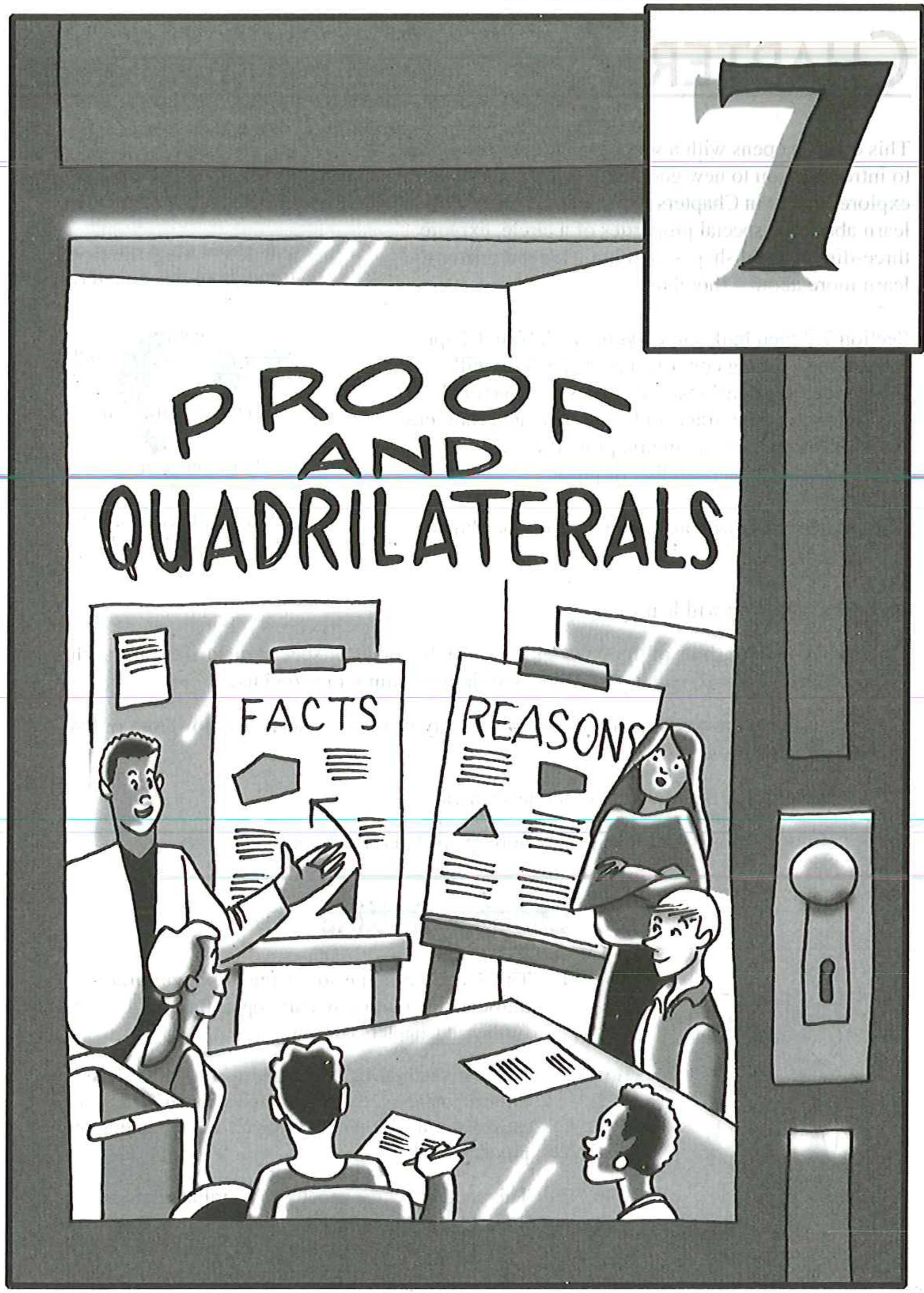

7
PROOF AND QUADRILATERALS
FACTS
REASONS

CHAPTER 7 Proof and Quadrilaterals

This chapter opens with a set of explorations designed to introduce you to new geometric topics that you will explore further in Chapters 8 through 12. You will learn about the special properties of a circle, explore three-dimensional shapes, and use a hinged mirror to learn more about a rhombus.

Section 7.2 then builds upon your work from Chapters 3 through 6. Using congruent triangles, you will explore the relationships of the sides and diagonals of a parallelogram, kite, trapezoid, rectangle, and rhombus. As you explore new geometric properties, you will formalize your understanding of proof.

This chapter ends with an exploration of coordinate geometry.

Guiding Questions

Think about these questions throughout this chapter:

What's the connection?

How can I prove it?

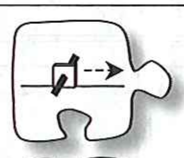
Is it convincing?

What tools can I use?

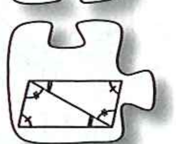
In this chapter, you will learn:

- The relationships of the sides, angles, and diagonals of special quadrilaterals, such as parallelograms, rectangles, kites, and rhombi (plural of rhombus).
- How to write a convincing proof in a variety of formats, such as a flowchart or two-column proof.
- How to find the midpoint of a line segment.
- How to use algebraic tools to explore quadrilaterals on coordinate axes.

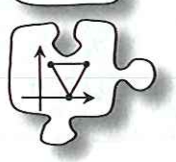
Chapter Outline



Section 7.1 This section contains four large **investigations** introducing you to geometry topics that will be explored further in Chapters 8 through 12.



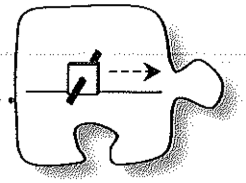
Section 7.2 While **investigating** what congruent triangles can inform you about the sides, angles, and diagonals of a quadrilateral, you will develop an understanding of proof.



Section 7.3 This section begins a focus on coordinate geometry, the study of geometry on coordinate axes. During this section, you will use familiar algebraic tools (such as slope) to make and justify conclusions about shapes.

7.1.1 Does it roll smoothly?

Properties of a Circle



In Chapters 1 through 6, you studied many different types of two-dimensional shapes, explored how they could be related, and developed tools to measure their lengths and areas. In Chapters 7 through 12, you will examine ways to extend these ideas to new shapes (such as polygons and circles) and will thoroughly **investigate** what we can learn about three-dimensional shapes.

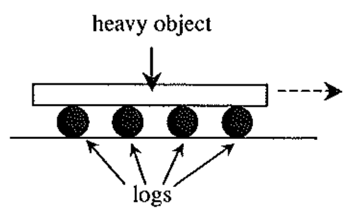
To start, Section 7.1 contains four key **investigations** that will touch upon the big ideas of these chapters. As you explore these lessons, take note of what mathematical tools from Chapters 1 through 6 you are using and think about what new directions this course will take. Generate "What if…" questions that can be answered later once new tools are developed.

Since much of the focus of Chapters 7 through 12 is on the study of circles, this lesson will first explore the properties of a circle. What makes a circle special? Today you are going to answer that question and, at the same time, explore other shapes with surprisingly similar qualities.

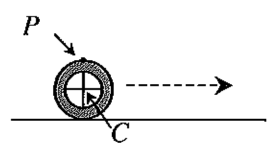
7-1. THE INVENTION OF THE WHEEL

One of the most important human inventions was the wheel. Many archeologists estimate that the wheel was probably first invented about 10,000 years ago in Asia. It was an important tool that enabled humans to transport very heavy objects long distances. Most people agree that impressive structures, such as the Egyptian pyramids, could not have been built without the help of wheels.

a. One of the earliest types of "wheels" used were actually logs. Ancient civilizations laid multiple logs on the ground, parallel to each other, under a heavy item that needed to be moved. As long as the logs had the same thickness (called **diameter**) and the road was even, the heavy object had a smooth ride. What is special about a circle that allows it to be used in this way? In other words, why do circles enable this heavy object to roll smoothly?



b. What happens to a point on a wheel as it turns? For example, as the wheel at right rolls along the line, what is the path of point *P*? Imagine a piece of gum stuck to a tire as it rolls. On your paper, draw the motion of point *P*. If you need help, find a coin or other round object and test this situation.

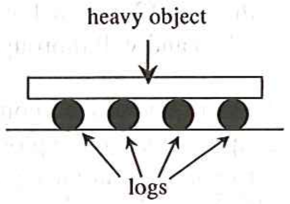


Problem continues on next page →

7-1. *Problem continued from previous page.*

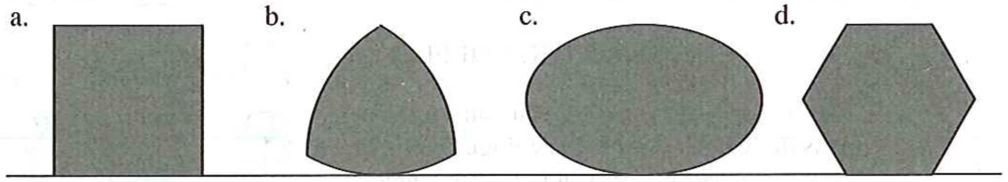
c. Now turn your attention to the center of the wheel (labeled C in the diagram above). As the wheel rolls along the line, what is the path of point C? Describe its motion. Why does that happen?

7-2. DO CIRCLES MAKE THE BEST WHEELS?

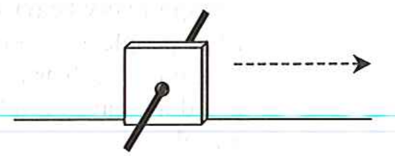


As you read in problem 7-1, ancient civilizations used circular logs to roll heavy objects. However, are circles the only shape they could have chosen? Are there any other shapes that could rotate between a flat road and a heavy object in a similar fashion?

Examine the shapes below. Would logs of any of these shapes be able to roll heavy objects in a similar fashion? Be prepared to defend your conclusion!



7-3. Stanley says that he has a tricycle with square wheels and claims that it can ride as smoothly as a tricycle with circular wheels! Rosita does not believe him. Analyze this possibility with your team as you answer the questions below.



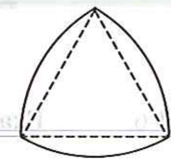
a. Is Stanley's claim possible? Describe what it would be like to ride a tricycle with square tires. What type of motion would the rider experience? Why does this happen?

b. When Rosita challenged him, Stanley confessed that he needed a special road so that the square wheels would be able to rotate smoothly and would keep Stanley at a constant height. What would his road need to look like? Draw an example on your paper.

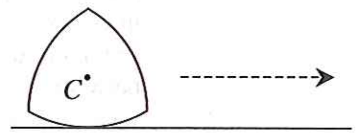
c. How would Stanley need to change his road to be able to ride a tricycle with rectangular (but non-square) wheels? Draw an example on your paper.

d. Read the Math Notes box for this lesson to see a picture of Stanley and his tricycle. Then explain why the square wheel needed a modified road to ride smoothly, while the circular wheel did not. What is different between the two shapes?

7-4. REULEAUX CURVES



Reuleaux curves (pronounced "roo **low**") are special because they have a constant diameter. That means that as a Reuleaux curve rotates, its height remains constant. Although the diagram at right is an example of a Reuleaux curve based on an equilateral triangle, these special curves can be based on any polygon with an odd number of sides, including scalene triangles and pentagons.



a. What happens to the center (point *C*) as the Reuleaux wheel at right rolls?

b. Since logs with a Reuleaux curve shape can also smoothly roll heavy objects, why are these shapes not used for bicycle wheels? In other words, what is the difference between a circle and a Reuleaux curve?

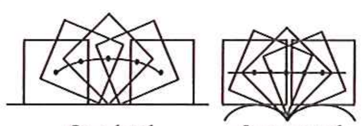
7-5. A big focus of Chapters 7 through 12 is on circles. What did you learn about circles today? Did you learn anything about other shapes that was new or that surprised you? Write a Learning Log entry explaining what you learned about the shapes of wheels. Title this entry "Shapes of Wheels" and include today's date.

LOOKING DEEPER

MATH NOTES

Square Tires

As the picture at right shows, square wheels are possible if the road is specially curved to accommodate the change in the radius of the wheel as it rotates. An example is shown below.

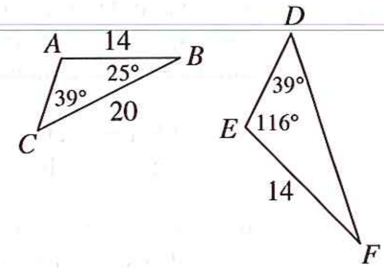


On a level surface *On a curved surface*

The picture above shows Stan Wagon riding his special square-wheeled tricycle. This tricycle is on display at Macalester College in St. Paul, Minnesota. *Reprinted with permission.*

Review & Preview

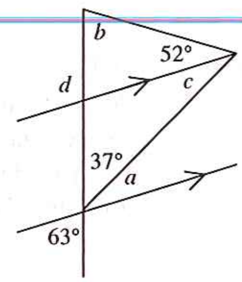
7-6. **Examine** $\triangle ABC$ and $\triangle DEF$ at right.



a. Assume the triangles are not drawn to scale. Using the information provided in each diagram, write a mathematical statement describing the relationship between the two triangles. **Justify** your conclusion.

b. Find AC and DF.

7-7. Use the relationships in the diagram at right to find the values of each variable. Name which geometric relationships you used.



7-8. A rectangle has one side of length 11 mm and a diagonal of 61 mm. Draw a diagram of this rectangle and find its width and area.

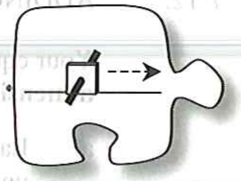
7-9. Troy is thinking of a shape. He says that it has four sides and that no sides have equal length. He also says that no sides are parallel. What is the best name for his shape?

7-10. Solve each system of equations below, if possible. If it is not possible, explain what the lack of an algebraic solution tells you about the graphs of the equations. Write each solution in the form (x, y). Show all work.

a. $y = -2x - 1$
$y = \frac{1}{2}x - 16$

b. $y = x^2 + 1$
$y = -x^2$

7.1.2 What can I build with a circle?



Building a Tetrahedron

In later chapters, you will learn more about polygons, circles, and 3-dimensional shapes. Later **investigations** will require that you remember key concepts you have already learned about triangles, parallel lines, and other angle relationships. Today you will have the opportunity to review some of the geometry you have learned while also beginning to think about what you will be studying in the future.

As you work with your team, consider the following focus questions:

Is there more than one way?

How can you be sure that is true?

What else can we try?

7-11. IS THERE MORE TO THIS CIRCLE?

Circles can be folded to create many different shapes. Today, you will work with a circle and use properties of other shapes to develop a three-dimensional shape. Be sure to have **reasons** for each conclusion you make as you work. Each person in your team should start by obtaining a copy of a circle from your teacher and cutting it out.

a. Fold the circle in half to create a crease that lies on a line of symmetry of the circle. Unfold the circle and then fold it in half again to create a new crease that is perpendicular to the first crease. Unfold your paper back to the full circle. How could you convince someone else that your creases are perpendicular? What is another name for the line segment represented by each crease?

b. On the circle, label the endpoints of one diameter A and B. Fold the circle so that point A touches the center of the circle and create a new crease. Then label the endpoints of this crease C and D. What appears to be the relationship between $\overline{AB}$ and $\overline{CD}$? Discuss and **justify** with your team. Be ready to share your **reasons** with the class.

c. Now fold the circle twice to form creases $\overline{BC}$ and $\overline{BD}$ and use scissors to cut out $\triangle BCD$. What type of triangle is $\triangle BCD$? How can you be sure? Be ready to convince the class.

7-12. ADDING DEPTH

Your equilateral triangle should now be flat (also called two-dimensional). **Two-dimensional** shapes have length and width, but not depth (or "thickness").

a. Label the vertices of ΔBCD if the labels were cut off. Then, with the unmarked side of the triangle facedown, fold and crease the triangle so that B touches the midpoint of $\overline{CD}$. Keep it in the folded position.

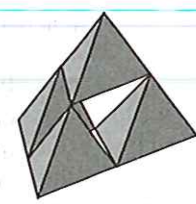
What does the resulting shape appear to be? What smaller shapes do you see inside the larger shape? **Justify** that your ideas are correct (for example, if you think that lines are parallel, you must provide evidence).

b. Open your shape again so that you have the large equilateral triangle in front of you. How does the length of a side of the large triangle compare to the length of the side of the small triangle formed by the crease? How many of the small triangles would fit inside the large triangle? In what ways are the small and large triangles related?

c. Repeat the fold in part (a) so that C touches the midpoint of $\overline{BD}$. Unfold the triangle and fold again so that D touches the midpoint of $\overline{BC}$. Create a three-dimensional shape by bringing points B, C, and D together. (A **three-dimensional** shape has length, width, and depth.) Use tape to hold your shape together.

d. Three-dimensional shapes formed with polygons have **faces** and **edges**, as well as **vertices**. Faces are the flat surfaces of the shape, while edges are the line segments formed when two faces meet. Vertices are the points where edges intersect. Discuss with your team how to use these words to describe your new shape. Then write a complete description. If you think you know the name of this shape, include it in your description.

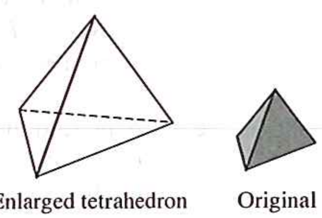
7-13. Your team should now have 4 three-dimensional shapes (called **tetrahedra**). (If you are working in a smaller team, you should quickly fold more shapes so that you have a total of four.



a. Put four tetrahedra together to make an enlarged tetrahedron like the one pictured at right. Is the larger tetrahedron similar to the small tetrahedron? How can you tell?

b. To determine the edges and faces of the new shape, pretend that it is solid. How many edges does a tetrahedron have? Are all of the edges the same length? How does the length of an edge of the team shape compare with the length of an edge of one of the small shapes?

c. How many faces of the small tetrahedral would it take to cover the face of the large tetrahedron? Remember to count gaps as part of a face. Does the area of the tetrahedron change in the same way as the length?

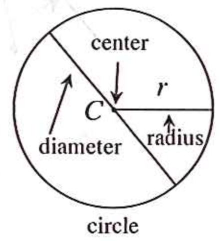


METHODS AND MEANINGS

MATH NOTES

Parts of a Circle

A **circle** is the set of all points that are the same distance from a fixed central point, C. This text will use the notation $\odot C$ to name a circle with **center** at point C.

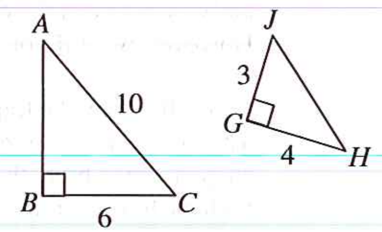


The distance from the center to the points on the circle is called the **radius** (usually denoted r), while the line segment drawn through the center of the circle with both endpoints on the circle is called a **diameter** (denoted d).

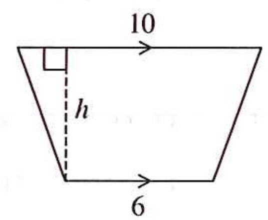
Notice that a diameter of a circle is always twice as long as the radius.

Review & Preview

7-14. What is the relationship of ΔABC and ΔGHJ at right? **Justify** your conclusion.

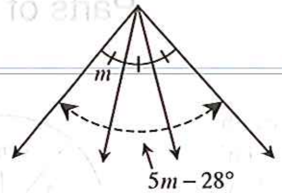


7-15. The area of the trapezoid at right is 56 un^2. What is h? Show all work.

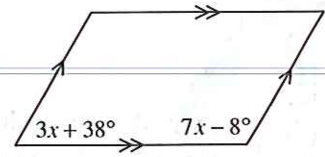


7-16. **Examine** the geometric relationships in each of the diagrams below. For each one, write and solve an equation to find the value of the variable. Name any geometric property or conjecture that you used.

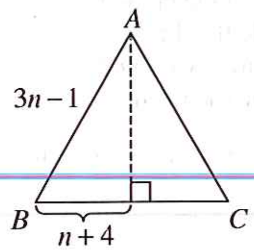
a.



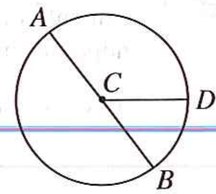
b.



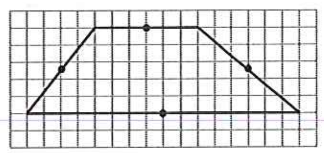
c. ΔABC below is equilateral.



d. C is the center of the circle below. $AB = 11x - 1$ and $CD = 3x + 12$



7-17. In the Shape Factory, you created many shapes by rotating triangles about the midpoint of its sides. (Remember that the **midpoint** is the point exactly halfway between the endpoints of the line segment.) However, what if you rotate a trapezoid instead?



Carefully draw the trapezoid above on graph paper, along with the given midpoints. Then rotate the trapezoid 180° about one of the midpoints and **examine** the resulting shape formed by both trapezoids (the original and its image). Continue this process with each of the other midpoints, until you discover all the shapes that can be formed by a trapezoid and its image when rotated 180° about the midpoint of one of its sides.

7-18. On graph paper, plot the points $A(-5, 7)$ and $B(3, 1)$.

a. Find AB (the length of $\overline{AB}$).

b. Locate the midpoint of $\overline{AB}$ and label it C. What are the coordinates of C?

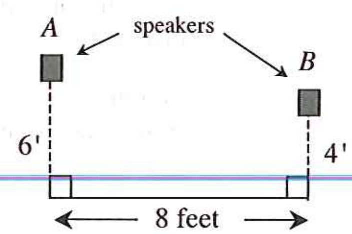
c. Find AC. Can you do this without using the Pythagorean Theorem?

7.1.3 What's the shortest distance?

Shortest Distance Problems

Questions such as, "What length will result in the largest area?" or "When was the car traveling the slowest?" concern *optimization*. To **optimize** a quantity is to find the "best" possibility. Calculus is often used to solve optimization problems, but sometimes geometric tools offer surprisingly simple and elegant solutions.

7-19. INTERIOR DESIGN



Laura needs your help. She needs to order expensive wire to connect her stereo to her built-in speakers and would like your help to save her money.

She plans to place her stereo somewhere on a cabinet that is 8 feet wide. Speaker A is located 6 feet above one end of the cabinet, while speaker B is located 4 feet above the other end. She will need wire to connect the stereo to speaker A, and additional wire to connect the stereo to speaker B.

Where should she place her stereo so that she needs the least amount of wire?

Your Task: Before you discuss this with your team, make your own guess. What does your intuition tell you? Then, using the Lesson 7.1.3 Resource Page, work with your team to determine where on the cabinet the stereo should be placed. How can you be sure that you found the best answer? In other words, how do you know that the amount of wire you found is the least amount possible?

Discussion Points

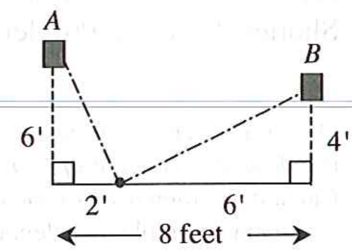
What is this problem about? What are you supposed to find?

What is a reasonable estimate of the total length of speaker wire?

What mathematical tools could be helpful to solve this problem?

Further Guidance

7-20. To help solve problem 7-19, first collect some data.



a. Calculate the total length of wire needed if the stereo is placed 2 feet from the left edge of the cabinet (the edge below Speaker A), as shown in the diagram at right.

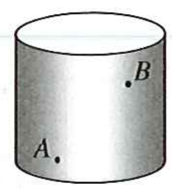
b. Now calculate the total length of wire needed if the stereo is placed 3 feet from the same edge. Does this placement require more or less wire than that from part (a)?

c. Continue testing placements for the stereo and create a table with your results. Where should the stereo be placed to minimize the amount of wire?

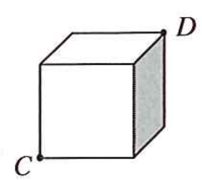
7-21. This problem reminds Bradley of problem 3-93, *You Are Getting Sleepy…*, in which you and a partner created two triangles by standing and gazing into a mirror. He remembered that the only way two people could see each others' eyes in the mirror was when the triangles were similar. **Examine** your solution to problem 7-19. Are the two triangles created by the speaker wires similar? **Justify** your conclusion.

7-22. Bradley enjoyed solving problem 7-19 so much that he decided to create other "shortest distance" problems. For each situation below, first predict where the shortest path lies using visualization and intuition. Then find a way to determine whether the path you chose is, in fact, the shortest.

a. In this first puzzle, Bradley decided to test what would happen on the side of a cylinder, such as a soup can. On a can provided by your teacher, find points *A* and *B* labeled on the outside of the can. With your team, determine the shortest path from point *A* to point *B* along the surface of the can. (In other words, no part of your path can go inside the can.) Describe how you found your solution.



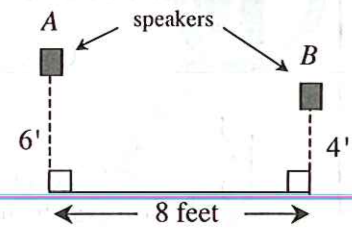
b. What if the shape is a cube? Using a cube provided by your teacher, predict which path would be the shortest path from opposite corners of the cube (labeled points *C* and *D* in the diagram at right). Then test your prediction. Describe how you found the shortest path.



7-23. MAKING CONNECTIONS

As Bradley looked over his answer from problem 7-19, he couldn't help but wonder if there is a way to change this problem into a straight-line problem like those in problem 7-22.

a. On the Lesson 7.1.3 Resource Page, reflect one of the speakers so that when the two speakers are connected with a straight line, the line passes through the horizontal cabinet.

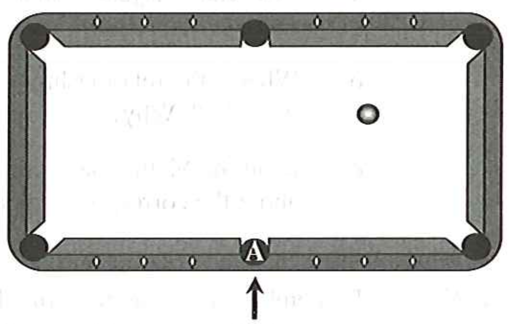


b. When the speakers from part (a) are connected with a straight line, two triangles are formed. How are the two triangles related? **Justify** your conclusion.

c. Use the fact that the triangles are similar to find where the stereo should be placed. Did your answer match that from problem 7-19?

7-24. TAKE THE SHOT

While playing a game of pool, "Montana Mike" needed to hit the last remaining ball into pocket A, as shown in the diagram below. However, to show off, he decided to make the ball first hit at least one of the rails of the table.



Your Task: On the Lesson 7.1.3 Resource Page provided by your teacher, determine where Mike could bounce the ball off a rail so that it will land in pocket A. Work with your team to find as many possible locations as you can. Can you find a way he could hit the ball so that it would rebound twice before entering pocket A?

Be ready to share your solutions with the class.

7-25. Look over your work from problems 7-19 to 7-24. What mathematical ideas did you use? What connections, if any, did you find? Can any other problems you have seen so far be solved using a straight line? Describe the mathematical ideas you developed during this lesson in your Learning Log. Title this entry "Shortest Distance" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Congruent Triangles → Congruent Corresponding Parts

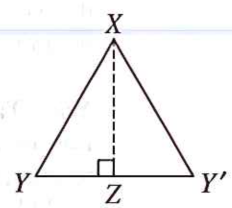
As you learned in Chapter 3, if two shapes are congruent, then they have exactly the same shape and the same size. This means that if you know two triangles are congruent, you can state that corresponding parts are congruent. This can be also stated with the arrow diagram:

$$\cong \Delta s \rightarrow \cong \text{ parts}$$

For example, if $\Delta ABC \cong \Delta PQR$, then it follows that $\measuredangle A \cong \measuredangle P$, $\measuredangle B \cong \measuredangle Q$, and $\measuredangle C \cong \measuredangle R$. Also, $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, and $\overline{BC} \cong \overline{QR}$.

Review & Preview

7-26. ΔXYZ is reflected across $\overline{XZ}$, as shown at right.



a. How can you **justify** that the points Y, Z, and Y' all lie on a straight line?

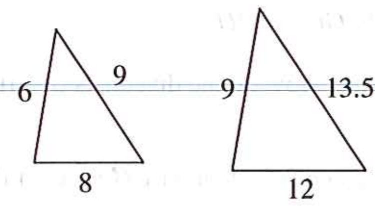
b. What is the relationship between ΔXYZ and $\Delta XY'Z$? Why?

c. Read the Math Notes box for this lesson. Then make all the statements you can about the corresponding parts of these two triangles.

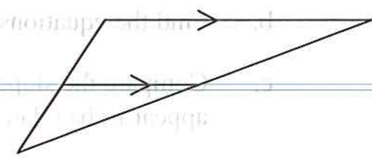
7-27. Remember that a midpoint of a line segment is the point that divides the segment into two segments of equal length. On graph paper, plot the points $P(0, 3)$ and $Q(0, 11)$. Where is the midpoint M if $PM = MQ$? Explain how you found your answer.

7-28. Recall the three similarity shortcuts for triangles: SSS ~, SAS ~ and AA ~. For each pair of triangles below, decide whether the triangles are congruent and/or similar. **Justify** each conclusion.

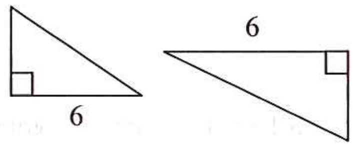
a.



b.



c.



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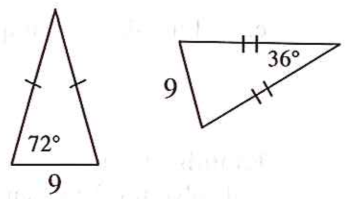


7-29. On graph paper, plot and connect the points $A(1, 1)$, $B(2, 3)$, $C(5, 3)$, and $D(4, 1)$ to form quadrilateral $ABCD$.

a. What is the best name for quadrilateral $ABCD$? **Justify** your answer.

b. Find and compare $m\measuredangle DAB$ and $m\measuredangle BCD$. What is their relationship?

c. Find the equations of diagonals $\overline{AC}$ and $\overline{BD}$. Are the diagonals perpendicular?

d. Find the point where diagonals $\overline{AC}$ and $\overline{BD}$ intersect.

7-30. Solve each system of equations below, if possible. If it is not possible, explain what having "no solution" tells you about the graphs of the equations. Write each solution in the form (x, y). Show all work.

a. $y=-\frac{1}{3}x+7$
$y=-\frac{1}{3}x-2$

b. $y=2x+3$
$y=x^2-2x+3$

7-31. How long is the longest line segment that will fit inside a square of area 50 square units? Show all work.

7-32. Graph and connect the points $G(-2, 2)$, $H(3, 2)$, $I(6, 6)$, and $J(1, 6)$ to form $GHIJ$.

a. What specific type of shape is quadrilateral $GHIJ$? **Justify** your conclusion.

b. Find the equations of the diagonals $\overline{GI}$ and $\overline{HJ}$.

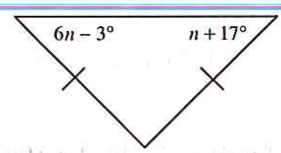
c. Compare the slopes of the diagonals. How do the diagonals of a rhombus appear to be related?

d. Find J' if quadrilateral $GHIJ$ is rotated 90° clockwise (↻) about the origin.

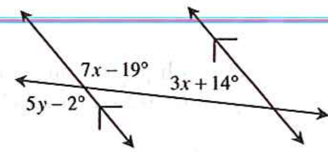
e. Find the area of quadrilateral $GHIJ$.

7-33. **Examine** the relationships in the diagrams below. For each one, write an equation and solve for the given variable(s). Show all work.

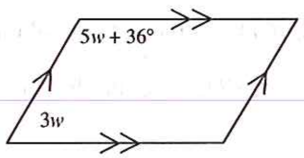
a.



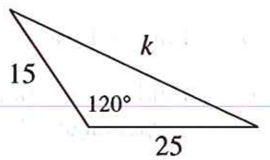
b.



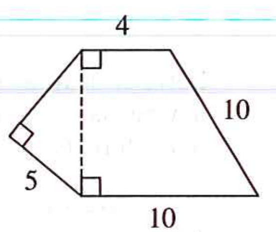
c.



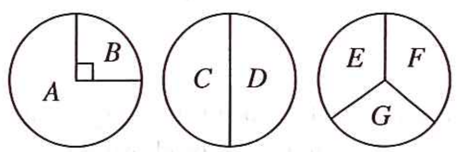
d.



7-34. Find the perimeter of the shape at right. Show all work.



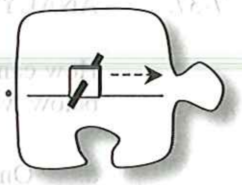
7-35. Three spinners are shown at right. If each spinner is randomly spun and if spinners #2 and #3 are each equally divided, find the following probabilities.



a. P(spinning A, C, and E)

b. P(spinning at least one vowel)

7.1.4 How can I create it?



Using Symmetry to Study Polygons

In Chapter 1, you used a hinged mirror to study the special angles associated with regular polygons. In particular, you **investigated** what happens as the angle formed by the sides of the mirror is changed. Today, you will use a hinged mirror to determine if there is more than one way to build each regular polygon using the principals of symmetry. And what about other types of polygons? What can a hinged mirror help you understand about them?

As your work with your study team, keep these focus questions in mind:

Is there another way?

What types of symmetry can I find?

What does symmetry tell me about the polygon?

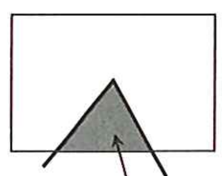
7-36. THE HINGED MIRROR TEAM CHALLENGE

Obtain a hinged mirror, a piece of unlined colored paper, and a protractor from your teacher.

With your team, spend five minutes reviewing how to use the mirror to create regular polygons. (Remember that a **regular polygon** has equal sides and angles). Once everyone remembers how the hinged mirror works, select a team member to read the directions of the task below.

Your Task: Below are four challenges for your team. Each requires you to find a creative way to position the mirror in relation to the colored paper. You can tackle the challenges in any order, but you must work together as a team on each. Whenever you successfully create a shape, do not forget to measure the angle formed by the mirror, as well as draw a diagram on your paper of the core region in front of the mirror. If your team decides that a shape is impossible to create with the hinged mirror, explain why.

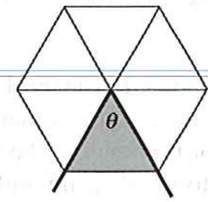
- Create a regular hexagon.
- Create an equilateral triangle at least <u>two</u> different ways.
- Create a rhombus that is <u>not</u> a square.
- Create a circle.



7-37. ANALYSIS

How can symmetry help you to learn more about shapes? Discuss each question below with the class.

a. One way to create a regular hexagon with a hinged mirror is with six triangles, as shown in the diagram at right. (Note: the gray lines represent reflections of the bottom edges of the mirrors and the edge of the paper, while the core region is shaded.)

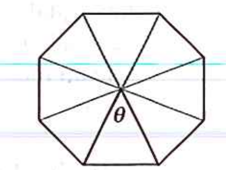


What is special about each of the triangles in the diagram? What is the relationship between the triangles? Support your conclusions. Would it be possible to create a regular hexagon with 12 triangles? Explain.

b. If you have not done so already, create an equilateral triangle so that the core region in front of the mirror is a right triangle. Draw a diagram of the result that shows the different reflected triangles like the one above. What special type of right triangle is the core region? Can all regular polygons be created with a right triangle in a similar fashion?

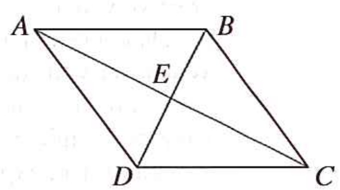
c. In problem 7-36, your team formed a rhombus that is not a square. On your paper, draw a diagram like the one above that shows how you did it. How can you be sure your resulting shape is a rhombus? Using what you know about the angle of the mirror, explain what must be true about the diagonals of a rhombus.

7-38. Use what you learned today to answer the questions below.



a. **Examine** the regular octagon at right. What is the measure of angle θ? Explain how you know.

b. Quadrilateral $ABCD$ at right is a rhombus. If $BD = 10$ units and $AC = 18$ units, then what is the perimeter of $ABCD$? Show all work.

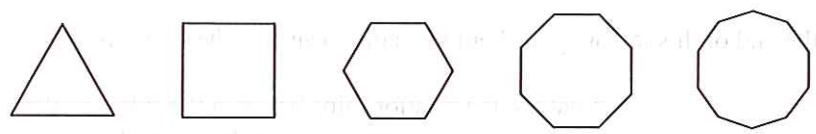


MATH NOTES

Methods and Meanings

Regular Polygons

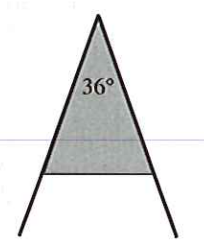
A polygon is **regular** if all its sides are congruent and its angles have equal measure. An equilateral triangle and a square are each regular polygons since they are both *equilateral* and *equiangular*. See the diagrams of common regular polygons below.



Equilateral Triangle | Square | Regular Hexagon | Regular Octagon | Regular Decagon

Review & Preview

7-39. Felipe set his hinged mirror so that its angle was 36° and the core region was isosceles, as shown at right.



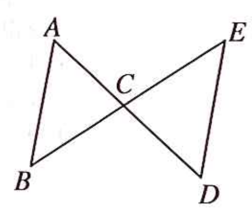
a. How many sides did his resulting polygon have? Show how you know.

b. What is another name for this polygon?

7-40. In problem 7-37 you learned that the diagonals of a rhombus are perpendicular bisectors. If $ABCD$ is a rhombus with side length 15 mm and if $BD = 24$ mm, then find the length of the other diagonal, $\overline{AC}$. Draw a diagram and show all work.

7-41. Joanne claims that (2, 4) is the midpoint of the segment connecting the points (–3, 5) and (7, 3). Is she correct? Explain how you know.

7-42. If $\triangle ABC \cong \triangle DEC$, which of the statements below must be true? **Justify** your conclusion. Note: More than one statement may be true.



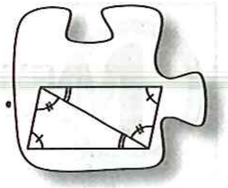
a. $\overline{AC} \cong \overline{DC}$ b. $m\measuredangle B = m\measuredangle D$ c. $\overline{AB} \parallel \overline{DE}$

d. $AD = BE$ e. None of these are true.

7-43. On graph paper, graph the points $A(2, 9)$, $B(4, 3)$, and $C(9, 6)$. Which point (A or C) is closer to point B? **Justify** your conclusion.

7.2.1 What can congruent triangles tell me?

Special Quadrilaterals and Proof



In earlier chapters you studied the relationships between the sides and angles of a triangle, and solved problems involving congruent and similar triangles. Now you are going to expand your study of shapes to quadrilaterals. What can triangles tell you about parallelograms and other special quadrilaterals?

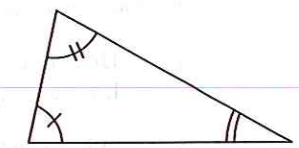
By the end of this lesson, you should be able to answer these questions:

What are the relationships between the sides, angles, and diagonals of a parallelogram?

How are congruent triangles useful?

7-44. Carla is thinking about parallelograms, and wondering if there are as many special properties for parallelograms as there are for triangles. She remembers that is it possible to create a shape that looks like a parallelogram by rotating a triangle about the midpoint of one of its sides.

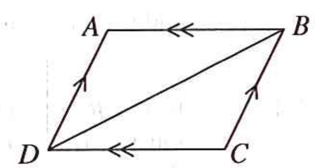
a. Carefully trace the triangle at right onto tracing paper. Be sure to copy the angle markings as well. Then rotate the triangle to make a shape that looks like a parallelogram.



b. Is Carla's shape truly a parallelogram? Use the angles to convince your teammates that the opposites sides must be parallel. Then write a convincing argument.

c. What else can the congruent triangles tell you about a parallelogram? Look for any relationships you can find between the angles and sides of a parallelogram.

d. Does this work for all parallelograms? That is, does the diagonal of a parallelogram always split the shape into two congruent triangles? Draw the parallelogram at right on your paper. Knowing only that the opposite sides of a parallelogram are parallel, create a flowchart to show that the triangles are congruent.



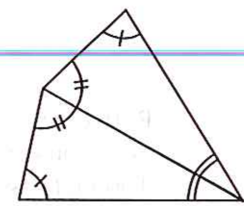
7-45. CHANGING A FLOWCHART INTO A PROOF

The flowchart you created for part (d) of problem 7-44 shows how you can conclude that if a quadrilateral is a parallelogram, then its each of its diagonals splits the quadrilateral into two congruent triangles.

However, to be convincing, the facts that you listed in your flowchart need to have **justification**. This shows the reader how you know the facts are true and helps to **prove** your conclusion.

Therefore, with the class or your team, decide how to add reasons to each statement (bubble) in your flowchart. You may need to add more bubbles to your flowchart to add **justification** and to make your proof more convincing.

7-46. Kip is confused. He put his two triangles from problem 7-44 together as shown at right, but he didn't get a parallelogram.



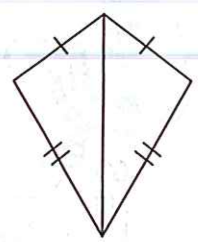
a. What shape did he make? **Justify** your conclusion.

b. What transformation(s) did Kip use to form his shape?

c. What do the congruent triangles tell you about the angles of this shape?

7-47. KITES

Kip shared his findings about his kite with his teammate, Carla, who wants to learn more about the diagonals of a kite. Carla quickly sketched the kite at right onto her paper with a diagonal showing the two congruent triangles.



a. **EXPLORE:** Trace this diagram onto tracing paper and carefully add the other diagonal. Then, with your team, consider how the diagonals may be related. Use tracing paper to help you explore the relationships between the diagonals. If you make an observation you think is true, move on to part (b) and write a conjecture.

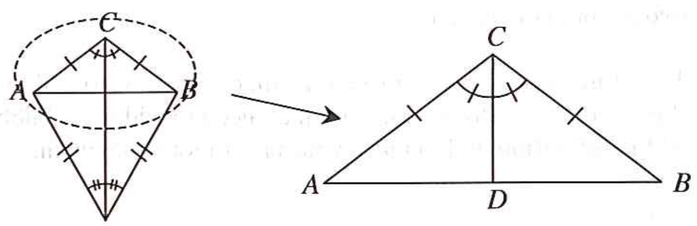
b. **CONJECTURE:** If you have not already done so, write a conjecture based on your observations in part (a).

Problem continues on next page →

7-47. *Problem continued from previous page.*

c. **PROVE:** When she drew the second diagonal, Carla noticed that four new triangles appeared. "If any of these triangles are congruent, then they may be able to help us prove our conjecture from part (b)," she said.

Examine $\triangle ABC$ below. Are $\triangle ACD$ and $\triangle BCD$ congruent? Create a flowchart proof like the one from problem 7-45 to **justify** your conclusion.



d. Now extend your proof from part (c) to prove your conjecture from part (b).

7-48. Reflect on all of the interesting facts about parallelograms and kites you have proven during this lesson. Obtain a Theorem Toolkit (Lesson 7.2.1A Resource Page) from your teacher or from www.cpm.org. In it, record each **theorem** (proven conjecture) that you have proven about the sides, angles, and diagonals of a parallelogram. Do the same for a kite. Be sure your diagrams contain appropriate markings to represent equal parts.

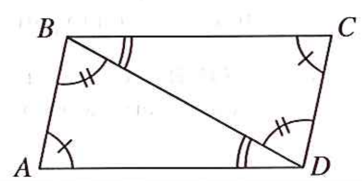
MATH NOTES

METHODS AND MEANINGS

Reflexive Property of Equality

In problem 7-44, you used the fact that two triangles formed by the diagonal of a parallelogram share a side of the same length to help show that the triangles were congruent.

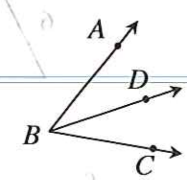
The **Reflexive Property of Equality** states that the measure of any side or angle is equal to itself. For example, in the parallelogram at right, $\overline{BD} \cong \overline{DB}$ because of the Reflexive Property.



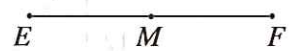
Review & Preview

7-49. Use the information given for each diagram below to solve for x. Show all work.

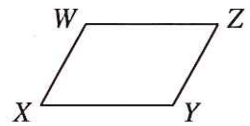
a. $\overrightarrow{BD}$ bisects $\measuredangle ABC$. (Remember that this means it divides the angle into two equal parts.) If $m\measuredangle ABD = 5x - 10°$ and $m\measuredangle ABC = 65°$, solve for x.



b. Point M is a midpoint of $\overline{EF}$. If $EM = 4x - 2$ and $MF = 3x + 9$, solve for x.



c. $WXYZ$ at right is a parallelogram. If $m\measuredangle W = 9x - 3°$ and $m\measuredangle Z = 3x + 15°$, solve for x.

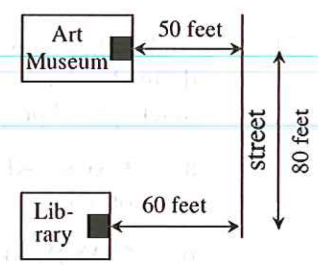


7-50. Jamal used a hinged mirror to create a regular polygon like he did in Lesson 7.1.4.

a. If his hinged mirror formed a 72° angle and the core region in front of the mirror was isosceles, how many sides did his polygon have?

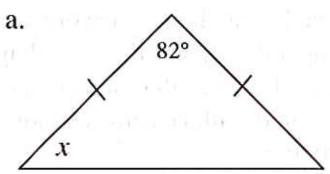
b. Now Jamal has decided to create a regular polygon with 9 sides, called a **nonagon**. If his core region is again isosceles, what angle is formed by his mirror?

7-51. Sandra wants to park her car so that she is the shortest distance possible from the entrances of both the Art Museum and the Public Library. Locate where on the street she should park so that her total distance directly to each building is the shortest.

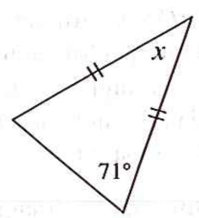


7-52. Use the geometric relationships in the diagrams below to solve for x.

a.

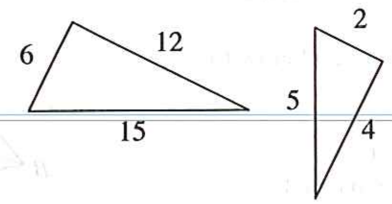


b.

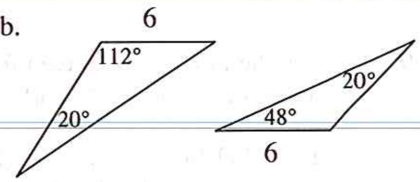


7-53. Which pairs of triangles below are congruent and/or similar? For each part, explain how you know. Note: The diagrams are not necessarily drawn to scale.

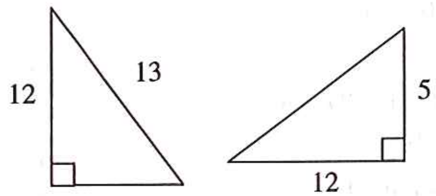
a.



b.



c.



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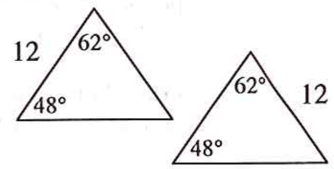
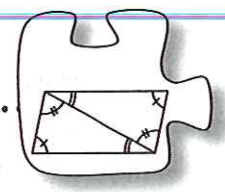


7.2.2 What is special about a rhombus?

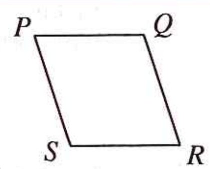


Properties of Rhombi

In Lesson 7.2.1, you learned that congruent triangles can be a useful tool to discover new information about parallelograms and kites. But what about other quadrilaterals? Today you will use congruent triangles to **investigate** and prove special properties of rhombi (the plural of rhombus). At the same time, you will continue to develop your ability to make conjectures and prove them convincingly.

7-54. Audrey has a favorite quadrilateral – the rhombus. Even though a rhombus is defined as having four congruent sides, she suspects that the sides of a rhombus have other special properties.

a. **EXPLORE:** Draw a rhombus like the one at right on your paper. Mark the side lengths equal.



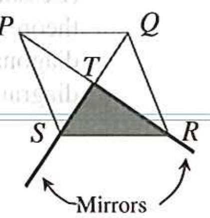
b. **CONJECTURE:** What else might be special about the sides of a rhombus? Write a conjecture.

c. **PROVE:** Audrey knows congruent triangles can help prove other properties about quadrilaterals. She starts by adding a diagonal $\overline{PR}$ to her diagram so that two triangles are formed. Add this diagonal to your diagram and prove that the triangles are congruent. Then use a flowchart with **reasons** to show your logic. Be prepared to share your flowchart with the class.

d. How can the triangles from part (c) help you prove your conjecture from part (b) above? Discuss with the class how to extend your flowchart to convince others. Be sure to **justify** any new statements with reasons.

7-55. Now that you know the opposite sides of a rhombus are parallel, what else can you prove about a rhombus? Consider this as you answer the questions below.

a. **EXPLORE:** Remember that in Lesson 7.1.4, you explored the shapes that could be formed with a hinged mirror. During this activity, you used symmetry to form a rhombus. Think about what you know about the reflected triangles in the diagram. What do you think is true about the diagonals $\overline{SQ}$ and $\overline{PR}$? What is special about $\overline{ST}$ and $\overline{QT}$? What about $\overline{PT}$ and $\overline{RT}$?

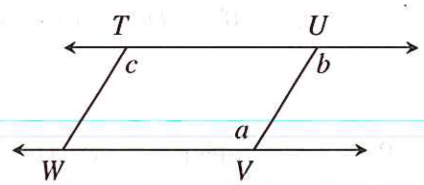


b. **CONJECTURE:** Use your observations from part (a) to write a conjecture on the relationship of the diagonals of a rhombus.

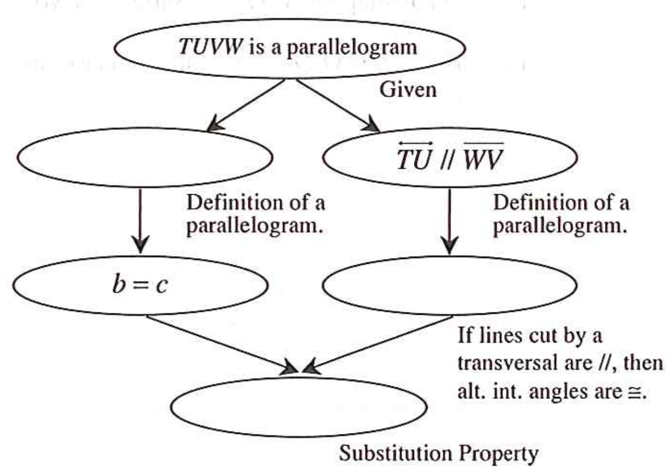
c. **PROVE:** Write a flowchart proof that proves your conjecture from part (b). Remember that to be convincing, you need to **justify** each statement with a reason. To help guide your discussion, consider the questions below.

- Which triangles should you use? Find two triangles that involve the lengths $\overline{ST}$, $\overline{QT}$, $\overline{PT}$ and $\overline{RT}$.
- How can you prove these triangles are congruent? Create a flowchart proof with **reasons** to prove these triangles must be congruent.
- How can you use the congruent triangles to prove your conjecture from part (b)? Extend your flowchart proof to include this **reasoning** and prove your conjecture.

7-56. There are often many ways to prove a conjecture. You have rotated triangles to create parallelograms and used congruent parts of congruent triangles to **justify** that opposite sides are parallel. But is there another way?



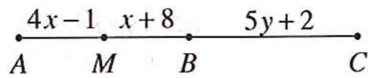
Ansel wants to prove the conjecture "*If a quadrilateral is a parallelogram, then opposite angles are congruent.*" He started by drawing parallelogram *TUVW* at right. Copy and complete his flowchart. Make sure that each statement has a **reason**.



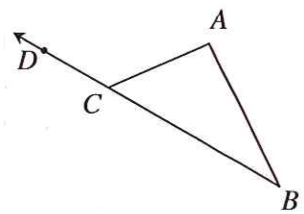
7-57. Think about the new facts you have proven about rhombi during this lesson. On your Theorem Toolkit (Lesson 7.2.1A Resource Page), record each new theorem you have proven about the angles and diagonals of a rhombus. Include clearly labeled diagrams to illustrate your findings.

Review & Preview

7-58. Point M is the midpoint of $\overline{AB}$ and B is the midpoint of $\overline{AC}$. What are the values of x and y? Show all work and **reasoning**.



7-59. In the diagram at right, $\measuredangle DCA$ is referred to as an **exterior angle** of $\triangle ABC$ because it lies outside the triangle and is formed by extending a side of the triangle.



a. If $m\measuredangle CAB = 46°$ and $m\measuredangle ABC = 37°$, what is $m\measuredangle DCA$? Show all work.

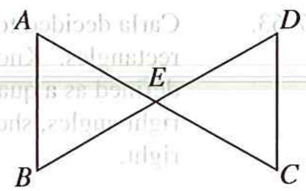
b. If $m\measuredangle DCA = 135°$ and $m\measuredangle ABC = 43°$, then what is $m\measuredangle CAB$?

7-60. On graph paper, graph quadrilateral $MNPQ$ if $M(-3, 6)$, $N(2, 8)$, $P(1, 5)$, and $Q(-4, 3)$.

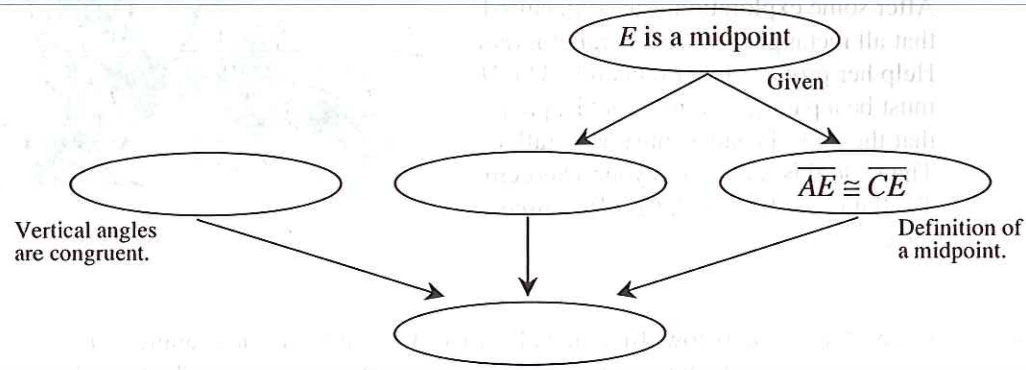
a. What shape is $MNPQ$? Show how you know.

b. Reflect $MNPQ$ across the x-axis to find $M'N'P'Q'$. What are the coordinates of P'?

7-61. Jester started to prove that the triangles at right are congruent. He is only told that point E is the midpoint of segments $\overline{AC}$ and $\overline{BD}$.



Copy and complete his flowchart below. Be sure that a **reason** is provided for every statement.



7-62. On graph paper, graph and shade the solutions for the inequality below.

$$y < -\frac{2}{3}x + 5$$

7.2.3 What else can be proved?

More Proof with Congruent Triangles

In Lessons 7.2.1 and 7.2.2, you used congruent triangles to learn more about parallelograms, kites, and rhombi. You now possess the tools to do the work of a geometrician: to discover and prove new properties about the sides and angles of shapes.

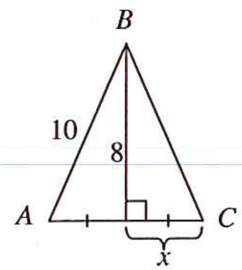
As you **investigate** these shapes, focus on proving your ideas. Remember to ask yourself and your teammates questions such as, "*Why does that work?*" and "*Is it always true?*" Decide whether your argument is convincing and work with your team to provide all of the necessary **justification**.

7-63. Carla decided to turn her attention to rectangles. Knowing that a rectangle is defined as a quadrilateral with four right angles, she drew the diagram at right.

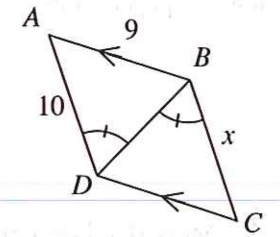
After some exploration, she conjectured that all rectangles are also parallelograms. Help her prove that her rectangle *ABCD* must be a parallelogram. That is, prove that the opposite sides must be parallel. Then add this theorem to your Theorem Toolkit (your Lesson 7.2.1A Resource Page).

7-64. For each diagram below, find the value of *x*, if possible. If the triangles are congruent, state which triangle congruence property was used. If the triangles are not congruent or if there is not enough information, state, "Cannot be determined."

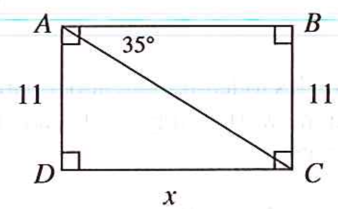
a. *ABC* below is a triangle.



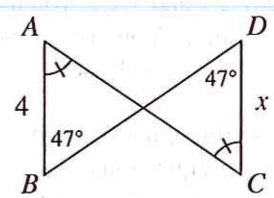
b.



c.



d. $\overline{AC}$ and $\overline{BD}$ are straight line segments.



7-65. With the class or your team, create a flowchart to prove your answer to part (b) of problem 7-64. That is, prove that $\overline{AD} \cong \overline{CB}$. Be sure to include a diagram for your proof and **reasons** for every statement. Make sure your argument is convincing and has no "holes."

MATH NOTES

METHODS AND MEANINGS

Definitions of Quadrilaterals

When proving properties of shapes, it is necessary to know exactly how a shape is defined. Below are the definitions of several quadrilaterals that you will study in this chapter and the chapters that follow.

Quadrilateral: A closed four-sided polygon.

Kite: A quadrilateral with two distinct pairs of consecutive congruent sides.

Trapezoid: A quadrilateral with at least one pair of parallel sides.

Parallelogram: A quadrilateral with two pairs of parallel sides.

Rhombus: A quadrilateral with four sides of equal length.

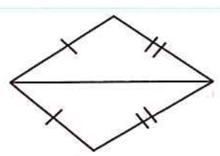
Rectangle: A quadrilateral with four right angles.

Square: A quadrilateral with four sides of equal length and four right angles.

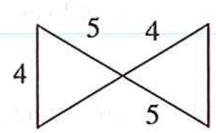
Review & Preview

7-66. Identify if each pair of triangles below is congruent or not. Remember that the diagram may not be drawn to scale. **Justify** your conclusion.

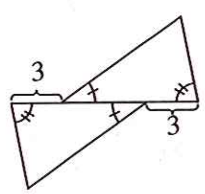
a.



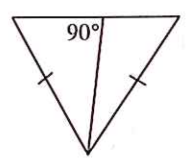
b.



c.

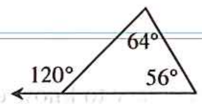


d.

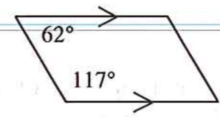


7-67. **Examine** the information provided in each diagram below. Decide if each figure is possible or not. If the figure is not possible, explain why.

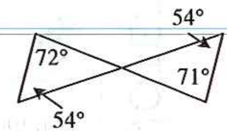
a.



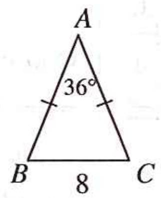
b.



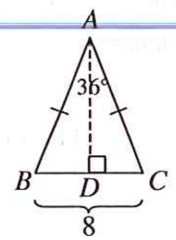
c.



7-68. Tromika wants to find the area of the isosceles triangle at right.



a. She decided to start by drawing a height from vertex A to side $\overline{BC}$ as shown below. Will the two smaller triangles be congruent? In other words, is $\Delta ABD \cong \Delta ACD$? Why or why not?



b. What is $m\measuredangle DAB$? BD?

c. Find AD. Show how you got your answer.

d. Find the area of ΔABC.

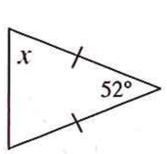
7-69. On graph paper, graph quadrilateral $ABCD$ if $A(0, 0)$, $B(6, 0)$, $C(8, 6)$, and $D(2, 6)$.

a. What is the best name for $ABCD$? **Justify** your answer.

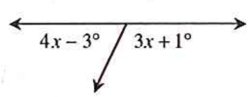
b. Find the equation of the lines containing each diagonal. That is, find the equations of lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$.

7-70. For each diagram below, solve for x. Show all work.

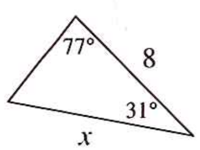
a.



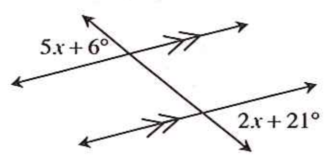
b.



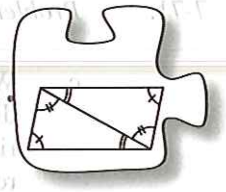
c.



d.



7.2.4 What else can I prove?



More Properties of Quadrilaterals

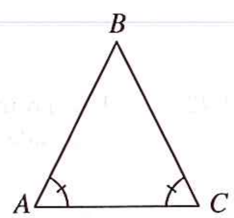
Today you will work with your team to apply what you have learned to other shapes. Remember to ask yourself and your teammates questions such as, "*Why does that work?*" and "*Is it always true?*" Decide whether your argument is convincing and work with your team to provide all of the necessary **justification**. By the end of this lesson, you should have a well-crafted mathematical argument proving something new about a familiar quadrilateral.

7-71. WHAT ELSE CAN CONGRUENT TRIANGLES TELL US?

Your Task: For each situation below, determine how congruent triangles can tell you more information about the shape. Then prove your conjecture using a flowchart. Be sure to provide a **reason** for each statement. For example, stating "$m\measuredangle A = m\measuredangle B$" is not enough. You must give a convincing reason, such as "*Because vertical angles are equal*" or "*Because it is given in the diagram.*" Use your triangle congruence properties to help prove that the triangles are congruent.

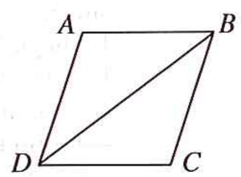
Later, your teacher will select one of these flowcharts for you to place on a poster. On your poster, include a diagram and all of your statements and reasons. Clearly state what you are proving and help the people who look at your poster understand your logic and **reasoning**.

a. In Chapter 1, you used symmetry of an isosceles triangle to show that the base angles must be congruent. But what if you only know that the base angles are congruent? Does the triangle have to be isosceles?



Assume that you know that the two base angles of ΔABC are congruent. With your team, decide how to split ΔABC into two triangles that you can show are congruent to show that $AB = CB$.

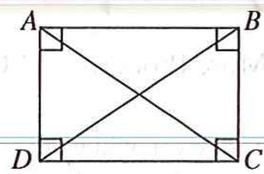
b. What can congruent triangles tell us about the diagonals and angles of a rhombus? **Examine** the diagram of the rhombus at right. With your team, decide how to prove that the diagonals of a rhombus bisect the angles. That is, prove that $\measuredangle ABD \cong \measuredangle CBD$.



Problem continues on next page →

7-71. *Problem continued from previous page.*

c. What can congruent triangles tell us about the diagonals of a rectangle? **Examine** the rectangle at right. Using the fact that the opposite sides of a rectangle are parallel (which you proved in problem 7-63), prove that the diagonals of the rectangle are congruent. That is, prove that $AC = BD$.

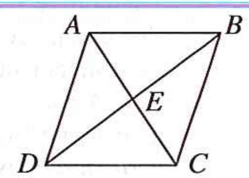


MATH NOTES

METHODS AND MEANINGS

Diagonals of a Rhombus

A **rhombus** is defined as a quadrilateral with four sides of equal length. In addition, you proved in problem 7-55 that the diagonals of a rhombus are perpendicular bisectors of each other.



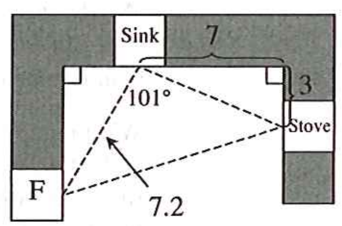
For example, in the rhombus at right, E is a midpoint of both $\overline{AC}$ and $\overline{DB}$. Therefore, $AE = CE$ and $DE = BE$. Also, $m\measuredangle AEB = m\measuredangle BEC = m\measuredangle CED = m\measuredangle DEA = 90°$.

In addition, you proved in problem 7-71 that the diagonals bisect the angles of the rhombus. For example, in the diagram above, $m\measuredangle DAE = m\measuredangle BAE$.

Review & Preview

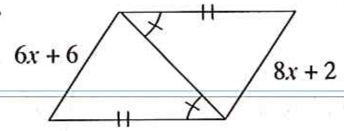
7-72. Use Tromika's method from problem 7-68 to find the area of an equilateral triangle with side length 12 units. Show all work.

7-73. The guidelines set forth by the National Kitchen & Bath Association recommends that the perimeter of the triangle connecting the refrigerator (F), stove, and sink of a kitchen be 26 feet or less. Lashayia is planning to renovate her kitchen and has chosen the design at right. (Note: All measurements are in feet.) Does her design conform to the National Kitchen and Bath Association's guidelines? Show how you got your answer.

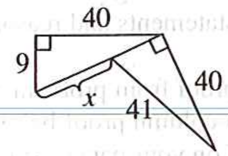


7-74. For each figure below, determine if the two smaller triangles in each figure are congruent. If so, explain why and solve for x. If not, explain why not.

a.



b.

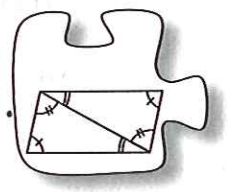


7-75. The diagonals of a rhombus are 6 units and 8 units long. What is the area of the rhombus? Draw a diagram and show all reasoning.

7-76. A hotel in Las Vegas is famous for its large-scale model of the Eiffel Tower. The model, built to scale, is 128 meters tall and 41 meters wide at its base. If the real tower is 324 meters tall, how wide is the base of the real Eiffel Tower?

7.2.5 How else can I write it?

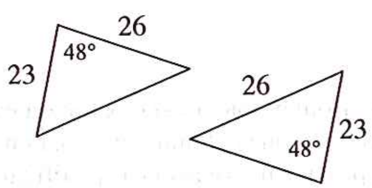
Two-Column Proofs



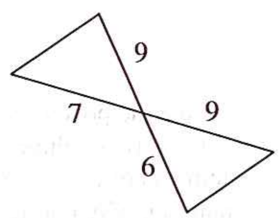
Today you will continue to work with constructing a convincing argument, otherwise known as writing a proof. You will use what you know about flowchart proofs to write a convincing argument using another format, called a "two-column" proof.

7-77. The following pairs of triangles are not necessarily congruent even though they appear to be. Use the information provided in the diagram to show why. **Justify** your statements.

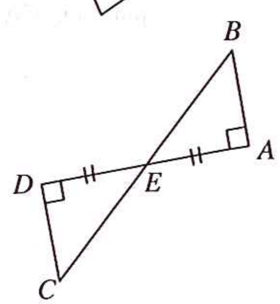
a.



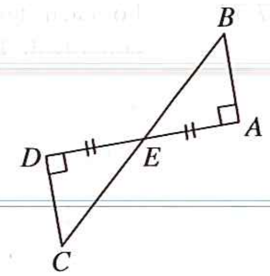
b.



7-78. Write a flowchart to prove that if E is the midpoint of $\overline{AD}$ and $\measuredangle A$ and $\measuredangle D$ are both right angles, then $\overline{AB} \cong \overline{DC}$.



7-79. Another way to organize a proof is called a **two-column proof**. Instead of using arrows to indicate the order of logical reasoning, this style of proof lists statements and reasons in a linear order.



The proof from problem 7-78 has been converted to a two-column proof below. Copy and complete the proof on your paper using your statements and **reasons** from problem 7-78.

If: E is the midpoint of $\overline{AD}$ and $\measuredangle A$ and $\measuredangle D$ are both right angles,
Prove: $\overline{AB} \cong \overline{DC}$

Statements	**Reasons** (This statement is true because…)
E is the midpoint of $\overline{AD}$ and $\measuredangle A$ and $\measuredangle D$ are both right angles	Given
$\measuredangle A \cong \measuredangle D$	Angles with the same measure are congruent.
	Definition of a midpoint
$\measuredangle DEC \cong \measuredangle AEB$	

7-80. Examine the posters of flowchart proofs from problem 7-71. Convert each flowchart proof to a two-column proof. Remember that one column must contain the statements of fact while the other must provide the **reason** (or **justification**) explaining why that fact must be true.

7-81. So far in Section 7.2, you have proven many special properties of quadrilaterals and other shapes. When a conjecture is proven, it is called a **theorem**. For example, once you proved the relationship between the sides of a right triangle, you were able to refer to that relationship as the Pythagorean Theorem. Find your Theorem Toolkit (Lesson 7.2.1A Resource Page) and make sure it contains all of the theorems you and your classmates have proven so far about various quadrilaterals. Be sure that your records include diagrams for each statement.

7-82. Reflect on the new proof format you learned today. Compare it to the flowchart proof format that you have used earlier. What are the strengths and weaknesses of each style of proof? Which format is easier for you to use? Which is easier to read? Title this entry "Two-Column Proofs" and include today's date.

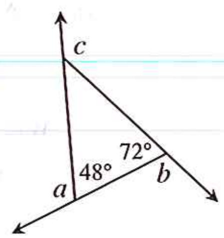
Review & Preview

7-83. Suppose you know that $\Delta TAP \cong \Delta DOG$ and that $TA = 14$, $AP = 18$, $TP = 21$, and $DG = 2y + 7$.

a. On your paper, draw a reasonable sketch of ΔTAP and ΔDOG.

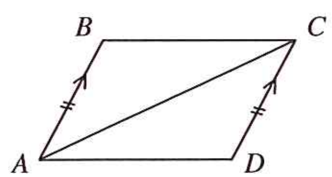
b. Find *y*. Show all work.

7-84. $\measuredangle a$, $\measuredangle b$, and $\measuredangle c$ are exterior angles of the triangle at right. Find $m\measuredangle a$, $m\measuredangle b$, and $m\measuredangle c$. Then find $m\measuredangle a + m\measuredangle b + m\measuredangle c$.

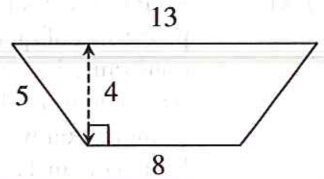


7-85. Prove that if a pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral must be a parallelogram.

For example, for the quadrilateral *ABCD* at right, given that $\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, show that $\overline{BC} \parallel \overline{AD}$. Organize your **reasoning** in a flowchart. Then record your theorem in your Theorem Toolkit (your Lesson 7.2.1A Resource Page).



7-86. Find the area and perimeter of the trapezoid at right.

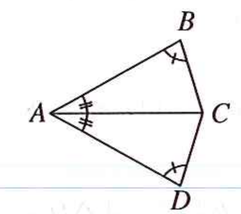


7-87. For each pair of triangles below, determine if the triangles are congruent. If the triangles are congruent,

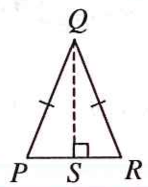
- complete the correspondence statement,
- state the congruence property,
- and record any other ideas you use that make your conclusion true.

Otherwise, explain why you cannot conclude that the triangles are congruent. Note that the figures are not necessarily drawn to scale.

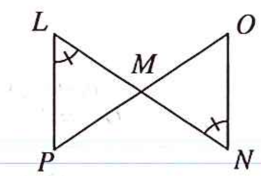
a. $\Delta ABC \cong \Delta$_____



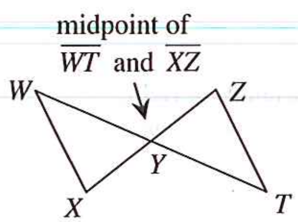
b. $\Delta SQP \cong \Delta$_____



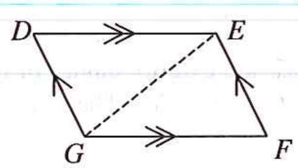
c. $\Delta PLM \cong \Delta$_____



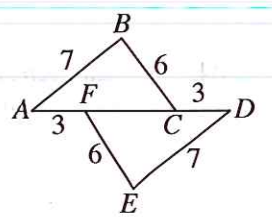
d. $\Delta WXY \cong \Delta$_____



e. $\Delta EDG \cong \Delta$_____

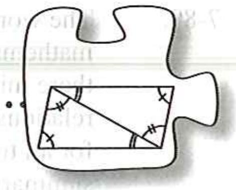


f. $\Delta ABC \cong \Delta$_____



7.2.6 What can I prove?

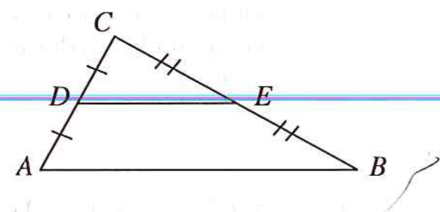
Explore-Conjecture-Prove



So far, congruent triangles have helped you to discover and prove many new facts about triangles and quadrilaterals. But what else can you discover and prove? Today your work will mirror the real work of professional mathematicians. You will **investigate** relationships, write a conjecture based on your observations, and then prove your conjecture.

7-88. TRIANGLE MIDSEGMENT THEOREM

As Sergio was drawing shapes on his paper, he drew a line segment that connected the midpoints of two sides of a triangle. (This is called the **midsegment** of a triangle.) "I wonder what we can find out about this midsegment," he said to his team. **Examine** his drawing at right.



a. **EXPLORE: Examine** the diagram of $\triangle ABC$, drawn to scale above. How do you think $\overline{DE}$ is related to $\overline{AB}$? How do their lengths seem to be related?

b. **CONJECTURE:** Write a conjecture about the relationship between segments $\overline{DE}$ and $\overline{AB}$.

c. **PROVE:** Sergio wants to prove that $AB = 2DE$. However, he does not see any congruent triangles in the diagram. How are the triangles in this diagram related? How do you know? Prove your conclusion with a flowchart.

d. What is the common ratio between side lengths in the similar triangles? Use this to write a statement relating lengths *DE* and *AB*.

e. Now Sergio wants to prove that $\overline{DE} \parallel \overline{AB}$. Use the similar triangles to find all the pairs of equal angles you can in the diagram. Then use your knowledge of angle relationships to make a statement about parallel segments.

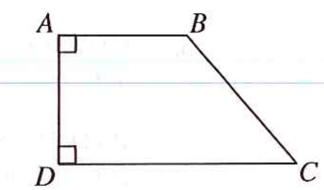
7-89. The work you did in problem 7-88 mirrors the work of many professional mathematicians. In the problem, Sergio **examined** a geometric shape and thought there might be something new to learn. You then helped him by finding possible relationships and writing a conjecture. Then, to find out if the conjecture was true for all triangles, you wrote a convincing argument (or proof). This process is summarized in the diagram below.

Explore		Conjecture		Prove
Use any tools that are available to explore a shape and discover any possible relationships.	⟹	Write a conditional statement or arrow diagram based on your observations from the investigation.	⟹	Convince yourself (and others) that your conjecture is always true with a convincing argument.

Discuss this process with the class and describe when you have used this process before (either in this class or outside of class). Why do mathematicians rely on this process?

7-90. RIGHT TRAPEZOIDS

Consecutive angles of a polygon occur at opposite ends of a side of the polygon. What can you learn about a quadrilateral with two consecutive right angles?



a. **EXPLORE: Examine** the quadrilateral at right with two consecutive right angles. What do you think is true of $\overline{AB}$ and $\overline{DC}$?

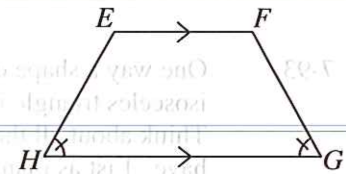
b. **CONJECTURE:** Write a conjecture about what type of quadrilateral has two consecutive right angles. Write your conjecture in conditional ("If…, then…") form.

c. **PROVE:** Prove that your conjecture from part (b) is true for all quadrilaterals with two consecutive right angles. Write your proof using the two-column format introduced in Lesson 7.2.4. (Hint: Look for angle relationships.)

d. The quadrilateral you worked with in this problem is called a **right trapezoid**. Are all quadrilaterals with two right angles a right trapezoid?

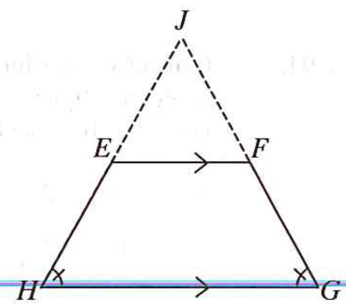
7-91. ISOSCELES TRAPEZOIDS

An **isosceles trapezoid** is a trapezoid with a pair of congruent base angles. What can you learn about the sides of an isosceles trapezoid?



a. **EXPLORE:** Examine *EFGH* at right. How do the side lengths appear to be related?

b. **CONJECTURE:** Write a conjecture about side lengths in an isosceles trapezoid. Write your conjecture in conditional ("If…, then…") form.



c. **PROVE:** Now prove that your conjecture from part (b) is true for all isosceles trapezoids. Write your proof using the two-column format introduced in Lesson 7.2.5. To help you get started, the isosceles trapezoid is shown at right with its sides extended to form a triangle.

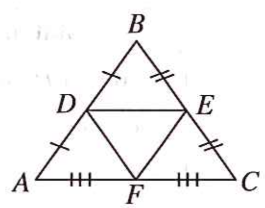
7-92. Add the theorems you have proved in this lesson to your Theorem Toolkit (your Lesson 7.2.1A Resource Page). Be sure that your records include diagrams for each statement.

METHODS AND MEANINGS

MATH NOTES

Triangle Midsegment Theorem

A **midsegment** of a triangle is a segment that connects the midpoints of any two sides of a triangle. Every triangle has three midsegments, as shown below.

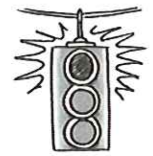


A midsegment between two sides of a triangle is half the length of and parallel to the third side of the triangle. For example, in ΔABC at right, $\overline{DE}$ is a midsegment, $\overline{DE} \parallel \overline{AC}$, and $DE = \frac{1}{2}AC$.

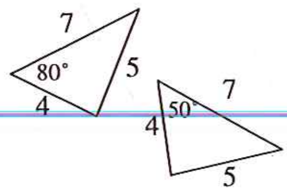
Review & Preview

7-93. One way a shape can be special is to have two congruent sides. For example, an isosceles triangle is special because it has a pair of sides that are the same length. Think about all the shapes you know and list the other special properties shapes can have. List as many as you can. Be ready to share your list with the class at the beginning of Lesson 7.3.1.

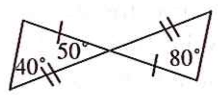
7-94. Carefully **examine** each diagram below and explain why the geometric figure cannot exist. Support your statements with **reasons**. If a line looks straight, assume that it is.



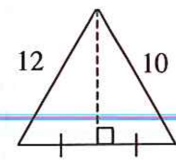
a.



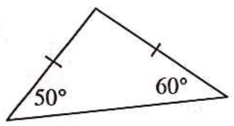
b.



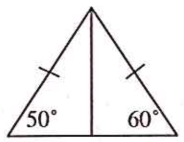
c.



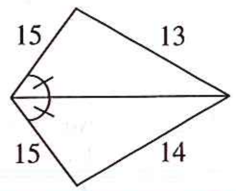
d.



e.



f.



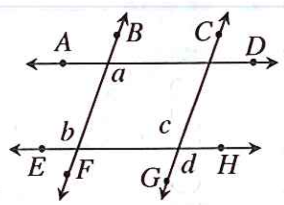
7-95. For each pair of numbers, find the number that is exactly halfway between them.

a. 9 and 15

b. 3 and 27

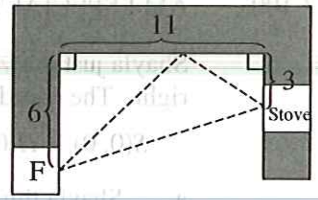
c. 10 and 21

7-96. Penn started the proof below to show that if $\overline{AD} \parallel \overline{EH}$ and $\overline{BF} \parallel \overline{CG}$, then $a = d$. Unfortunately, he did not provide reasons for his proof. Copy his proof and provide **justification** for each statement.

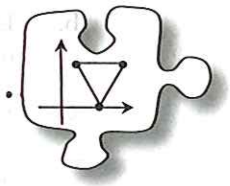


Statements	Reasons
1. $\overline{AD} \parallel \overline{EH}$ and $\overline{BF} \parallel \overline{CG}$	
2. $a = b$	
3. $b = c$	
4. $a = c$	
5. $c = d$	
6. $a = d$	

7-97. After finding out that her kitchen does not conform to industry standards, Lashayia is back to the drawing board. (See problem 7-73). Where can she locate her sink along her top counter so that its distance from the stove and refrigerator is as small as possible? And will this location keep her perimeter below 26 feet? Show all work.



7.3.1 What makes a quadrilateral special?



Studying Quadrilaterals on a Coordinate Grid

In Section 7.2 you **investigated** special types of quadrilaterals, such as parallelograms, kites, and rhombi. Each of these quadrilaterals has special properties you have proved: parallel sides, sides of equal length, equal opposite angles, bisected diagonals, etc.

But not all quadrilaterals have a special name. How can you tell if a quadrilateral belongs to one of these types? And if a quadrilateral doesn't have a special name, can it still have special properties? In Section 7.3 you will use both algebra and geometry to **investigate** quadrilaterals defined on coordinate grids.

7-98. PROPERTIES OF SHAPES

Think about the special quadrilaterals you have studied in this chapter. Each shape has many properties that make it special. For example, a rhombus has two diagonals that are perpendicular. With the class, brainstorm the other types of properties that a shape can have. You may want to refer to your work from problem 7-93. Be ready to share your list with the class.

7-99. Review some of the algebra **tools** you already have. On graph paper, draw $\overline{AB}$ given $A(0, 8)$ and $B(9, 2)$, and $\overline{CD}$ given $C(1, 3)$ and $D(9, 15)$.

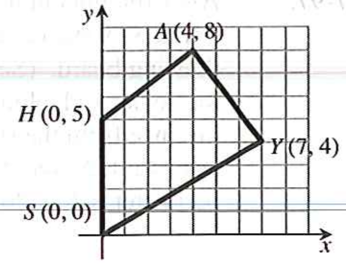
a. Draw these two segments on a coordinate grid. Find the length of each segment.

b. Find the equation of $\overleftrightarrow{AB}$ and the equation of $\overleftrightarrow{CD}$. Write both equations in $y = mx + b$ form.

c. Is $\overline{AB} \parallel \overline{CD}$? Is $\overline{AB} \perp \overline{CD}$? **Justify** your answer.

d. Use algebra to find the coordinates of the point where $\overline{AB}$ and $\overline{CD}$ intersect.

7-100. AM I SPECIAL?



Shayla just drew quadrilateral *SHAY*, shown at right. The coordinates of its vertices are:

S(0, 0)   *H*(0, 5)   *A*(4, 8)   *Y*(7, 4)

a. Shayla thinks her quadrilateral is a trapezoid. Is she correct? Be prepared to **justify** your answer to the class.

b. Does Shayla's quadrilateral look like it is one of the other kinds of special quadrilaterals you have studied? If so, which one?

c. Even if Shayla's quadrilateral doesn't have a special name, it may still have some special properties like the ones you listed in problem 7-98. Use algebra and geometry tools to **investigate** Shayla's quadrilateral and see if it has any special properties. If you find any special properties, be ready to **justify** your claim that this property is present.

7-101. THE MUST BE / COULD BE GAME

Mr. Quincey plays a game with his class. He says, "My quadrilateral has four right angles." His students say, "Then it *must be* a rectangle" and "It *could be* a square." For each description of a quadrilateral below, say what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no "must be"s, and some descriptions may have many "could be"s!

a. "My quadrilateral has four equal sides."

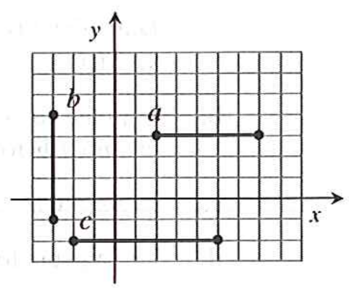
b. "My quadrilateral has two pairs of opposite parallel sides."

c. "My quadrilateral has two consecutive right angles."

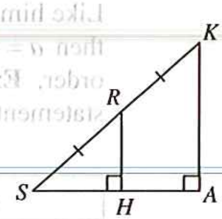
d. "My quadrilateral has two pairs of equal sides."

Review & Preview

7-102. The diagram at right shows three bold segments. Find the coordinates of the midpoint of each segment.



7-103. **Examine** the diagram at right.



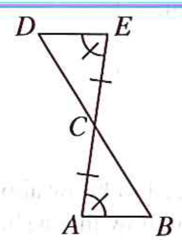
a. Are the triangles in this diagram similar? **Justify** your answer.

b. What is the relationship between the lengths of HR and AK? Between the lengths of SH and SA? Between the lengths of SH and HA?

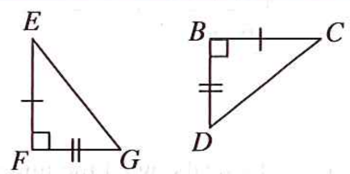
c. If $SK = 20$ units and $RH = 8$ units, what is DA?

7-104. For each pair of triangles below, determine if the triangles are congruent. If the triangles are congruent, state the congruence property that **justifies** your conclusion. If you cannot conclude that the triangles are congruent, explain why not.

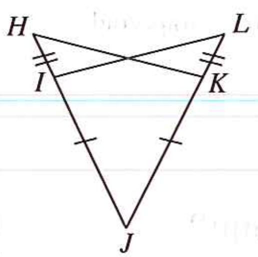
a. $\Delta CAB \cong \Delta$_____



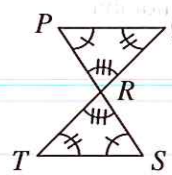
b. $\Delta CBD \cong \Delta$_____



c. $\Delta LJI \cong \Delta$_____

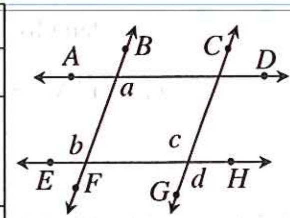


d. $\Delta PRQ \cong \Delta$_____



7-105. Carolina compared her proof to that of Penn in problem 7-96. Like him, she wanted to prove that if $\overline{AD} \parallel \overline{EH}$ and $\overline{BF} \parallel \overline{CG}$, then $a = d$. Unfortunately, her statements were in a different order. **Examine** her proof below and help her decide if her statements are in a logical order in order to prove that $a = d$.

Statements	Reasons
1. $\overline{AD} \parallel \overline{EH}$ and $\overline{BF} \parallel \overline{CG}$	Given
2. $a = b$	If lines are parallel, alternate interior angles are equal.
3. $a = c$	Substitution
4. $b = c$	If lines are parallel, corresponding angles are equal.
5. $c = d$	Vertical angles are equal.
6. $a = d$	Substitution



7-106. Describe what the minimum information you would need to know about the shapes below in order to identify it correctly. For example, to know that a shape is a square, you must know that it has four sides of equal length and at least one right angle. Be as thorough as possible.

a. rhombus

b. trapezoid

7.3.2 How can I find the midpoint?

Coordinate Geometry and Midpoints

In Lesson 7.3.1, you applied your existing algebraic tools to analyze geometric shapes on a coordinate grid. What other algebraic processes can help us analyze shapes? And what else can be learned about geometric shapes?

7-107. Cassie wants to confirm her theorem on midsegments (from Lesson 7.2.6) using a coordinate grid. She started with ΔABC, with $A(0, 0)$, $B(2, 6)$, and $C(7, 0)$.

a. Graph ΔABC on graph paper.

b. With your team, find the coordinates of P, the midpoint of $\overline{AB}$. Likewise, find the coordinates of Q, the midpoint of $\overline{BC}$.

c. Verify that the length of the midsegment, $\overline{PQ}$, is half the length of $\overline{AC}$. Also verify that $\overline{PQ}$ is parallel to $\overline{AC}$.

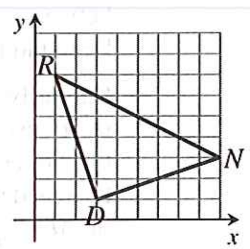
7-108. As Cassie worked on problem 7-107, her teammate, Esther, had difficulty finding the midpoint of $\overline{BC}$. The study team decided to try to find another way to find the midpoint of a line segment.

a. As part of her team, Cassie wants you to draw $\overline{AM}$, with $A(3, 4)$ and $M(8, 11)$, on graph paper. Then extend the line segment to find a point B so that M is the midpoint of $\overline{AB}$. **Justify** your location of point B by drawing and writing numbers on the graph.

b. Esther thinks she understands how to find the midpoint on a graph. "I always look for the middle of the line segment. But what if the coordinates are not easy to graph?" she asks. With your team, find the midpoint of $\overline{KL}$ if $K(2, 125)$ and $L(98, 15)$. Be ready to share your method with the class.

c. Test your team's method by verifying that the midpoint between $(-5, 7)$ and $(9, 4)$ is $(2, 5.5)$.

7-109. Randy has decided to study the triangle graphed at right.



a. Consider all the special properties this triangle can have. Without using any algebra tools, predict the best name for this triangle.

b. For your answer to part (a) to be correct, what is the minimum amount of information that must be true about ΔRND?

c. Use your algebra tools to verify each of the properties you listed in part (b). If you need, you may change your prediction of the shape of ΔRND.

d. Randy wonders if there is anything special about the midpoint of $\overline{RN}$. Find the midpoint M, and then find the lengths of $\overline{RM}$, $\overline{DM}$, and $\overline{MN}$. What do you notice?

7-110. Tomika remembers that the diagonals of a rhombus are perpendicular to each other.

a. Graph on *ABCD* if *A*(1, 4), *B*(6, 6), *C*(4, 1), and *D*(–1, –1). Is *ABCD* a rhombus? Show how you know.

b. Find the equation of the lines on which the diagonals lie. That is, find the equations of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$.

c. Compare the slopes of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$. What do you notice?

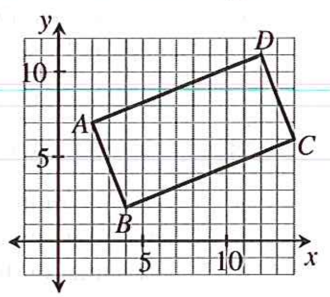
7-111. In your Learning Log, explain what a midpoint is and the method you prefer for finding midpoints of a line segment when given the coordinates of its endpoints. Include any diagram or example that helps explain why this method works. Title this entry "Finding a Midpoint" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Coordinate Geometry

Coordinate geometry is the study of geometry on a coordinate grid. Using common algebraic and geometric tools, you can learn more about a shape such as the if it has a right angle or if two sides are the same length.



One useful tool is the Pythagorean Theorem. For example, the Pythagorean Theorem could be used to determine the length of side $\overline{AB}$ of *ABCD* at right. By drawing the slope triangle between points *A* and *B*, the length of $\overline{AB}$ can be found to be $\sqrt{2^2+5^2}=\sqrt{29}$ units.

Similarly, slope can help analyze the relationships between the sides of a shape. If the slopes of two sides of a shape are equal, then those sides are **parallel.** For example, since the slope of $\overline{BC}=\frac{2}{5}$ and the slope of $\overline{AD}=\frac{2}{5}$, then $\overline{BC} \parallel \overline{AD}$.

Also, if the slopes of two sides of a shape are opposite reciprocals, then the sides are **perpendicular** (meaning they form a 90° angle). For example, since the slope of $\overline{BC}=\frac{2}{5}$ and the slope of $\overline{AB}=-\frac{5}{2}$, then $\overline{BC}\perp\overline{AB}$.

By using multiple algebraic and geometric tools, you can identify shapes. For example, further analysis of the sides and angles of *ABCD* above shows that $AB=DC$ and $BC=AD$. Furthermore, all four angles measure 90°. These facts together indicate that *ABCD* must be a rectangle.

Review & Preview

7-112. Find another valid, logical order for the statements for Penn's proof from problem 7-96. Explain how you know that changing the order the way you did does not affect the logic.

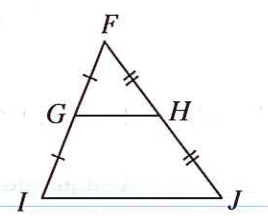
7-113. Each of these numberlines shows a segment in bold. Find the midpoint of the segment in bold. (Note that the diagrams are *not* to scale.)

a. 

b. 

c. 

7-114. **Examine** the diagram at right.



a. Are the triangles in this diagram similar? Explain.

b. Name all the pairs of congruent angles in this diagram you can.

c. Are $\overline{GH}$ and $\overline{IJ}$ parallel? Explain how you know.

d. If $GH = 4x - 3$ and $IJ = 3x + 14$, find x. Then find the length of $\overline{GH}$.

7-115. Consider ΔABC with vertices $A(2, 3)$, $B(6, 3)$, and $C(6, 10)$.

a. Draw ΔABC on graph paper. What kind of triangle is ΔABC?

b. Reflect ΔABC across $\overline{AC}$. Where is B'? And what shape is $ABCB'$?

7-116. MUST BE / COULD BE

Here are some more challenges from Mr. Quincey. For each description of a quadrilateral below, say what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no "must be"s, and some descriptions may have many "could be"s!

a. "My quadrilateral has a pair of equal sides and a pair of parallel sides."

b. "The diagonals of my quadrilateral bisect each other."

7.3.3 What kind of quadrilateral is it?

Quadrilaterals on a Coordinate Plane

Today you will use algebra tools to **investigate** the properties of a quadrilateral and then will use those properties to identify the type of quadrilateral it is.

7-117. MUST BE / COULD BE

Mr. Quincey has some new challenges for you! For each description below, decide what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no "must be"s, and some descriptions may have many "could be"s!

a. "My quadrilateral has three right angles."

b. "My quadrilateral has a pair of parallel sides."

c. "My quadrilateral has two consecutive equal angles."

7-118. THE SHAPE FACTORY

You just got a job in the Quadrilaterals Division of your uncle's Shape Factory. In the old days, customers called up your uncle and described the quadrilaterals they wanted over the phone: "I'd like a parallelogram with…". "But nowadays," your uncle says, "customers using computers have been emailing orders in lots of different ways." Your uncle needs your team to help analyze his most recent orders listed below to identify the quadrilaterals and help the shape-makers know what to produce.

Your Task: For each of the quadrilateral orders listed below,

- Create a diagram of the quadrilateral on graph paper.
- Decide if the quadrilateral ordered has a special name. To help the shape-makers, your name must be as specific as possible. (Don't just call a shape a rectangle when it's also a square!)
- Record and be ready to present a **justification** that the quadrilateral ordered must be the kind you say it is. It is not enough to say that a quadrilateral *looks* like it is of a certain type or *looks* like it has a certain property. Customers will want to be sure they get the type of quadrilateral they ordered!

Problem continues on next page →

7-118. *Problem continued from previous page.*

Discussion Points

What special properties might a quadrilateral have?

What algebra tools could be useful?

What types of quadrilaterals might be ordered?

The orders:

a. A quadrilateral formed by the intersection of these lines:

$y=-\frac{3}{2}x+3$ $\quad$ $y=\frac{3}{2}x-3$ $\quad$ $y=-\frac{3}{2}x+9$ $\quad$ $y=\frac{3}{2}x+3$

b. A quadrilateral with vertices at these points:

$A(0, 2)$ $\quad$ $B(1, 0)$ $\quad$ $C(7, 3)$ $\quad$ $D(4, 4)$

c. A quadrilateral with vertices at these points:

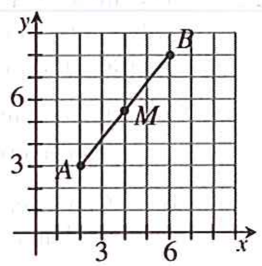
$W(0, 5)$ $\quad$ $X(2, 7)$ $\quad$ $Y(5, 7)$ $\quad$ $Z(5, 1)$

METHODS AND MEANINGS

MATH NOTES

Finding a Midpoint

A **midpoint** is a point that divides a line segment into two parts of equal length. For example, M is the midpoint of $\overline{AB}$ at right.



There are several ways to find the midpoint of a line segment if the coordinates of the endpoints are known. One way is to average the x-coordinates and to average the y-coordinates. Thus, if $A(2, 3)$ and $B(6, 8)$, then the x-coordinate of M is $\frac{2+6}{2}=4$ and the y-coordinate is $\frac{3+8}{2}=5.5$. So M is at $(4, 5.5)$.

Review & Preview

7-119. Each problem below gives the endpoints of a segment. Find the coordinates of the midpoint of the segment. If you need help, consult the Math Notes box for this lesson.

a. (5, 2) and (11, 14)

b. (3, 8) and (10, 4)

c. (–3, 11) and (5, 6)

d. (–4, –1) and (8, 9)

7-120. Below are the equations of two lines and the coordinates of three points. For each line, determine which of the points, if any, lie on that line. (There may be more than one!)

a. $y = \frac{1}{3}x + 15$ $\quad X(0, 15) \quad Y(3, 16) \quad Z(7, 0)$

b. $y - 16 = -4(x - 3)$

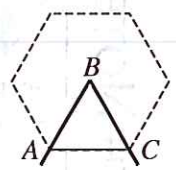
7-121. MUST BE / COULD BE

Here are some more challenges from Mr. Quincey. For each description of a quadrilateral below, say what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no "must be"s, and some descriptions may have many "could be"s!

a. "My quadrilateral has two right angles."

b. "The diagonals of my quadrilateral are perpendicular."

7-122. The angle created by a hinged mirror when forming a regular polygon is called a **central angle**. For example, $\measuredangle ABC$ in the diagram at right is the central angle of the regular hexagon.



a. If the central angle of a regular polygon measures 18°, how many sides does the polygon have?

b. Can a central angle measure 90°? 180°? 13°? For each angle measure, explain how you know.

7-123. On graph paper, graph lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ if $\overleftrightarrow{AB}$ can be represented by $y = -\frac{4}{3}x + 5$ and $\overleftrightarrow{CD}$ can be represented by $y = \frac{3}{4}x - 1$. Label their intersection *E*.

a. What is the relationship between the lines? How do you know?

b. If *E* is a midpoint of $\overline{CD}$, what type of quadrilateral could *ABCD* be? Is there more than one possible type? Explain how you know.

Chapter 7 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following three topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

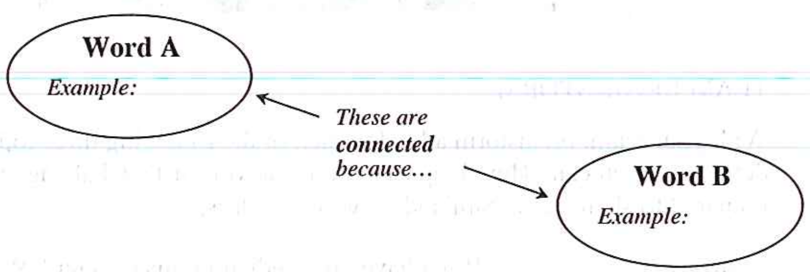
Connections: How are the topics, ideas, and words that you learned in previous courses are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

bisect	**central angle**	circle
congruent	conjecture	**coordinate geometry**
diagonal	**diameter**	**exterior angle**
kite	**midpoint**	**midsegment**
opposite	parallel	parallelogram
perpendicular	**proof**	**quadrilateral**
radius	rectangle	**Reflexive Property**
regular polygon	**rhombus**	square
theorem	**three-dimensional**	trapezoid
two-column proof	**two-dimensional**	

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this. Your teacher may give you a "GO" page to work on.

To use the Quadrilateral "GO" (or Graphic Organizer), complete the diagram to show the relationships between special quadrilaterals. For each type of quadrilateral, draw a diagram and list the special properties (if any) that it has. Use words to explain how one type of quadrilateral is related to the others. The diagram is started below for you. You will need to add: **quadrilateral, kite, square, rectangle**, and **trapezoid**. Each quadrilateral should be connected to at least one other type of quadrilateral, but some can be related to more than one.

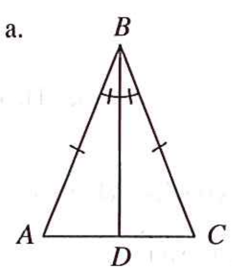
④ WHAT HAVE I LEARNED?

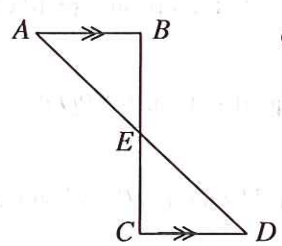
This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

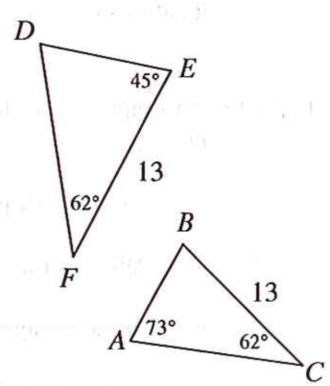
Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 7-124. **Examine** the triangle pairs below, which are not necessarily drawn to scale. For each pair, determine:

- if they must be congruent, and state the congruence property (such as SAS ≅) and give a correct congruence statement (such as $\triangle PQR \cong \triangle STU$)
- if there is not enough information, and explain why.
- if they cannot be congruent, and explain why.

a. 

b. 

c. 

CL 7-125. Complete the following statements.

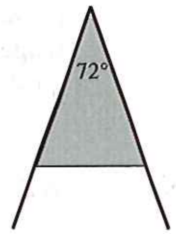
a. If $\Delta YSR \cong \Delta NVD$, then $\overline{DV} \cong$ __?__ and $m\measuredangle RYS =$ __?__

b. If $\overline{AB}$ bisects $\measuredangle DAC$, then __?__ $\cong$ __?__ ?

c. In ΔWQY, if $\measuredangle WQY \cong \measuredangle QWY$, then __?__ $\cong$ __?__ .

d. If $ABCD$ is a parallelogram, and $m\measuredangle B = 148°$, then $m\measuredangle C =$ __?__ .

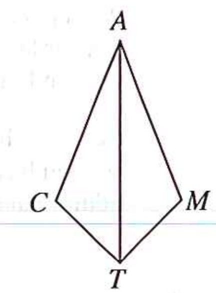
CL 7-126. Julius set his hinged mirror so that its angle was 72° and the core region was isosceles, as shown at right.



a. How many sides did his resulting polygon have? Show how you know.

b. What is another name for this polygon?

CL 7-127. Kelly started the proof below to show that if $\overline{TC} \cong \overline{TM}$ and $\overline{AT}$ bisects $\angle CTM$, then $\overline{CA} \cong \overline{MA}$. Copy and complete her proof.



Statements	**Reasons**
1. $\overline{TC} \cong \overline{TM}$ and $\overline{AT}$ bisects $\angle CTM$	
2.	Definition of bisect
3. $\overline{AT} \cong \overline{AT}$	
4.	
5.	$\cong \Delta s \rightarrow \cong$ parts

CL 7-128. $ABCD$ is a parallelogram. If $A(3, -4)$, $B(6, 2)$, $C(4, 6)$, then what are the possible locations of point D? Draw a graph and **justify** your answer.

CL 7-129. On graph paper, draw quadrilateral $MNPQ$ if $M(1, 7)$, $N(-2, 2)$, $P(3, -1)$, and $Q(6, 4)$.

a. Find the slopes of $\overline{MN}$ and $\overline{NP}$. What can you conclude about $\measuredangle MNP$?

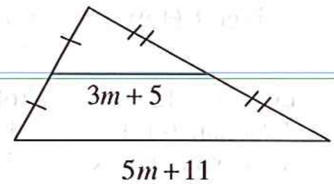
b. What is the best name for $MNPQ$? **Justify** your answer.

c. Which diagonal is longer? Explain how you know your answer is correct.

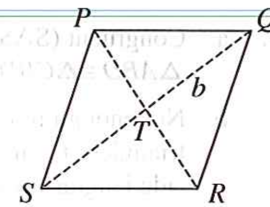
d. Find the midpoint of $\overline{MN}$.

CL 7-130. **Examine** the geometric relationships in each of the diagrams below. For each one, write and solve an equation to find the value of the variable. Name any geometric property or conjecture that you used.

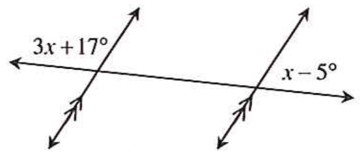
a.



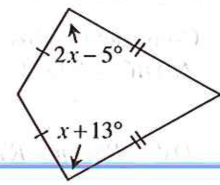
b. *PQRS* is a rhombus with perimeter = 28 units. *PR* = 8 units.



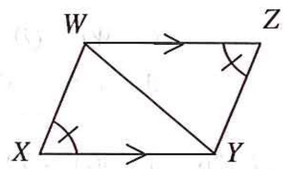
c.



d.



CL 7-131. Given the information in the diagram at right, prove that $\triangle WXY \cong \triangle YZW$ using either a flowchart or a two-column proof.



CL 7-132. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: investigating, examining, reasoning & justifying, visualizing, and choosing a strategy/tool. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

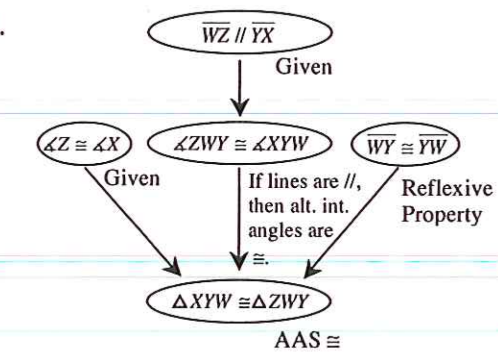
Choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 7-124.	a. Congruent (SAS ≅), $\triangle ABD \cong \triangle CBD$ b. Not enough information (the triangles are similar (AA ~), but no side lengths are given to know if they are the same size.) c. Congruent (ASA ≅ or AAS ≅), $\triangle ABC \cong \triangle DEF$	Lessons 2.1.4, 2.2.1, and 6.1.3 Math Notes boxes	Problems 7-6, 7-14, 7-28, 7-42, 7-53, 7-66, 7-87, 7-104
CL 7-125.	a. $\overline{DV} \cong \overline{RS}$, $m\measuredangle RYS = m\measuredangle DNV$ b. $\measuredangle DAB \cong \measuredangle CAB$ c. $\overline{WY} \cong \overline{QY}$ d. $m\measuredangle C = 32°$	Lessons 2.1.4, 3.1.4, and 7.1.3 Math Notes boxes, problems 7-49 and 7-71	Problems 7-26, 7-42, 7-53, 7-87
CL 7-126.	a. 360° ÷ 72° = 5 sides b. regular pentagon	Lesson 7.1.4 Math Notes box, problems 7-37 and 7-38	Problems 7-39, 7-50, 7-122
CL 7-127.	(see proof table below)	Lessons 6.1.3, 7.1.3, and 7.2.1 Math Notes boxes, problems 7-45 and 7-79	Problems 7-61, 7-78, 7-85, 7-87, 7-96, 7-104, 7-105
CL 7-128.	Point D is at (1, 0) or at (5, –8).	Lessons 7.2.3 and 7.3.2 Math Notes boxes	Problems 7-29, 7-60, 7-69

CL 7-127.

Statements	Reasons
1. $\overline{TC} \cong \overline{TM}$ and $\overline{AT}$ bisects $\angle CTM$	Given
2. $\measuredangle CTA \cong \measuredangle MTA$	Definition of bisect
3. $\overline{AT} \cong \overline{AT}$	Reflexive Property
4. $\triangle CAT \cong \triangle MAT$	SAS ≅
5. $\overline{CA} \cong \overline{MA}$	≅ $\triangle s$ → ≅ parts

Problem	Solution	Need Help?	More Practice
CL 7-129.	a. Slope of $\overline{MN} = \frac{5}{3}$ and $\overline{NP} = -\frac{3}{5}$, $\measuredangle MNP$ is a right angle. b. It is a square because all sides are equal and all angles are right angles. c. The diagonals have equal length. Each is $\sqrt{68}$ units long. d. $(-\frac{1}{2}, \frac{9}{2})$	Lessons 3.2.4, 7.2.3, 7.3.2, and 7.3.3 Math Notes boxes, problem 7-40	Problems 7-18, 7-27, 7-29, 7-32, 7-38, 7-40, 7-41, 7-43, 7-69, 7-99, 7-109, 7-110, 7-118, 7-119
CL 7-130.	a. $2(3m+5) = 5m+11$, $m = 1$ b. $b^2 + 4^2 = 7^2$, so $b = \sqrt{33} \approx 5.74$ units c. $3x + 17° + x - 5° = 180°$, so $x = 42°$ d. $2x - 5° = x + 13°$, so $x = 18°$	Lessons 2.1.4, 7.2.4, and 7.2.6 Math Notes boxes, problem 7-49	Problems 7-16, 7-33, 7-49, 7-52, 7-70
CL 7-131.		Lessons 3.2.4, 6.1.3, and 7.2.1 Math Notes boxes, problems 7-45 and 7-56	Problems 7-61, 7-78, 7-85, 7-87, 7-96, 7-104, 7-105