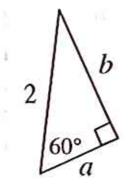
b. Without a calculator, how could Estelle find $\sin 67°$? Explain.

5.2.1 Is there a shortcut?

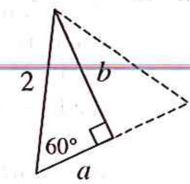
Special Right Triangles

You now know when triangles are similar and how to find missing side lengths in similar triangles. Today you will be using both of those ideas to **investigate** patterns within two types of special right triangles. These patterns will allow you to use a shortcut whenever you are finding side lengths in these particular types of right triangles.

5-41. Darren wants to find the side lengths of the triangle at right. The only problem is that he left his calculator at home and he does not remember the value of $\cos 60°$.



a. "That's okay," says his teammate, Jan. "I think I see a short-cut." Using tracing paper, she created the diagram at right by reflecting the triangle across the side with length b. What is the resulting shape? How do you know?

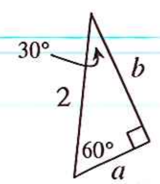


b. What if Jan had reflected across a different side? Would the result still be an equilateral triangle? Why or why not?

c. Use Jan's diagram to find the value of a without using a trig ratio.

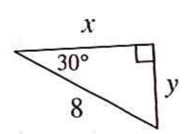
d. Now find the length of b without a calculator. Leave your answer in **exact form**. In other words, do not approximate the height with a decimal.

5-42. Darren's triangle is an example of a half-equilateral triangle, also known as a **30°- 60°- 90° triangle** because of its nice angle measures. Darren is starting to understand Jan's short-cut, but he still has some questions. Help Darren by answering his questions below.



a. "Will this approach work on all triangles? In other words, can you always form an equilateral triangle by reflecting a right triangle?" Explain your **reasoning**.

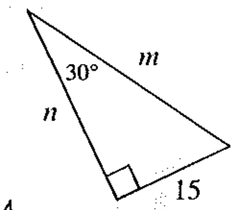
b. "What if the triangle looks different?" Use tracing paper to show how Darren can reflect the triangle at right to form an equilateral triangle. Then find the lengths of x and y without a calculator.



c. "Is the longer leg of a 30°- 60°- 90° triangle always going to be the length of the shorter leg multiplied by $\sqrt{3}$? Why or why not?"

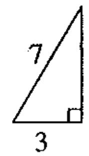
Problem continues on next page →

5-42. *Problem continued from previous page.*

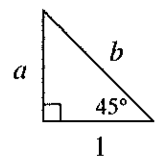


d. "What if I only know the length of the shorter leg?" Consider the triangle at right. **Visualize** the equilateral triangle. Then find the values of n and m without a calculator.

e. Darren drew the triangle at right and is wondering if it also is a 30°- 60°- 90° triangle. What do you think? How do you know?



5-43. Darren wonders if he can find a similar pattern in another special triangle he knows, shown at right.

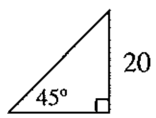


a. Use what you know about this triangle to help Darren find the lengths of a and b without a trig tool or a calculator. Leave your answer in exact form.

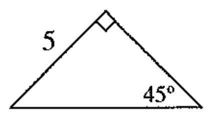
b. What should Darren name this triangle?

c. Use the fact that all 45°- 45°- 90° triangles are similar to find the missing side lengths in the right triangles below. Leave your answers in exact, radical form.

(1)

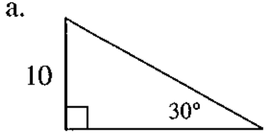


(2)

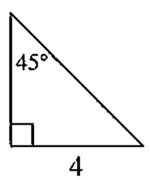


5-44. Use your new 30°- 60°- 90° and 45°- 45°- 90° triangle patterns to quickly find the lengths of the missing sides in each of the triangles below. Do not use a calculator. Leave answers in exact form. Note: The triangles are not necessarily drawn to scale.

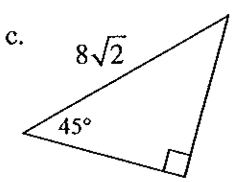
a.



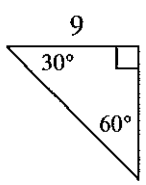
b.



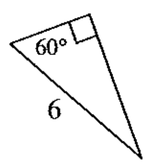
c.



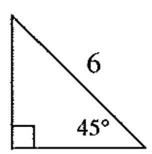
d.



e.



f.



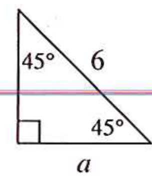
5-45. In your Learning Log, explain what you know about 30°- 60°- 90° and 45°- 45°- 90° triangles. Include diagrams of each. Label this entry "Special Right Triangles" and include today's date.

LOOKING DEEPER

MATH NOTES

Rationalizing a Denominator

In Lesson 5.2.1, you developed some shortcuts to help find the lengths of the sides of a 30°- 60°- 90° and 45°- 45°- 90° triangle. This will enable you to solve similar problems in the future without a calculator or a Trig Table.



However, sometimes using the shortcuts leads to some strange looking answers. For example, when finding the length of a in the triangle at right, you will get the expression $\frac{6}{\sqrt{2}}$.

A number with a radical in the denominator is difficult to estimate. Therefore, it is sometimes beneficial to **rationalize the denominator** so that no radical remains in the denominator. Study the example below.

Example: Simplify $\frac{6}{\sqrt{2}}$.

Example

$$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$$
$$= 3\sqrt{2}$$

First, multiply the numerator and denominator by the radical in the denominator. Since $\frac{\sqrt{2}}{\sqrt{2}} = 1$, this does not change the value of the expression.

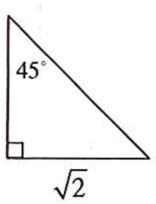
After multiplying, notice that the denominator no long has a radical, since $\sqrt{2} \cdot \sqrt{2} = 2$.

Often, the product can be further simplified. Since 2 divides evenly into 6, the expression $\frac{6\sqrt{2}}{2}$ can be rewritten as $3\sqrt{2}$.

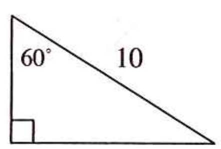
Review & Preview

5-46. For each triangle below, use your triangle shortcuts from this lesson to find the missing side lengths. Then find the area and perimeter of the triangle.

a.

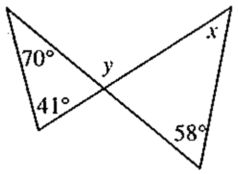


b.

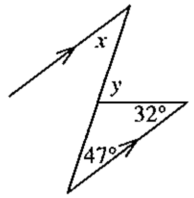


5-47. Use the relationships found in each of the diagrams below to solve for x and y. Assume the diagrams are not drawn to scale. State which geometric relationships you used.

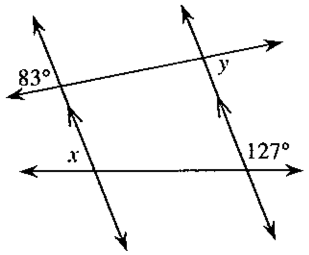
a.



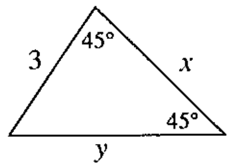
b.



c.



d.



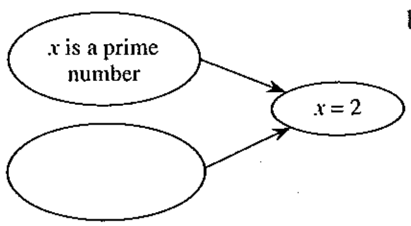
5-48. On graph paper, graph $\overline{AB}$ if $A(1, 6)$ and $B(5, 2)$.

a. Find AB (the length of $\overline{AB}$). Leave your answer in **exact form**. That is, do not approximate with a decimal. Explain your method.

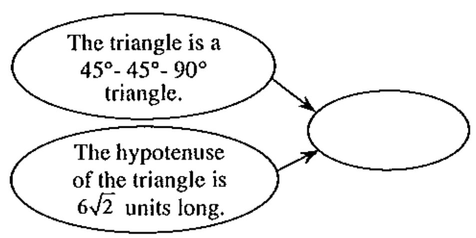
b. Reflect $\overline{AB}$ across the y-axis to create $\overline{A'B'}$. What type of shape is $ABB'A'$ if the points are connected in order? Then find the area of $ABB'A'$.

5-49. Fill in the blank ovals below so that each flowchart is correct.

a.

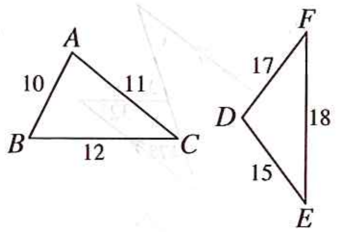


b.

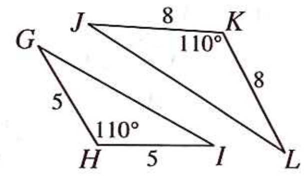


5-50. Decide if each pair of triangles below are similar. If they are similar, show a flowchart that organizes your **reasoning**. If they are not similar, explain how you know.

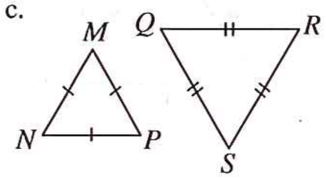
a.



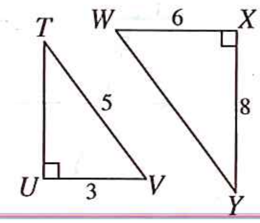
b.



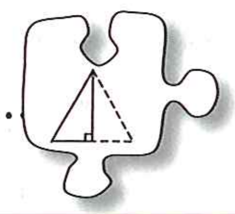
c.



d.



5.2.2 How can I use similar triangles?

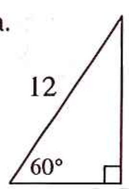


Pythagorean Triples

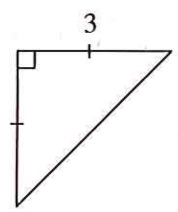
In Lesson 5.2.1, you developed shortcuts that will help you quickly find the lengths of the sides of certain right triangles. What other shortcuts can be helpful? As you work today with your study team, look for patterns and connections between triangles.

5-51. Use the tools you have developed to find the lengths of the missing sides of the triangles below. If you know of a shortcut, share it with your team. Be ready to share your strategies with the class.

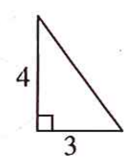
a.



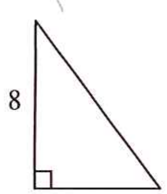
b.



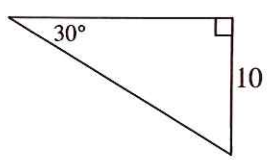
c.



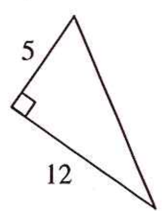
d.



e.



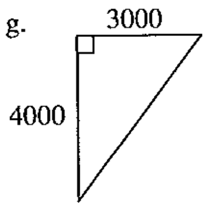
f.



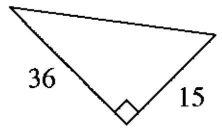
Problem continues on next page →

5-51. *Problem continued from previous page.*

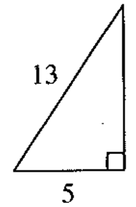
g.



h.



i.



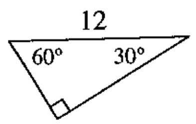
5-52. Karl noticed some patterns as he was finding the sides of the triangles in problem 5-46. He recognized that the triangles in parts (c), (d), (f), (g), (h), and (i) are integers. He also noticed that knowing the triangle in part (c) can help find the hypotenuse in parts (d) and (g).

a. Groups of numbers like 3, 4, 5 and 5, 12, 13 are called **Pythagorean Triples.** Why do you think they are called Pythagorean Triples?

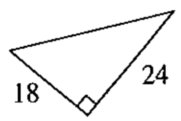
b. What other sets of numbers are also Pythagorean Triples? How many different sets can you find?

5-53. With your team, find the missing side lengths for each triangle below. Try to use your new shortcuts when possible.

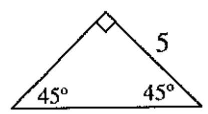
a.



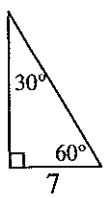
b.



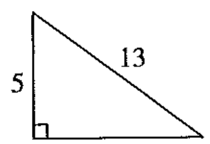
c.



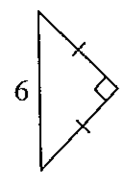
d.



e.

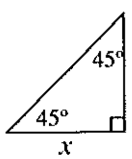


f.

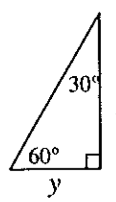


5-54. Diana looked at the next problems and thought she could not do them. Erik pointed out that they are the same as the previous triangles, but instead of numbers, each side length is given in terms of a variable. Use Erik's idea to help you find the missing side lengths of the triangles below:

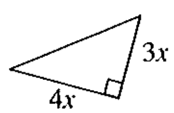
a.



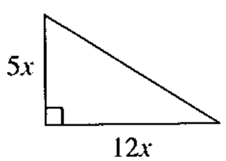
b.



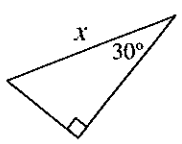
c.



d.

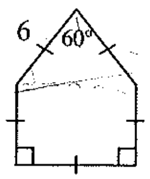


e.

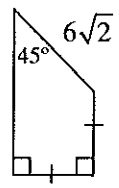


5-55. Use your shortcuts to find the area and perimeter of each shape below:

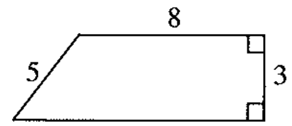
a.



b.



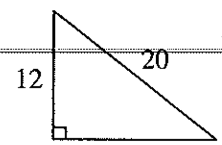
c.



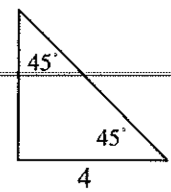
Review & Preview

5-56. The sides of each of the triangles below can be found using one of the shortcuts from Section 5.2. Try to find the missing lengths using your patterns. Do not use a calculator.

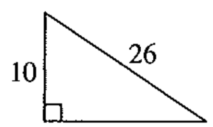
a.



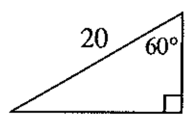
b.



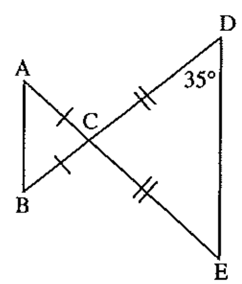
c.



d.



5-57. Copy the diagram at right onto your paper.

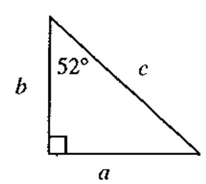


a. Find the measures of all the angles in the diagram.

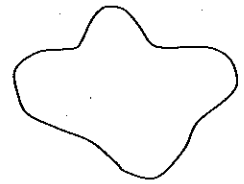
b. Make a flowchart showing that the triangles are similar.

c. Cheri and Roberta noticed their similarity statements for part (b) were not the same. Cheri had stated $\Delta ABC \sim \Delta DEC$, while Roberta maintained that $\Delta ABC \sim \Delta EDC$. Who is correct? Or are they both correct? Explain your **reasoning**.

5-58. Using the triangle at right, write an expression representing $\cos 52°$. Then write an expression for $\tan 52°$ and $\cos 38°$.



5-59. Hadrosaurs, a family of duck-billed, plant-eating dinosaurs, were large creatures with thick, strong tails. It has recently been determined that hadrosaurs probably originated in North America.



Example of a hadrosaur footprint.

Scientists in Alaska recently found a hadrosaur footprint like the one at right that measured 14 inches across. It is believed that the footprint was created by a young dinosaur that was approximately 27 feet long. Adult hadrosaurs have been known to be 40 feet long. How wide would you expect a footprint of an adult hadrosaur to be? Show your **reasoning**.

5-60. Jeynysha has a Shape Bucket with a trapezoid, right triangle, scalene triangle, parallelogram, square, rhombus, pentagon, and kite. If she reaches in the bucket and randomly selects a shape, find:

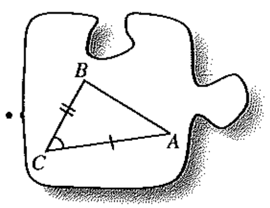
a. P(at least one pair of parallel sides)

b. P(hexagon)

c. P(not a triangle)

d. P(has at least 3 sides)

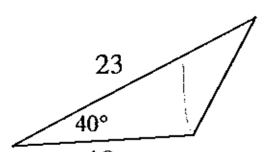
5.3.1 What triangle tools do I still need?



Finding Missing Parts of Triangles

When do you have enough information to find all of the angle measures and side lengths of a triangle? For example, can you find all of the side lengths if you are only informed about the three angles? Does it matter if the triangle has a right angle or not? Today you will organize your triangle knowledge so that you know what tools you have and for which triangles you can accurately find all of the angle measures and side lengths.

5-61. How many ways can three pieces of information about a triangle be given? For example, the three given measurements could be one angle and two side lengths, as shown in the triangle at right. List as many other combinations of three pieces of information about a triangle as you can.



5-62. HOW MUCH INFORMATION DO YOU NEED?

So far in this course you have developed several tools to find missing parts of triangles. But how complete is your Triangle Toolkit? Are there more tools that you need to develop?

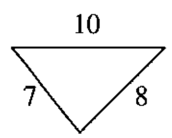
Your Task: With your team, find the missing angles and sides of the triangles below. (Also printed on the Lesson 5.3.1 Resource Page). Notice that each triangle has three given pieces of information about its angles and sides. If there is not enough information or if you do not yet have the tools to find the missing information, explain why.

Discussion Points

- Are there any triangles that look familiar or that you already have a **strategy** for?
- What tools do you have to solve for parts of triangles? For what types of triangles do these tools work?
- Would it be helpful to subdivide any of the triangles into right triangles?

GIVEN: 3 Sides

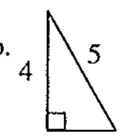
a.



Three sides

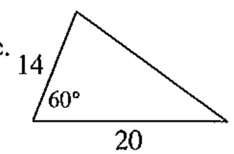
GIVEN: 2 Sides and 1 Angle

b.



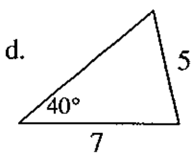
A right angle and two sides

c.



Two sides and an angle between them

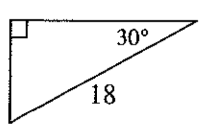
d.



Two sides and an angle not between them

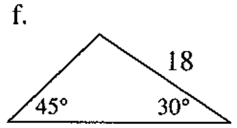
GIVEN: 1 Side and 2 Angles

e.



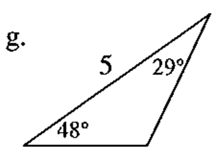
A right angle, another angle, and a side

f.



Two angles and a side not between them

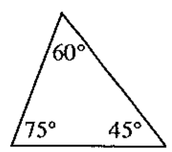
g.



Two angles and a side between them

GIVEN: 3 Angles

h.



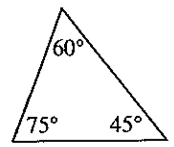
Three angles

Further Guidance

5-63. While looking for different strategies to use, advice from a teammate can often help.

a. Angelo thinks that the Pythagorean Theorem is a useful tool. For which types of triangles is the Pythagorean Theorem useful? Look for these types of triangles in problem 5-62 and use the theorem to solve for any missing sides.

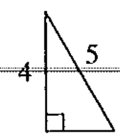
b. Tomas remembers using trigonometric ratios to find the missing sides and angles of a triangle. Which triangles from problem 5-62 can be analyzed using this **strategy**?



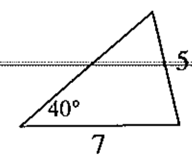
Three angles

c. Ngoc thinks that more than one triangle exists with the angles at right. Is she correct? If so, how are these triangles related?

d. Does it matter if the triangle is a right triangle? For example, both of these triangles (from parts (b) and (d)) give an angle and two sides. Can you use the same tool for both? Why or why not?



A right angle and two sides



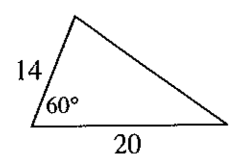
Two sides and an angle not between them

Further Guidance section ends here.

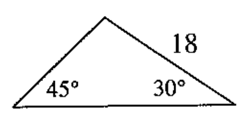
5-64. WHAT IF IT DOES NOT HAVE A RIGHT ANGLE?

If your team needs help on parts (c) and (f) of problem 5-62, Leila has an idea. She knows that she has some tools to use with right triangles but noticed that some of the triangles in problem 5-62 are <u>not</u> right triangles. Therefore, she thinks it is a good idea to split a triangle into two right triangles.

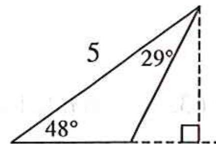
a. Discuss with your team how to change the diagram at right so that the triangle is divided into two right triangles. Then use your right triangle tools to solve for the missing sides and angles.



b. Leila wonders if her method would work for other triangles too. Test her method on the triangle from part (f) of problem 5-62 (also shown at right). Does her method work?



5-65. Ryan liked Leila's idea so much, he looked for a way to create a right triangle in the triangle from part (g) of problem 5-62. He decided to draw a height <u>outside</u> the triangle, forming a large right triangle. Use the right triangle to help you find the missing side lengths of the original triangle.



5-66. LEARNING LOG

Return to problem 5-62 and **examine** all of the ways three pieces of information can be given about a triangle. For which triangle(s) were you able to find missing side lengths and angles? For which triangle(s) do you not have enough given information? For which triangle(s) do you need a new **strategy**? Reflect on the strategies you have developed so far. Title this entry "Strategies to find Sides and Angles of a Triangle" and include today's date.

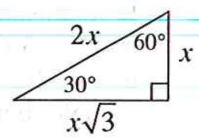
METHODS AND MEANINGS

MATH NOTES

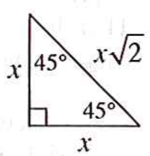
Special Right Triangles

So far in this chapter, you have learned about several special right triangles that will reappear throughout the rest of this course. Being able to recognize these triangles will enable you to quickly find the lengths of the sides and will save you time and effort in the future.

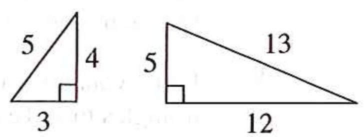
The half-equilateral triangle is also known as the **30°- 60°- 90° triangle.** The sides of this triangle are always in the ratio $1:\sqrt{3}:2$, as shown at right.



Another special triangle is the **45°- 45°- 90° triangle.** This triangle is also commonly known as an isosceles right triangle. The ratio of the sides of this triangle is always $1:1:\sqrt{2}$.



You also discovered several **Pythagorean triples**. A Pythagorean triple is any set of 3 positive integers a, b, and c for which $a^2+b^2=c^2$. Two of the common Pythagorean triples that you will see throughout this course are shown at right.

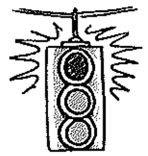


Review & Preview

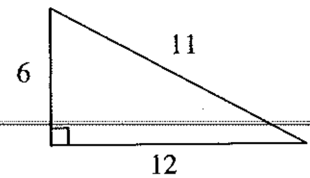
5-67. To paint a house, Travis leans a ladder against the wall. If the ladder is 16 feet long and it makes contact with the house 14 feet above ground, what angle does the ladder make with the ground? Draw a diagram of this situation and show all work.

5-68. WACKY DIAGRAMS

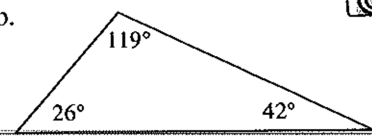
After drawing some diagrams on his paper, Jason thinks there is something wrong. **Examine** each diagram below and decide whether or not the triangle could exist. If it cannot exist, explain why not.



a.



b.



5-69. William thinks that the hypotenuse must be the longest side of a right triangle, but Chad does not agree. Who is correct? Support your answer with an explanation and a counterexample, if possible.

5-70. Plot $\triangle ABC$ on graph paper with points $A(3, 3)$, $B(1, 1)$, and $C(6, 1)$.

a. Reflect $\triangle ABC$ across the x-axis. Then translate the result to the left 6 and down 3. Name the coordinates of the result.

b. Rotate $\triangle ABC$ 90° counterclockwise (↺) about the origin. Then reflect the result across the y-axis. Name the coordinates of the result.

5-71. Solve the equations below, if possible. If there is no solution, explain why.

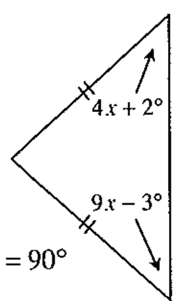
a. $\frac{8-x}{x} = \frac{3}{2}$

b. $-2(5x-1)-3=-10x$

c. $x^2+8x-33=0$

d. $\frac{2}{3}x-12=180$

5-72. **Multiple Choice:** Based on the relationships provided in the diagram, which of the equations below is correct? **Justify** your solution.



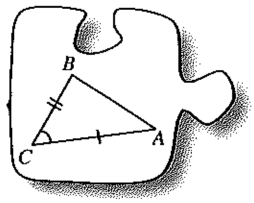
a. $4x+2°+9x-3°=90°$

b. $4x+2°=9x-3°$

c. $4x+2°+9x-3°=180°$

d. $(4x+2°)(9x-3°)=90°$

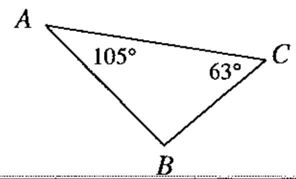
5.3.2 Is there a faster way?



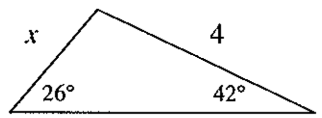
Law of Sines

In problem 6-64, you used a complicated **strategy** to find the lengths of sides and measures of angles for a non-right triangle. Is there a tool you can use to find angles and side lengths of non-right triangles directly, using fewer steps? Today you will explore the relationships that exist among the sides and angles of triangles and will develop a new tool called the Law of Sines.

5-73. Is there a relationship between a triangle's side and the angle opposite to it? For example, assume that the diagram for $\triangle ABC$, shown at right, is not drawn to scale. Based on the angle measures provided in the diagram, which side must be longest? Which side must be shortest? How do you know?



5-74. When Madelyn **examined** the triangle at right, she said, "I don't think this diagram is drawn to scale because I think the side labeled x has to be longer than 4."

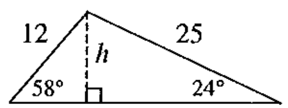


a. Do you agree with Madelyn? Why or why not?

b. Leila thinks that x can be found by using right triangles. Review what you learned in Lesson 5.3.1 by finding the value of x.

c. Find the area of the triangle.

5-75. Thui and Ivan came up with two different ways to find the height of the triangle at right.



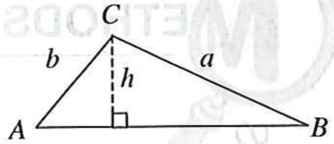
- Using the right triangle on the left, Thui wrote: $\sin 58° = \frac{h}{12}$.
- Ivan also used the sine function, but his equation looked like this: $\sin 24° = \frac{h}{25}$.

a. Which triangle did Ivan use?

b. Calculate h using Thui's equation and again using Ivan's equation. How do their answers compare?

5-76. LAW OF SINES

Edwin wonders if Thui's and Ivan's methods can help find a way to relate the sides and angles of a non-right triangle. To find the height, Ivan and Thui each used the sine ratio with an acute angle and the hypotenuse of a right triangle.

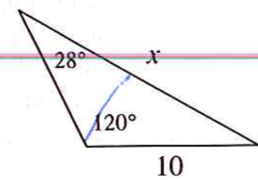


a. Use the triangle above to find **two expressions** for h using the individual right triangles like you did in problem 5-75.

b. Use your expressions from part (a) to show that $\frac{\sin(m\measuredangle B)}{b} = \frac{\sin(m\measuredangle A)}{a}$.

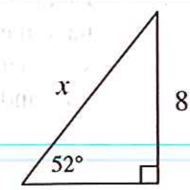
c. Describe where $\measuredangle B$ is located in relation to the side labeled b. How is $\measuredangle A$ related to the side labeled a?

d. The relationship $\frac{\sin(m\measuredangle B)}{b} = \frac{\sin(m\measuredangle A)}{a}$ is called the **Law of Sines**. Read the Math Notes box for this lesson to learn more about this relationship. Then use this relationship to solve for x in the triangle at right.



5-77. EXTENSION

Does the Law of Sines work for a right triangle as well? Test this idea by solving for x in the triangle at right <u>twice</u>: once using the Law of Sines and again using right-triangle trigonometry (such as sine, cosine, or tangent). What happened? If it worked, do you think it will work for all right triangles?



5-78. LEARNING LOG

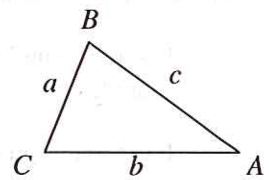
Reflect on what you have learned during this lesson about the sides and angles of a triangle. What is the Law of Sines and when can it be used? Include an example. Title this entry "Law of Sines" and include today's date.

Methods and Meanings

MATH NOTES

Law of Sines

For any ΔABC, the ratio of the sine of an angle to the length of the side opposite the angle is constant. This means that:



$$\frac{\sin(m\measuredangle A)}{a} = \frac{\sin(m\measuredangle B)}{b},$$

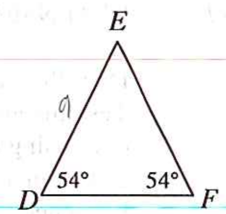
$$\frac{\sin(m\measuredangle B)}{b} = \frac{\sin(m\measuredangle C)}{c}, \text{ and}$$

$$\frac{\sin(m\measuredangle A)}{a} = \frac{\sin(m\measuredangle C)}{c}.$$

This property is called the **Law of Sines**. This is a powerful tool because it allows you to use the sine ratio to solve for measures of angles and lengths of sides of *any* triangle, not just right triangles. The law works for angle measures between 0° and 180°.

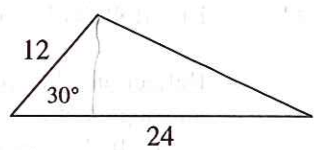
Review & Preview

5-79. Lizzie noticed that two angles in ΔDEF, shown at right, have the same measure. Based on this information, what statement can you make about the relationship between $\overline{ED}$ and $\overline{EF}$?

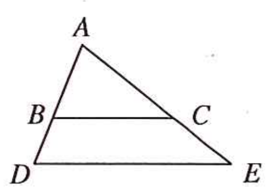


5-80. Find the length of $\overline{DF}$ in the diagram from problem 5-79 if $DE = 9$ units.

5-81. Find the area of the triangle at right. Show all work.



5-82. In the diagram at right, ΔABC and ΔADE are similar. If $AB = 5$, $BD = 4$, and $BC = 7$, then what is DE?



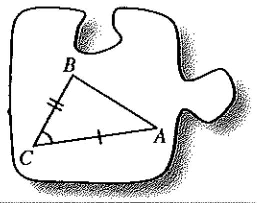
5-83. A particular spinner only has two regions: green and purple. If the spinner is randomly spun twice, the probability of it landing on green twice is 16%. What is the probability of the spinner landing on purple twice?

5-84. Solve the system of equations below. Write your solution as a point in (x, y) form. Check your solution.

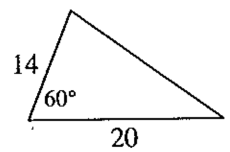
$$y = -3x - 2$$
$$2x + 5y = 16$$

5.3.3 How can I complete my triangle toolkit?

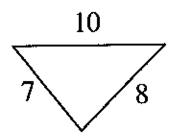
Law of Cosines



So far, you have three tools that will help you solve for missing sides and lengths of a triangle. In fact, one of those tools, the Law of Sines, even helps when the triangle is not a right triangle. There are still two triangles from our exploration in Lesson 5.3.1 that you cannot solve directly with any of your existing tools. Today you will develop a tool to help find missing side lengths and angle measures for triangles such as those shown at right.



Two sides and an angle between them

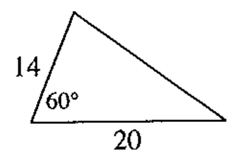


Three sides

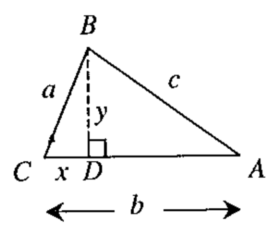
By the end of this lesson, you will have a complete set of tools to help solve for the side lengths and angle measures of *any* triangle, as long as enough information is given.

5-85. LAW OF COSINES

Leila remembers that in problem 5-64, she solved for the side lengths and missing angles of the triangle at right by dividing the triangle into two right triangles. She thinks that using two right triangles may help find a tool that works for any triangle with two given sides and a given angle between them. Help Leila generalize this process by answering the questions below.



a. **Examine** $\triangle ABC$ at right. Assume that you know the lengths of sides a and b and the measure of $\measuredangle C$. Notice how the side opposite $\measuredangle A$ is labeled a and the side opposite $\measuredangle B$ is labeled b, and so on. Line segment $\overline{BD}$ is drawn so that $\triangle ABC$ is divided into two right triangles. If $CD = x$, then what is DA?



Problem continues on next page →

5-85. *Problem continued from previous page.*

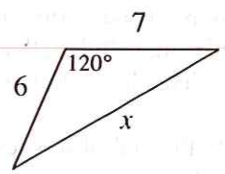
b. Write an equation relating a, x, and y. Likewise, write an equation relating the side lengths of $\triangle BDA$.

c. Leila noticed that both expressions from part (b) have a y^2-term. "Can we combine these equations so that we have one equation that links sides a, b, and c?" she asked. With your team, use algebra to combine these two equations so that y^2 is eliminated. Then simplify the resulting equation as much as possible.

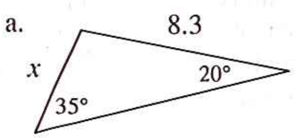
d. The equation from part (c) still has an x-term. Using only a and $m\measuredangle C$, find an expression for x using the left-hand triangle. Solve your equation for x, then substitute this expression into your equation from part (c) for x.

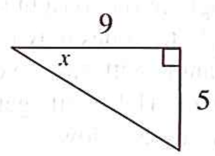
e. Solve your equation from part (d) for c^2. You have now found an equation that links the lengths of two sides and the measure of the angle between them to find the length of the side opposite the angle. This relationship is called the **Law of Cosines**. Read the Math Notes box for this lesson to learn more about the Law of Cosines.

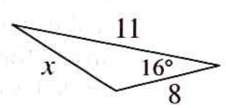
f. Use the Law of Cosines to solve for x in the triangle at right.



5-86. You now have many tools to solve for missing side lengths and angle measures. Decide which tool to use for each of the triangles below and solve for x. Decide if your answer is reasonable based on the diagram.

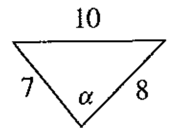
a. 

b. 

c. 

5-87. EXTENSION

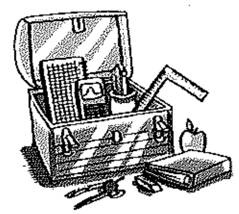
Not only can the Law of Cosines be used to solve for side lengths, but it can also be used to solve for angles. Consider the triangle from Lesson 5.3.1, shown at right.



a. Write an equation that relates the three sides and the angle α. Then solve the equation for α.

b. Now solve for the other two angles using any method. Be sure to name which tool(s) you used!

5-88. You have now completed your Triangle Toolkit and can find the missing side lengths and angle measures for *any* triangle, provided that enough information is given. Add the Law of Sines and Law of Cosines to your Triangle Toolkit for reference later in this course.

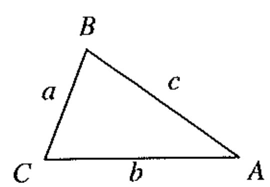


METHODS AND MEANINGS

MATH NOTES

Law of Cosines

Just like the Law of Sines, the **Law of Cosines** represents a relationship between the sides and angles of a triangle.



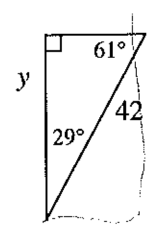
Specifically, when given the lengths of any two sides, such as a and b, and the angle between them, $\measuredangle C$, the length of the third side, in this case c, can be found using this relationship:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Similar equations can be used to solve for a and b.

Review & Preview

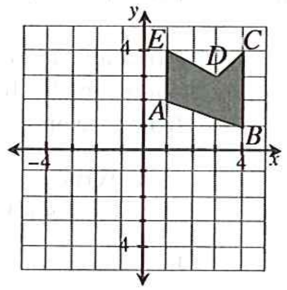
5-89. Eugene wants to use the cosine ratio to find y on this triangle.



a. Which angle should he use for his slope angle? Why?

b. Set up an equation, and solve for y using cosine.

5-90. Copy the graph at right onto graph paper.



a. If the shape $ABCDE$ were rotated about the origin 180°, where would point A' be?

b. If the shape $ABCDE$ were reflected across the x-axis, where would point C' be?

c. If the shape $ABCDE$ were translated so that each point (x, y) corresponds to $(x - 1, y + 3)$, where would point B' be?

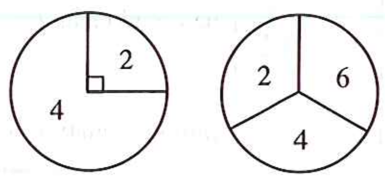
5-91. Jerry was trying to use a flowchart to describe how his friend Marcy feels about Whizzbangs candy. **Examine** his flowchart below.

a. How do you know Jerry's flowchart is incorrect?

b. Make a flowchart on your paper with the same three ovals, but with arrows drawn in so the flowchart makes sense. Explain why your flowchart makes more sense than the one at right.

5-92. On graph paper, graph the line $y = \frac{3}{4}x + 6$. Then find the slope angle (the acute angle the line makes with the x-axis).

5-93. The spinners at right are spun and the results are added.

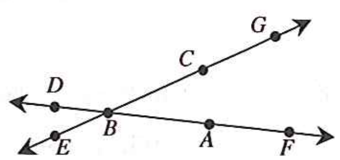


Spinner #1 Spinner #2

a. Find P(sum is 4).

b. Find P(sum is 8).

5-94. **Examine** the diagram at right. Which angle below is another name for $\measuredangle ABC$? Note: More than one solution is possible.



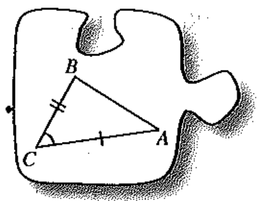
a. $\measuredangle ABE$ b. $\measuredangle GBD$

c. $\measuredangle FBG$ d. $\measuredangle EBC$

e. None of these

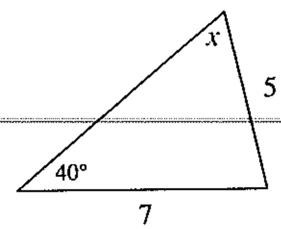
5.3.4 Is there more than one possible triangle?

Optional: Ambiguous Triangles



Now that you have completed your Triangle Toolkit, you can solve for the missing angles or sides of any triangle, provided that enough information is given. But how do you know if you have enough information? What if there is more than one possible triangle? Today you will explore situations where your tools may not be adequate to solve for the missing side lengths and angle measures of a triangle.

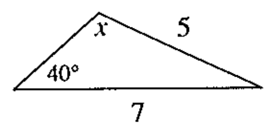
5-95. **Examine** the triangle at right from problem 5-62. Notice that two side lengths and an angle measure not between the labeled sides are given. (This situation is sometimes referred to as **SSA**.)



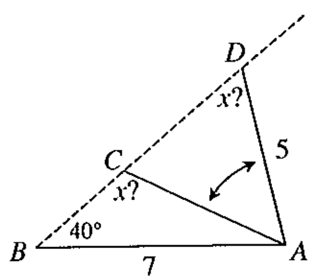
a. Assuming that the triangle is not drawn to scale, what do you know about x? Is it more than 40° or less than 40°? **Justify** your conclusion.

b. Solve for x. Was your conclusion from part (a) correct?

c. "Hold on!" proclaims your teammate, Missy. "That's not what I got. I found out that $x \approx 115.9°$." She then drew the triangle at right. Do you agree with Missy? Use the Law of Sines to test her answer. That is, find out if the ratios of $\frac{\sin(angle)}{\text{opposite side}}$ are equal for each angle and its opposite side.



d. What happened? How can there be two possible answers for x? **Examine** the diagram at right for clues.



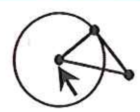
e. What is the relationship between the two solution angles? Do you think this relationship always exists? **Examine** the diagram above, which shows the two angle solutions, and use it to explain how the solutions are related.

5-96. In problem 5-95, you determined that it was possible to create two triangles because you were given only two side lengths and an angle not between them. When this happens, we call this **triangle ambiguity** since we cannot tell which triangle was the one we were supposed to find. Will there always be two possible triangles? Can there ever be more than two possible triangles? Think about this as you answer the questions below.

a. Obtain the Lesson 5.3.4 Resource Page and some linguini (or other flat manipulative) from your teacher. Prepare pieces of linguini that are 1 inch, 1.5 inches, 2 inches, 2.5 inches, and 3 inches long.

b. For each length of linguini, place one end at point A in the diagram on the resource page. Determine if you can form a triangle by connecting the linguini with the dashed side to close the triangle. If you can make a triangle, label the third vertex C and label $\overline{AC}$ with its length. Can you form more than one triangle with the same side length? Is a triangle always possible? Record any conjectures you make.

c. If the technology is available, test your conjectures from part (b) with a dynamic geometry tool. Try to learn everything you can about SSA triangles. Use the questions below to guide your **investigation**.

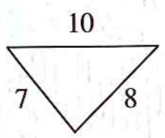


- Can you find a way to create three possible triangles with one set of SSA information?
- Is it ever impossible to form a triangle?
- Is it possible to choose SSA information that will create only one triangle? How?

5-97. Now that Alex knows that SSA (two sides and an angle not between them) can result in more than one possible triangle, he wants to know if other types of given information can also create ambiguous results. For example, when given three side lengths, is more than one triangle possible?

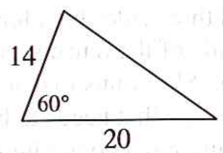
Examine each of the diagrams below (from problem 5-62) and determine if any other types of triangles are also ambiguous. You may want to imagine building the shapes with linguini. Remember that the given information cannot change – thus, if a side length is given, it cannot be lengthened or shortened.

a.



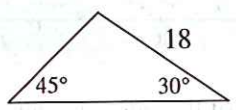
Three sides (SSS)

b.



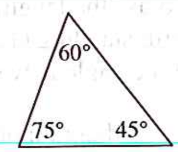
Two sides and an angle between them (SAS)

c.



Two angles and a side not between them (AAS)

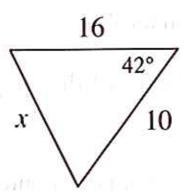
d.



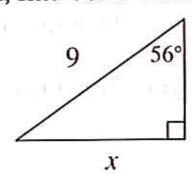
Three angles (AAA)

5-98. You now have many tools to use when solving for missing sides lengths and angle measures. Decide which tool to use for each of the triangles below and solve for x. If there is more than one solution, find both. Name the tool you use.

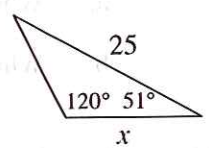
a.



b.

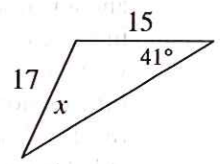


c.



5-99. EXTENSION

While examining the triangle at right, Alex stated, "Well, I know that there can be at most one solution."



a. **Examine** the information given in the triangle. What do you know about x? Is it more than 41° or less? How can you tell?

b. Alex remembered that if there were two solutions, then they had to be supplementary. Explain why this triangle cannot have two different values for x.

Review & Preview

5-100. Farmer Jill has a problem. She lives on a triangular plot of land that is surrounded on all three sides by a fence. Yesterday, one side of the fence was torn down in a storm. She wants to determine the length of the side that needs to be rebuilt so she can purchase enough lumber. Since the weather is still too poor for her to go outside and measure the distance, she decides to use the lengths of the two sides that are still standing (116 feet and 224 feet) and the angle between them (58°).

a. Draw a diagram of this situation. Label all of the sides and angles that Farmer Jill has measurements for.

b. Find the length of the fence that needs to be replaced. Show all work. Which tool did you use?

5-101. Mr. Miller has informed you that two shapes are similar.

a. What does this tell you about the angles in the shapes?

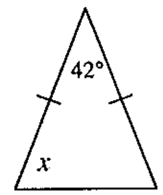
b. What does this tell you about the lengths of the sides of the shapes?

5-102. Find the equation of the line that has a slope angle of 25° and a y-intercept of (0, 4). Sketch a graph of this line. Assume the slope of the line is positive.

5-103. Two sides of a triangle have lengths 9 and 14 units. Describe what you know about the length of the third side.

5-104. Tehachapi High School has 839 students and is increasing by 34 students per year. Meanwhile, Fresno High School has 1644 students and is decreasing by 81 students per year. In how many years will the two high schools have the same number of students? Be sure to show all work.

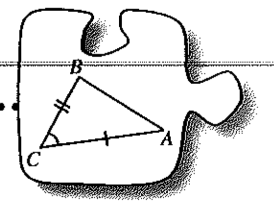
5-105. **Multiple Choice:** In the triangle at right, x must be:



a. 42° b. 69° c. 21°

d. 138° e. none of these

5·3·5 Which tool should I use?

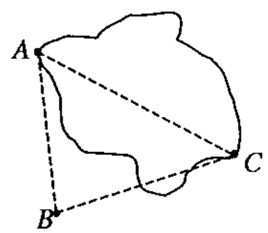


Choosing a Tool

During this section, you have developed new tools such as the Law of Sines and the Law of Cosines to find the lengths of sides and the measures of angles of a triangle. These strategies are very useful because they work with all triangles, not just right triangles. But when is each **strategy** the best one to use? Today you will focus on which **strategy** is most effective to use in different situations. You will also apply your **strategies** to triangles in different contexts.

As you work on these problems, keep in mind that good communication and a joint brainstorming of ideas will greatly enhance your team's ability to **choose a strategy** and to solve these problems.

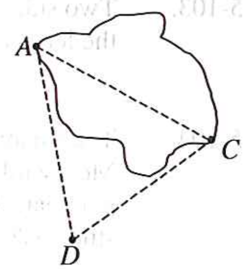
5-106. LAKE TOFTEE, Part One



A bridge is being designed to connect two towns along the shores of Lake Toftee in Minnesota (one at point A and the other at point C). Lavanne has been given the responsibility of determining the length of the bridge.

Since he could not accurately measure across the lake (AC), he measured the only distance he could by foot (AB). He drove a stake into the ground at point B and found that $AB = 684$ feet. He also used a protractor to determine that $m\angle B = 79°$ and $m\angle C = 53°$. How long will the bridge need to be?

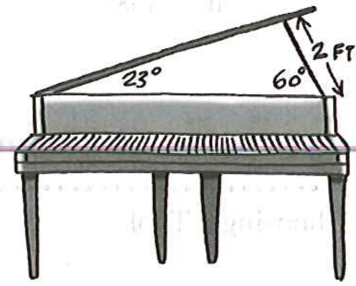
5-107. LAKE TOFTEE, Part Two



Lavanne was not convinced that his measurements from problem 5-106 were correct. He decided to measure the distance between towns A and C again using a different method to verify his results.

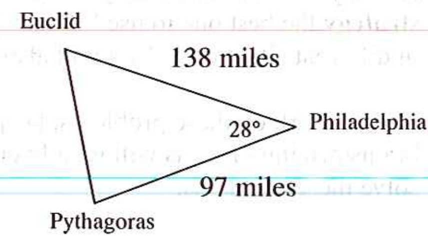
This time, he decided to drive a stake in the ground at point D, which is 800 feet from town A and 694 feet from town C. He also determined that $m\measuredangle D = 68°$. Using these measurements, how wide is the lake between points A and C? Does this confirm the results from problem 5-106?

5-108. The lid of a grand piano is propped open by a supporting arm, as shown in the diagram at right. Carson knows that the supporting arm is 2 feet long and makes a 60° angle with the piano. He also knows that the piano lid makes a 23° angle with the piano. How wide is the piano?



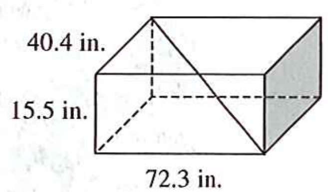
5-109. PENNANT RACE

Your basketball team has made it to the semi-finals and now needs to win only two more games to go to the finals. Your plan is to leave Philadelphia, travel 138 miles to the town of Euclid, and then play the team there. Then you will leave Euclid, travel to Pythagoras, and play that team. Finally, you will travel 97 miles to return home.

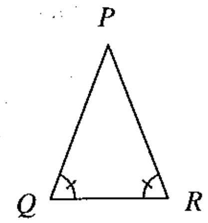


Your team bus can travel only 300 miles on one tank of gas. Assuming that all of the roads connecting the three towns are straight and that the two roads that connect in Philadelphia form a 28° angle, will one tank of gas be enough for the trip? **Justify** your solution.

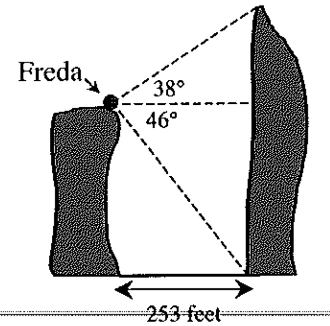
5-110. Shonte is buying pipes to install sprinklers for her front lawn. She needs to fit the pipes into the bed of her pickup truck to get them home from the store. She knows that the longest dimension in her truckbed is from the top of one corner to the bottom of the bed at the opposite corner. After measuring the truckbed, she drew the diagram at right. What is the longest pipe she can buy?



5-111. In problem 5-105 from homework, you used your intuition to state that if a triangle has two angles that are equal, then the triangle has two sides that are the same length. Now use your triangle tools to show that your intuition was correct. For example, for ΔPQR, show that if $m\measuredangle Q = m\measuredangle R$, then $PQ = PR$.



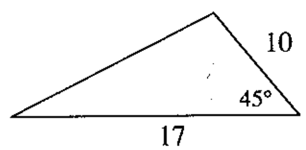
5-112. As Freda gazes at the edge of the Grand Canyon, she decides to try to determine the height of the wall opposite her. Using her trusty clinometer, she determines that the top of the wall is at a 38° angle above her, while the bottom is at a 46° angle below her, as shown in the diagram at right. If the base of the wall is 253 feet from the point on the ground directly below Freda, determine the height of the wall opposite her.



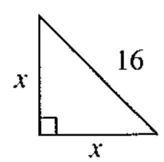
5-113. While facing north, Lisa and Aaron decide to hike to their campsite. Lisa plans to hike 5 miles due north to Lake Toftee before she goes to the campsite. Aaron plans to turn 38° east and hike 7.4 miles directly to the campsite. How far will Lisa have to hike from the lake to meet Aaron at the campsite? Start by drawing a diagram of the situation, then calculate the distance.

Review & Preview

5-114. Solve for the missing side lengths and angles in the triangle at right.



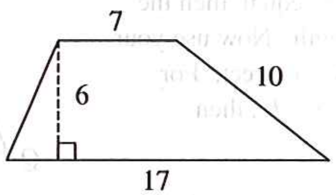
5-115. **Examine** the triangle shown at right. Solve for x twice, using two different methods. Show your work for each method clearly.



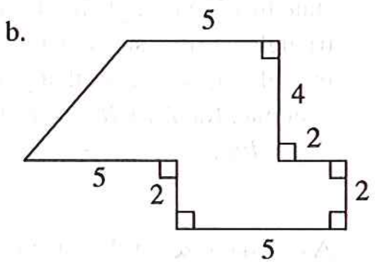
5-116. In Chapter 1 you learned that all rectangles are parallelograms because they all have two pairs of opposite parallel sides. Does that mean that all parallelograms are rectangles? Why or why not? Support your statements with **reasons**.

5-117. Find the area and perimeter of each shape below. Show all work.

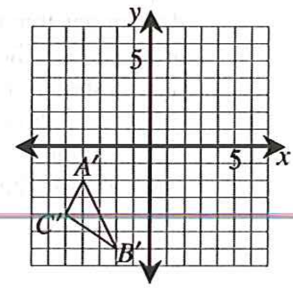
a.



b.



5-118. ΔABC was reflected across the x-axis, and then that result was rotated 90° clockwise about the origin to result in $\Delta A'B'C'$, shown at right. Find the coordinates of points A, B, and C of the original triangle.

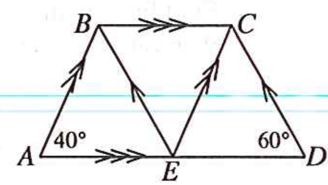


5-119. Find the equation of a line parallel to the line $y = \frac{3}{4}x - 5$ that passes through the point (–4, 1). Show how you found your answer.

5-120. **Examine** trapezoid $ABCD$ at right.

a. Find the measures of all the angles in the diagram.

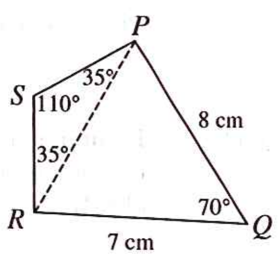
b. What is the sum of the angles that make up the trapezoid $ABCD$? That is, what is $m\measuredangle A + m\measuredangle ABC + m\measuredangle BCD + m\measuredangle D$?



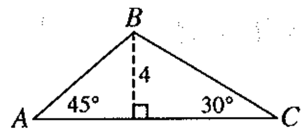
5-121. Use your triangle tools to solve the problems below.

a. Find PR in the diagram at right.

b. Find the perimeter of quadrilateral $PQRS$.



5-122. Find the area and perimeter of ΔABC at right. Give approximate (decimal) answers, not exact answers.



5-123. Earl still hates to wash the dishes and take out the garbage. (See problem 5-9.) He found his own weighted coin, one that would randomly land on heads 30% of the time. He will flip a coin once for each chore and will perform the chore if the coin lands on heads.

a. What is the probability that Earl will get out of doing both chores?

b. What is the probability that Earl will have to take out the garbage, but will not need to wash the dishes?

5-124. On graph paper, draw ΔABC if $A(3, 2)$, $B(-1, 4)$, and $C(0, -2)$.

a. Find the perimeter of ΔABC.

b. Dilate ΔABC from the origin by a factor of 2 to create $\Delta A'B'C'$. What is the perimeter of $\Delta A'B'C'$?

c. If ΔABC is rotated 90° clockwise (↻) about the origin to form $\Delta A''B''C''$, name the coordinates of C''.

5-125. Solve each equation below for x. Check your solution if possible.

a. $\frac{4}{5}x - 2 = 7$

b. $3x^2 = 300$

c. $\frac{4x-1}{2} = \frac{x+5}{3}$

d. $x^2 - 4x + 6 = 0$

Chapter 5 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned in this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following two subjects. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Connections: How are the topics, ideas, and words that you learned in previous courses connected to the new ideas in this chapter? Again, make your list as long as you can.

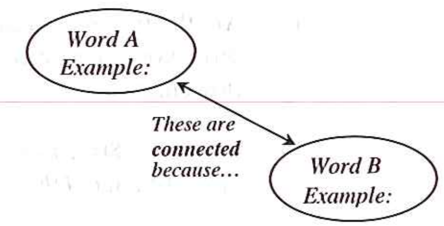
② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

adjacent	**ambiguous**	angle
approximate	**cosine**	equilateral
exact answer	hypotenuse	**inverse sin, cos, tan**
law of cosines	**law of sines**	leg
opposite	**Pythagorean triple**	ratio
right triangle	**sine**	slope
tangent	theta (θ)	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the example below. A word can be connected to any other word as long as there is a justified connection. For each key word or idea, provide a sketch of an example.

Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.



While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

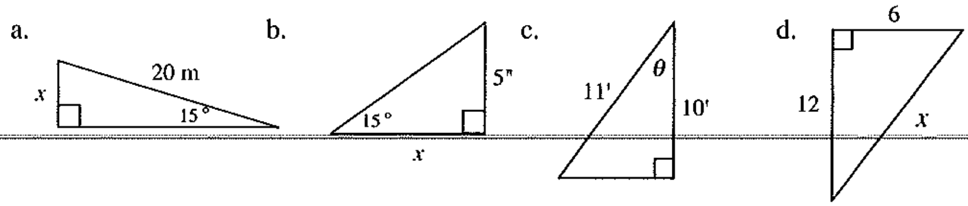
This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will direct you how to do this. Your teacher may give you a "GO" page to work on. "GO" stands for "Graphic Organizer," a tool you can use to organize your thoughts and communicate your ideas clearly.

④ WHAT HAVE I LEARNED?

This section will help you recognize those types of problems you feel comfortable with and those you need more help with. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

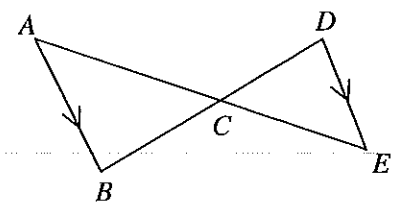
CL 5-126. For each diagram, write an equation and solve to find the value for each variable.



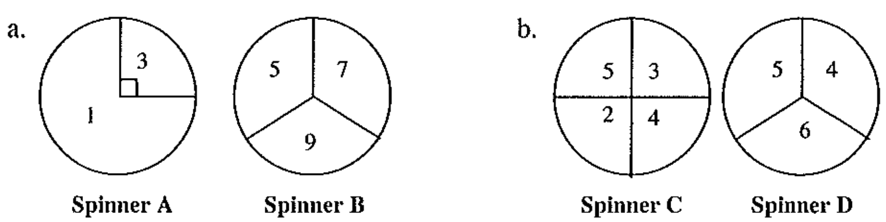
CL 5-127. Copy the diagram at right onto your paper.

a. Are the triangles similar? If so, show your **reasoning** with a flowchart.

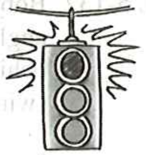
b. If $m\measuredangle B = 80°$, $m\measuredangle ACB = 29°$, $AB = 14$, and $DE = 12$, find CE.



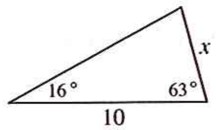
CL 5-128. Cynthia is planning a party. For entertainment, she has designed a game that involves spinning two spinners. If the sum of the numbers on the spinners is 10 or greater, the guests can choose a prize from a basket of candy bars. If the sum is less than 10, then the guest will be thrown in the pool. She has two possible pairs of spinners, shown below. For each pair of spinners, determine the probability of getting tossed in the pool. Assume that Spinners B, C, and D are equally subdivided.



CL 5-129. While working on homework, Zachary was finding the value of each variable in the diagrams below. His first step for each problem is shown under the diagram. If his first step is correct, continue solving the problem to find the solution. If his first step is incorrect, explain his mistake and solve the problem correctly.

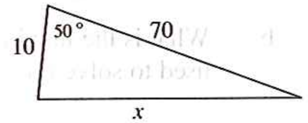


a.



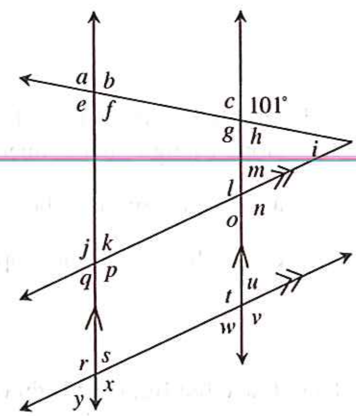
$$\sin 16° = \frac{x}{10}$$

b.



$$x^2 = 10^2 + 70^2 - 2(10)(70)\cos 50°$$

CL 5-130. **Examine** the diagram at right.



a. Find the measures of each of the angles below, if possible. If it is not possible, explain why it is not possible. If it is possible, state your **reasoning**.

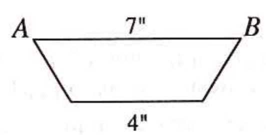
(1) $m\measuredangle b$ (2) $m\measuredangle f$

(3) $m\measuredangle m$ (4) $m\measuredangle g$

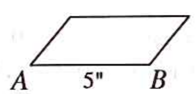
(5) $m\measuredangle h$ (6) $m\measuredangle i$

b. If $m\measuredangle p = 130°$, can you now find the measures of any of the angles from part (a) that you couldn't before? Find the measures for all that you can. Be sure to justify your **reasoning**.

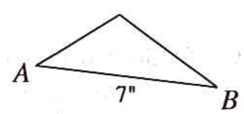
CL 5-131. Trace each figure at right onto your paper. The side labeled $\overline{AB}$ is the base in each figure. Then:



Trapezoid



Parallelogram



Triangle

a. Draw a height for each figure to side $\overline{AB}$.

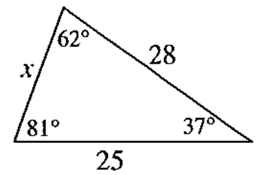
b. Find the area of each figure assuming that the height of each shape is 4" long.

CL 5-132. Bob is hanging a swing from a pole high off the ground so that it can swing a total angle of 120°. Since there is a bush 5 feet in front of the swing and a shed 5 feet behind the swing, Bob wants to ensure that no one will get hurt when they are swinging. What is the maximum length of chain that Bob can use for the swing?

a. Draw a diagram of this situation.

b. What is the maximum length of chain that Bob can use? State what tools you used to solve this problem.

CL 5-133. **Examine** the triangle at right. Solve for x twice using two different methods.



CL 5-134. Graph the points $(3, -4)$ and $(7, 2)$ on graph paper and draw the line segment and a slope triangle that connects the points. Find:

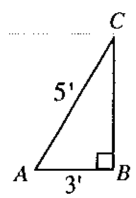
a. The length of the segment

b. The slope of the line segment

c. The area of the slope triangle

d. The measure of the slope angle

CL 5-135. Trace the figure at right onto your paper and then perform all of the transformations listed below on the same diagram. Then find the perimeter of the final shape.



- Reflect $\triangle ABC$ across $\overline{AB}$.
- Rotate $\triangle ABC$ 180° around the midpoint of $\overline{BC}$.
- Reflect $\triangle ABC$ across $\overline{AC}$.

CL 5-136. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: investigating, examining, reasoning and justifying, visualizing, and choosing a strategy/tool. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **choosing a strategy/tool**. Read the description of this Way of Thinking at right.

Choosing a Strategy/Tool

To choose a strategy means to think about what you know about a problem and match that information with methods and processes for solving problems. As you develop this way of thinking you will learn how to choose ways of solving problems based in given information. You think this way when you ask/answer questions like "What strategy might work for…?" or, "How can I use this information to answer…?" When you catch yourself looking for a method to answer a problem, you are choosing a strategy/tool.

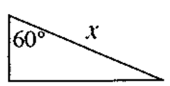
Think about the problems you have worked on in this chapter. When did you need to think about what method you would use to solve a problem? What helped you decide how to approach a problem? Were there times when more than one **strategy** seemed most useful? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss these ideas with the class.

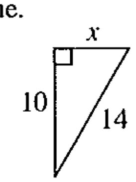
Once your discussion is complete, think about the way you think as you answer the questions below.

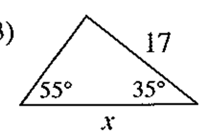
a. List all the triangle tools that you have learned so far in this course. For each tool, find or create a problem that can be solved with this **strategy**.

b. Sometimes, the key to being able to **choose a strategy or tool** is to recognize that different tools can be used on the same problem, but that sometimes some tools are more efficient than others.

Solve for x in each diagram below twice. Each time, use a different **strategy** or tool. Then decide which method was easiest for that problem or state that both methods were of equal value.

(1) 

(2) 

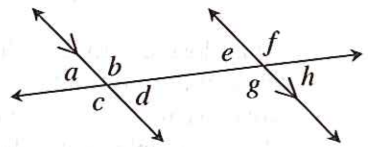
(3) 

Problem continues on next page →

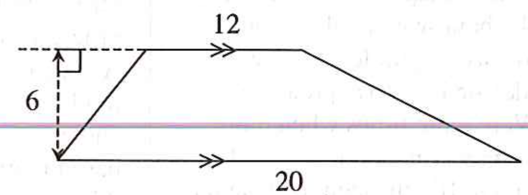
⑤ *Problem continued from previous page.*

c. While you have focused most of this chapter on developing strategies for measuring triangles, you have developed strategies for several other topics so far in this course. Consider your **strategy** options as you answer the questions below.

(1) **Examine** the diagram at right. Assume you know the measure of $\measuredangle b$. How could you find $m\measuredangle h$? Describe two different **strategies.**



(2) Now consider the trapezoid below. Find the area of the shape twice, using two different **strategies.**



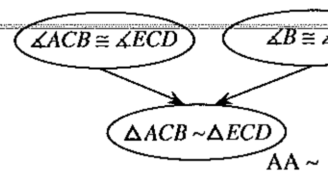
(3) While shopping at the Two Tired Bike Shop, Barry notices that he may choose from many bicycles. Of the bikes at the store, $\frac{1}{4}$ were mountain bikes, while the rest were racing bikes. Also, $\frac{1}{2}$ of the bikes were blue, $\frac{1}{3}$ of the bikes were red, and $\frac{1}{6}$ of the bikes were purple. Barry decides that he will randomly choose a bicycle from the store.

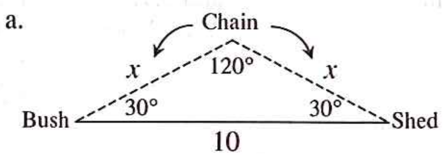
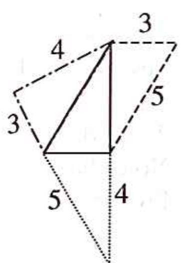
Choose a probability model to represent this situation. Then find the probability that he chooses a purple racing bike. **[Students can use a scaled area model, generic area model, or a tree diagram; $\frac{1}{6} \cdot \frac{3}{4} = \frac{1}{8}$]**

d. What about multiple strategies that you have learned in an earlier class? For example, you have multiple ways to approach a problem involving a system of equations. Consider the system below. Solve the system <u>twice</u>: once by graphing and again algebraically. Which **strategy** seemed most efficient? Why? **[(10, –2)]**

$$y = -x + 8$$
$$y = \frac{1}{2}x - 7$$

Answers and Support for
Closure Activity #4
What Have I Learned?

Problem	Solutions	Need Help?	More Practice
CL 5-126.	a. $\sin 15° = \frac{x}{20}$; $x \approx 5.176$ m b. $\tan 15° = \frac{5}{x}$; $x \approx 18.66$ in c. $\cos\theta = \frac{10}{11}$; $\theta \approx 24.62°$ d. $6^2 + 12^2 = x^2$; $x \approx 13.416$ un	Lessons 2.2.3, 4.1.4, 5.1.2, and 5.1.4 Math Notes boxes	Problems 4-11, 4-12, 4-16, 4-22, 4-25, 4-31, 4-34, 4-36, 4-46, 5-5, 5-7, 5-12, 5-16, 5-23, 5-89
CL 5-127.	a.  b. $x \approx 24.38$	Lessons 3.1.2, 3.1.3, 3.2.1, 3.2.5, and 5.3.2 Math Notes boxes	Problems 3-64, 3-65, 3-83, 3-84, 3-89, 4-7, 4-17, 4-38, 4-39, 4-83, 5-20, 5-49, 5-50, 5-57, 5-76, 5-86, 5-91
CL 5-128.	a. $\frac{7}{12}$ b. $\frac{9}{12} = \frac{3}{4}$	Lesson 4.2.4 Math Notes box	Problems 4-52, 4-60, 4-62, 4-68, 4-69, 4-70, 4-71, 4-77, 4-78, 4-87, 5-9, 5-18, 5-28, 5-39, 5-83, 5-123
CL 5-129.	a. Cannot use sine in this manner in a non-right triangle. Use Law of Sines instead: $\frac{x}{\sin(16°)} = \frac{10}{\sin(101°)}$; $x \approx 2.807$ b. $x \approx 64.03$ un.	Lessons 5.3.2 and 5.3.3 Math Notes boxes	Problems 5-76, 5-77, 5-80, 5-81, 5-86, 5-98, 5-106, 5-107, 5-109, 5-114, 5-121
CL 5-130.	a. (1) $b = 101°$, corres. angles are equal (2) $f = 79°$, supplementary angles (3) not enough information (4) $g = 101°$, vertical angles are equal (5) $h = 79°$, supplementary angles (6) not enough information b. $m = 50°$; $v = 130°$; $i = 51°$	Lessons 2.1.1, 2.1.4, and 2.2.1 Math Notes boxes, Problem 2-3	Problems 2-13, 2-16, 2-17, 2-23, 2-24, 2-25, 2-31, 2-32, 2-38, 2-49, 2-51, 2-62, 2-72, 2-111, 3-20, 3-31, 3-60, 3-70, 3-96, 4-6, 4-29, 4-44, 4-85, 4-93, 5-47

Problem	Solutions	Need Help?	More Practice
CL 5-131.	Trapezoid Area: 22 square in. Parallelogram Area: 20 square in. Triangle Area: 14 square in.	Lessons 1.1.3 and 2.2.4 Math Notes boxes	Problems 2-66, 2-68, 2-71, 2-75, 2-76, 2-78, 2-79, 2-82, 2-83, 2-90, 2-120, 3-51, 3-81, 3-97, 4-43, 4-67, 4-92, 5-10, 5-29, 5-34, 5-55, 5-81, 5-117, 5-122
CL 5-132.	a.  b. $x = \frac{10\sqrt{3}}{3} \approx 5.77$ ft.	Lessons 5.1.2, 5.3.2, and 5.3.3 Math Notes boxes	Problems 5-17, 5-76, 5-77, 5-80, 5-81, 5-86, 5-98, 5-106, 5-107, 5-109, 5-112, 5-114, 5-121
CL 5-133.	Both the Law of Sines and the Law of Cosines will work, as does dividing the triangle into two right triangles; $x \approx 17.06$ un.	Lessons 5.1.2, 5.3.2, and 5.3.3 Math Notes boxes	Problems 5-16, 5-17, 5-23, 5-76, 5-77, 5-80, 5-81, 5-86, 5-98, 5-106, 5-107, 5-109, 5-112, 5-114, 5-121
CL 5-134.	a. $\sqrt{52} \approx 7.21$ un. b. $\frac{3}{2}$ c. 12 sq. un. d. 56.31°	Lessons 1.2.5, 2.3.3, 4.1.1, 4.1.4, 5.1.4 Math Notes boxes	Problems 2-116, 5-2, 5-37, 5-92, 5-102
CL 5-135.	The final shape is shown below:  Perimeter: 24 feet	Lessons 1.1.3, 1.2.2 and 1.2.3 Math Notes boxes	Problems 1-50, 1-51, 1-64, 1-69, 1-73, 1-85, 1-96, 2-11, 2-19, 2-20, 2-33, 2-64, 2-113, 3-96, 5-41, 5-70, 5-90, 5-118, 5-124