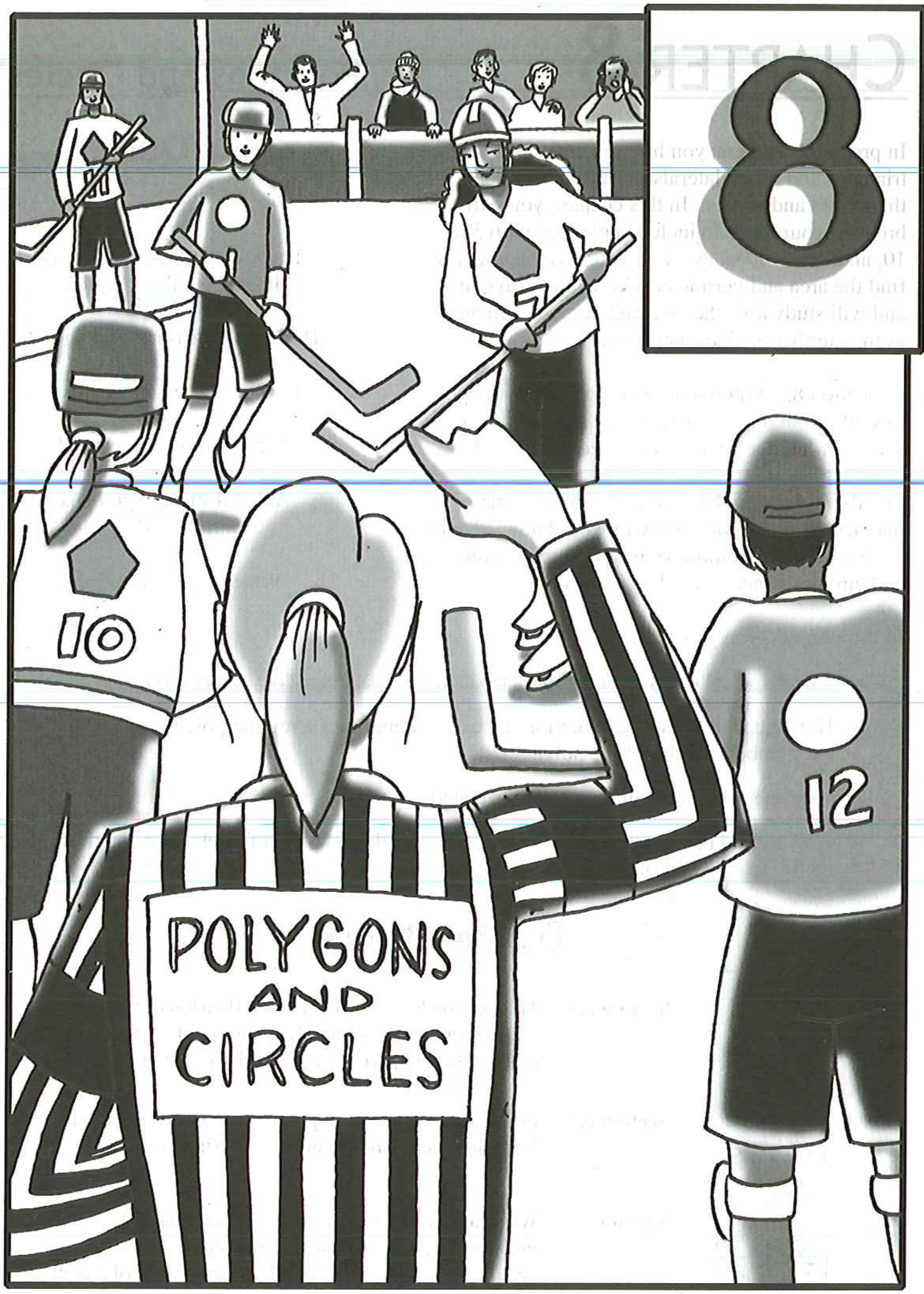
CHAPTER
8
POLYGONS
AND
CIRCLES
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7

CHAPTER 8 Polygons and Circles

In previous chapters, you have extensively studied triangles and quadrilaterals to learn more about their sides and angles. In this chapter, you will broaden your focus to include polygons with 5, 8, 10, and even 100 sides. You will develop a way to find the area and perimeter of a regular polygon and will study how the area and perimeter changes as the number of sides increases.

In Section 8.2, you will re-examine similar shapes to study what happens to the area and perimeter of a shape when the shape is enlarged or reduced.

Finally, in Section 8.3, you will connect your understanding of polygons with your knowledge of the area ratios of similar figures to find the area and circumference of circles of all sizes.

Guiding Questions

Think about these questions throughout this chapter:

How can I measure a polygon?

How does the area change?

Is there another method?

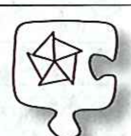
What if the polygon has infinite sides?

What's the connection?

In this chapter, you will learn:

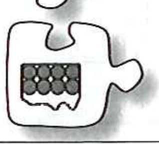
- About special types of polygons, such as regular and non-convex polygons.
- How the measures of the interior and exterior angles of a regular polygon are related to the number of sides of the polygon.
- How the areas of similar figures are related.
- How to find the area and circumference of a circle and parts of circles and use this ability to solve problems in various contexts.

Chapter Outline



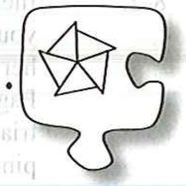
Section 8.1 This section begins with an **investigation** of the interior and exterior angles of a polygon and ends with a focus on the area and perimeter of regular polygons.

Section 8.2 In this section, similar figures are revisited in order to **investigate** the ratio of the areas of similar figures.



Section 8.3 While answering the question, "*What if the polygon has an infinite number of sides?*", a process will be developed to find the area and circumference of a circle.

8.1.1 How can I build it?



Pinwheels and Polygons

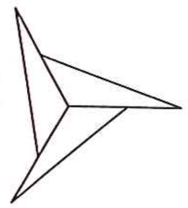
In previous chapters, you have studied triangles and quadrilaterals. In Chapter 8, you will broaden your focus to include all polygons and will study what triangles can tell us about shapes with 5, 8, or even 100 sides.

By the end of this lesson, you should be able to answer these questions:

How can you use the number of sides of a regular polygon to find the measure of the central angle?

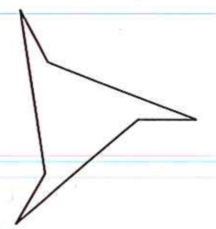
What type of triangle is needed to form a regular polygon?

8-1. PINWHEELS AND POLYGONS



Inez loves pinwheels. One day in class, she noticed that if she put three congruent triangles together so that one set of corresponding angles are adjacent, she could make a shape that looks like a pinwheel.

a. Can you determine any of the angles of her triangles? Explain how you found your answer.



b. The overall shape (outline) of Inez's pinwheel is shown at right. How many sides does it have? What is another name for this shape?

c. Inez's shape is an example of a **polygon** because it is a closed, two-dimensional figure made of straight line segments connected end-to-end. As you study polygons in this course, it is useful to use the names below because they identify how many sides a particular polygon has. Some of these words may be familiar, while others may be new. On your paper, draw an example of a *heptagon.*

Name of Polygon	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7

Name of Polygon	Number of Sides
Octagon	8
Nonagon	9
Decagon	10
11-gon	11
n-gon	*n*

8-2. Inez is very excited. She wants to know if you can build a pinwheel using *any* angle of her triangle. Obtain a Lesson 8.1.1 Resource Page from your teacher and cut out Inez's triangles. Then work with your team to build pinwheels and polygons by placing different corresponding angles together at the center. You will need to use the triangles from all four team members together to build one shape. Be ready to share your results with the class.

8-3. Jorge likes Inez's pinwheels but wonders, "Will all triangles build a pinwheel or a polygon?"

a. If you have not already done so, cut out the remaining triangles on the Lesson 8.1.1 Resource Page. Work together to determine which congruent triangles can build a pinwheel (or polygon) when corresponding angles are placed together at the center. For each successful pinwheel, answer the questions below.

- How many triangles did it take to build the pinwheel?
- Calculate the measure of a **central angle** of the pinwheel. (Remember that a central angle is an angle of a triangle with a vertex at the center of the pinwheel.)
- Is the shape familiar? Does it have a name? If so, what is it?

b. Explain why one triangle may be able to create a pinwheel or polygon while another triangle cannot.

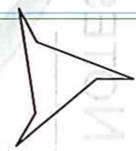
c. Jorge has a triangle with angle measures 32°, 40°, and 108°. Will this triangle be able to form a pinwheel? Explain.

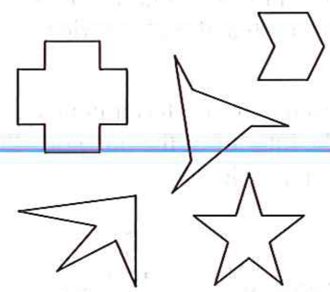
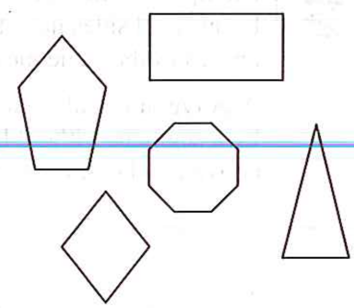
8-4. Jasmine wants to create a pinwheel with equilateral triangles.

a. How many equilateral triangles will she need? Explain how you know.

b. What is the name for the polygon she created?

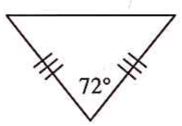
c. Jasmine's shape is an example of a **convex polygon**, while Inez's shape, shown at right, is **non-convex**. Study the examples of convex and non-convex polygons below and then write a definition of a convex polygon on your paper.

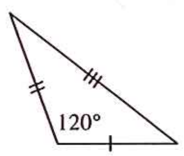


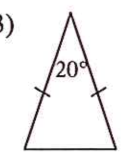
Examples of Non-Convex Polygons	Examples of Convex Polygons
	

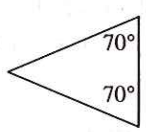
8-5. When corresponding angles are placed together, why do some triangles form convex polygons while others result in non-convex polygons? Consider this as you answer the following questions.

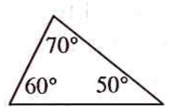
a. Carlisle wants to build a convex polygon using congruent triangles. He wants to select one of the triangles below to use. Which triangle(s) will build a convex polygon if multiple congruent triangles are placed together so that they share a common vertex and do not overlap? Explain how you know.

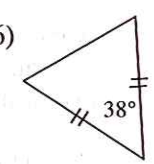
(1)


(2)


(3)


(4)


(5)


(6)


b. For each triangle from part (a) that creates a convex polygon, how many sides would the polygon have? What name is most appropriate for the polygon?

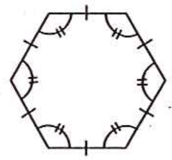
MATH NOTES

METHODS AND MEANINGS

Convex and Non-Convex Polygons

A **polygon** is defined as a two-dimensional closed figure made up of straight line segments connected end-to-end. These segments may not cross (intersect) at any other points.

A polygon is referred to as a **regular polygon** if it is equilateral (all sides have the same length) and equiangular (all interior angles have equal measure). For example, the hexagon shown at right is a regular hexagon because all sides have the same length and each interior angle has the same measure.

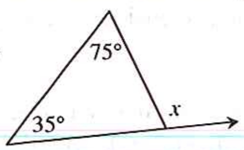


A polygon is called **convex** if each pair of interior points can be connected by a segment without leaving the interior of the polygon. See the example of convex and non-convex shapes in problem 8-4.

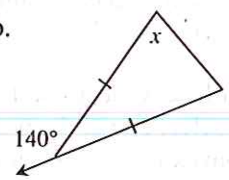
Review & Preview

8-6. Solve for x in each diagram below.

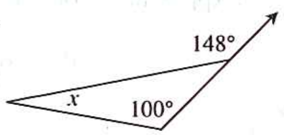
a.



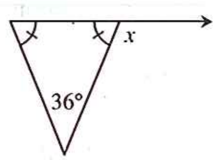
b.



c.



d.



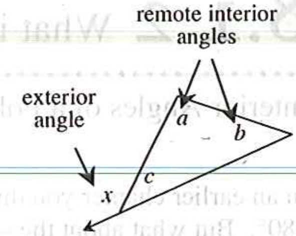
8-7. After solving for x in each of the diagrams in problem 8-6, Jerome thinks he sees a pattern. He notices that the measure of an exterior angle of a triangle is related to two of the angles of a triangle.

a. Do you see a pattern? To help find a pattern, study the results of problem 8-6.

Problem continues on next page →

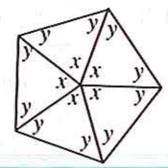
8-7. *Problem continued from previous page.*

b. In the example at right, angles a and b are called **remote interior angles** of the given exterior angle because they are not adjacent to the exterior angle. Write a conjecture about the relationships between the remote interior and exterior angles of a triangle.



c. Prove that the conjecture you wrote for part (b) is true for all triangles. Your proof can be written in any form, as long as it is convincing and provides **reasons** for all statements.

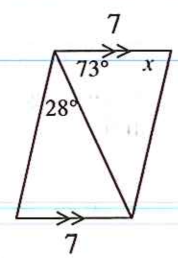
8-8. **Examine** the geometric relationships in the diagram at right. Show all of the steps in your solutions for x and y.



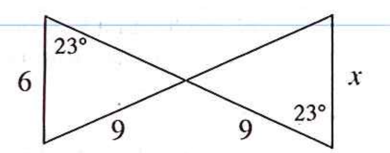
8-9. Steven has 100 congruent triangles that each has an angle measuring 15°. How many triangles would he need to use to make a pinwheel? Explain how you found your answer.

8-10. Find the value of x in each diagram below, if possible. If the triangles are congruent, state which triangle congruence property was used. If the triangles are not congruent or if there is not enough information, state, "Cannot be determined."

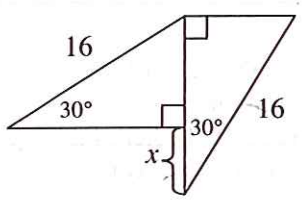
a.



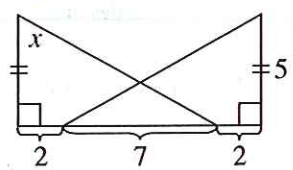
b.



c.



d.



8-11. Decide if the following statements are true or false. If a statement is false, provide a diagram of a counterexample.

a. All squares are rectangles.

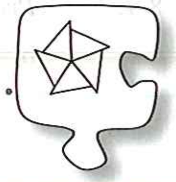
b. All quadrilaterals are parallelograms.

c. All rhombi are parallelograms.

d. All squares are rhombi.

e. The diagonals of a parallelogram bisect the angles.

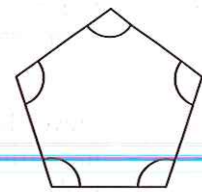
8.1.2 What is its measure?



Interior Angles of a Polygon

In an earlier chapter you discovered that the sum of the interior angles of a triangle is always 180°. But what about the sum of the interior angles of other polygons, such as hexagons or decagons? Does it matter if the polygon is convex or not? Consider these questions today as you **investigate** the angles of a polygon.

8-12. Copy the diagram of the regular pentagon at right onto your paper. Then, with your team, find the sum of the measures of the interior angles *as many ways as you can*. You may want to use the fact that the sum of the angles of a triangle is 180°. Be prepared to share your team's methods with the class.



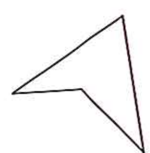
8-13. SUM OF THE INTERIOR ANGLES OF A POLYGON

In problem 8-12, you found the sum of the angles of a regular pentagon. But what about other polygons?

a. Obtain a Lesson 8.1.2 Resource Page from your teacher. Then use one of the methods from problem 8-12 to find the sum of the interior angles of other polygons. Complete the table (also shown below) on the resource page.

Number of Sides of the Polygon	3	4	5	6	7	8	9	10	12
Sum of the Interior Angles of the Polygon	180°								

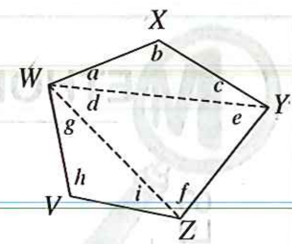
b. Does the interior angle sum depend on whether the polygon is convex? Test this idea by drawing a few non-convex polygons (like the one at right) on your paper and determine if it matters whether the polygon is convex. Explain your findings.



c. Find the sum of the interior angles of a 100-gon. Explain your **reasoning**.

d. In your Learning Log, write an expression that represents the sum of the interior angles of an n-gon. Title this entry "Interior Angles of a Polygon" and include today's date.

8-14. The pentagon at right has been dissected (broken up) into three triangles with the angles labeled as shown. Use the three triangles to prove that the sum of the interior angles of **any** pentagon is always 540°. If you need help, answer the questions below.



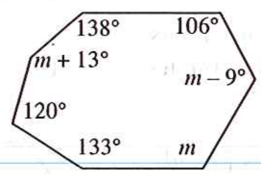
a. What is the sum of the angles of a triangle? Use this fact to write three equations based on the triangles in the diagram.

b. Add the three equations to create one long equation that represents the sum of all nine angles.

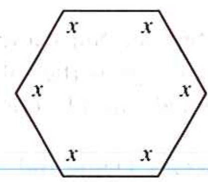
c. Substitute the three-letter name for each angle of the pentagon for the lower case letters at each vertex of the pentagon. For example, $m\measuredangle XYZ = c + e$.

8-15. Use the angle relationships in each of the diagrams below to solve for the given variables. Show all work.

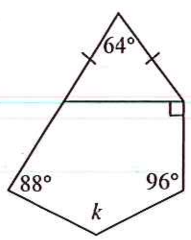
a.



b.



c.



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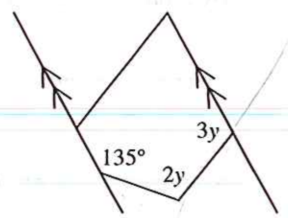


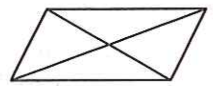
METHODS AND MEANINGS

MATH NOTES

Special Quadrilateral Properties

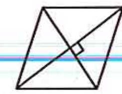
In Chapter 7, you examined several special quadrilaterals and proved conjectures regarding many of their special properties. Review what you learned below.

Parallelogram: Opposite sides of a parallelogram are congruent and parallel. Opposite angles are congruent. Also, since the diagonals dissect the parallelogram into four congruent triangles, the diagonals bisect each other.



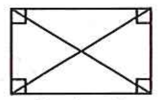
Parallelogram

Rhombus: Since a rhombus is a parallelogram, it has all of the properties of a parallelogram. In addition, its diagonals are perpendicular bisectors and bisect the angles of the rhombus.



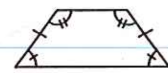
Rhombus

Rectangle: Since a rectangle is a parallelogram, it has all of the properties of a parallelogram. In addition, its diagonals must be congruent.



Rectangle

Isosceles Trapezoid: The base angles (angles joined by a base) of an isosceles trapezoid are congruent.



Isosceles Trapezoid

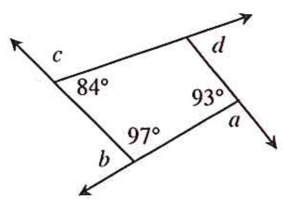
Review & Preview

8-16. On graph paper, graph $\triangle ABC$ if $A(3, 0)$, $B(2, 7)$, and $C(6, 4)$.

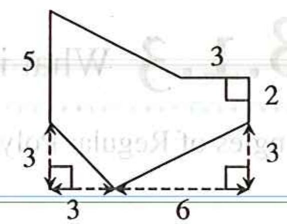
a. What is the best name for this triangle? **Justify** your answer using slope and/or lengths of sides.

b. Find $m\measuredangle A$. Explain how you found your answer.

8-17. The exterior angles of a quadrilateral are labeled a, b, c, and d in the diagram at right. Find the measures of a, b, c, and d and then find the sum of the exterior angles.

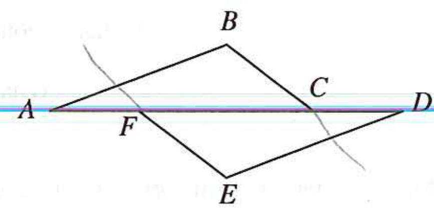


8-18. Find the area and perimeter of the shape at right. Show all work.



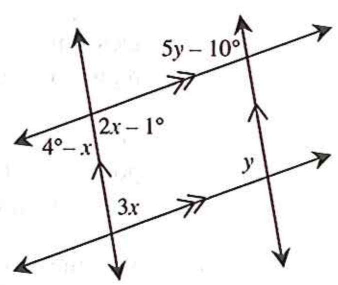
8-19. Crystal is amazed! She graphed ΔABC using the points $A(5, -1)$, $B(3, -7)$, and $C(6, -2)$. Then she rotated ΔABC 90° counterclockwise (↺) about the origin to find $\Delta A'B'C'$. Meanwhile, her teammate took a different triangle (ΔTUV) and rotated it 90° clockwise (↻) about the origin to find $\Delta T'U'V'$. Amazingly, $\Delta A'B'C'$ and $\Delta T'U'V'$ ended up using exactly the same points! Name the coordinates of the vertices of ΔTUV.

8-20. Suzette started to set up a proof to show that if $\overline{BC} \parallel \overline{EF}$, $\overline{AB} \parallel \overline{DE}$, and $AF = DC$, then $\overline{BC} \cong \overline{EF}$. **Examine** her work below. Then complete her missing statements and **reasons**.



Statements	Reasons
1. $\overline{BC} \parallel \overline{EF}$, $\overline{AB} \parallel \overline{DE}$, and $AF = DC$	1.
2. $m\measuredangle BCF = m\measuredangle EFC$ and $m\measuredangle EDF = m\measuredangle CAB$	2.
3.	3. Reflexive Property
4. $AF + FC = CD + FC$	4. Additive Property of Equality (adding the same amount to both sides of an equation keeps the equation true)
5. $AC = DF$	5. Segment addition
6. $\Delta ABC \cong \Delta DEF$	6.
7.	7. $\cong \Delta s \rightarrow \cong$ parts

8-21. **Multiple Choice:** Which equation below is **not** a correct statement based on the information in the diagram?



a. $3x + y = 180°$

b. $2x - 1° = 4° - x$

c. $2x - 1° = 5y - 10°$

d. $2x - 1° + 3x = 180°$

e. None of these is correct

8.1.3 What if it is a regular polygon?

Angles of Regular Polygons

In Lesson 8.1.2 you discovered how to determine the sum of the interior angles of a polygon with any number of sides. But what more can you learn about a polygon? Today you will focus on the interior and exterior angles of regular polygons.

As you work today, keep the following focus questions in mind:

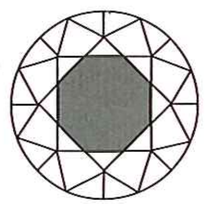
Does it matter if the polygon is regular?

Is there another way to find the answer?

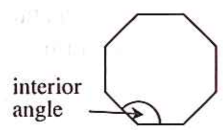
What's the connection?

8-22. Diamonds, the most valuable naturally-occurring gem, have been popular for centuries because of their beauty, durability, and ability to reflect a spectrum of light. In 1919, a diamond cutter from Belgium, Marcel Tolkowsky, used his knowledge of geometry to design a new shape for a diamond, called the "round brilliant cut" (top view shown at right). He discovered that when diamonds are carefully cut with flat surfaces (called "facets" or "faces") in this design, the angles maximize the brilliance and reflective quality of the gem.

Notice that at the center of this design is a **regular octagon** with equal sides and equal interior angles. For a diamond cut in this design to achieve its maximum value, the octagon must be cut carefully and accurately. One miscalculation, and the value of the diamond can be cut in half!



a. Determine the measure of each interior angle of a regular octagon. Explain how you found your answer.



b. What about the interior angles of other regular polygons? Find the interior angles of a regular nonagon and a regular 100-gon.

c. Will the process you used for part (a) work for any regular polygon? Write an expression that will calculate the interior angle of an n-gon.

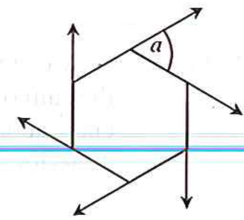
8-23. Fern states, "If a triangle is equilateral, then all angles have equal measure and it must be a regular polygon." Does this logic work for polygons with more than three sides?

a. If all of the sides of a polygon (such as a quadrilateral) are equal, does that mean that the angles must be equal? If you can, draw a counterexample.

b. What if all the angles are equal? Does that force a polygon to be equilateral? Explain your thinking. Draw a counterexample on your paper if possible.

8-24. Jeremy asks, "What about exterior angles? What can we learn about them?"

a. **Examine** the regular hexagon shown at right. Angle *a* is an example of an **exterior angle** because it is formed on the outside of the hexagon by extending one of its sides. Are all of the exterior angles of a regular polygon equal? Explain how you know.



b. Find *a*. Be prepared to share how you found your answer.

c. This regular hexagon has six exterior angles, as shown in the diagram above. What is the sum of the exterior angles of a regular hexagon?

d. What about the exterior angles of other regular polygons? Explore this with your team. Have each team member choose a different shape from the list below to analyze. For each shape:

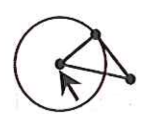
- find the measure of one exterior angle of that shape
- find the sum of the exterior angles.

(1) equilateral triangle (2) regular octagon

(3) regular decagon (4) regular dodecagon (12-gon)

e. Compare your results from part (d). As a team, write a conjecture about the exterior angles of polygons based on your observations. Be ready to share your conjecture with the rest of the class.

f. Is your conjecture from part (e) true for all polygons or for only regular polygons? Does it matter if the polygon is convex? Explore these questions using a dynamic geometric tool or obtain the Lesson 8.1.3 Resource Page and tracing paper from your teacher. Write a statement explaining your findings.



8-25. Use your understanding of polygons to answer the questions below, if possible. If there is no solution, explain why not.

a. Gerardo drew a regular polygon that had exterior angles measuring 40°. How many sides did his polygon have? What is the name for this polygon?

b. A polygon has an interior angle sum of 2,520°. How many sides does it have?

c. A quadrilateral has four sides. What is the measure of each of its interior angles?

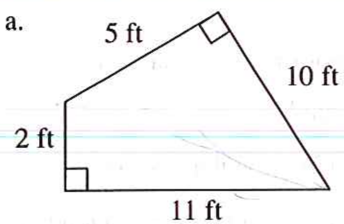
d. What is the measure of an interior angle of a regular 360-gon? Is there more than one way to find this answer?

8-26. How can you find the interior angle of a regular polygon? What is the sum of the exterior angles of a polygon? Write a Learning Log entry about what you learned during this lesson. Title this entry "Interior and Exterior Angles of a Polygon" and include today's date.

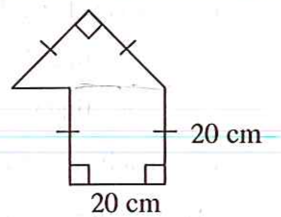
Review & Preview

8-27. Find the area and perimeter of each shape below. Show all work.

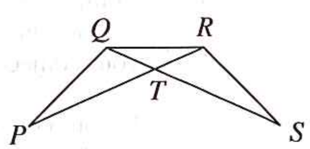
a.



b.



8-28. In the figure at right, if $PQ = RS$ and $PR = SQ$, prove that $\measuredangle P \cong \measuredangle S$. Write your proof either in a flowchart or in two-column proof form.



8-29. Joey used 10 congruent triangles to create a regular decagon.

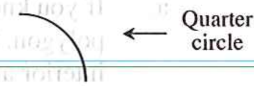
a. What kind of triangles is he using?

b. Find the three angle measures of one of the triangles. Explain how you know.

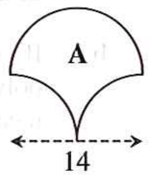
c. If the area of each triangle is 14.5 square inches, then what is the area of the regular decagon? Show all work.

8-30. On graph paper, plot $A(2, 2)$ and $B(14, 10)$. If C is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{AC}$, and E is the midpoint of $\overline{CD}$, find the coordinates of E.

8-31. The arc at right is called a **quarter circle** because it is one-fourth of a circle.

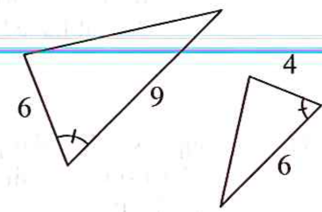


a. Copy Region A at right onto your paper. If this region is formed using four quarter circles, can you find another shape that must have the same area as Region A? **Justify** your conclusion.



b. Find the area of Region A. Show all work.

8-32. **Multiple Choice:** Which property below can be used to prove that the triangles at right are similar?



a. AA ~ b. SAS ~

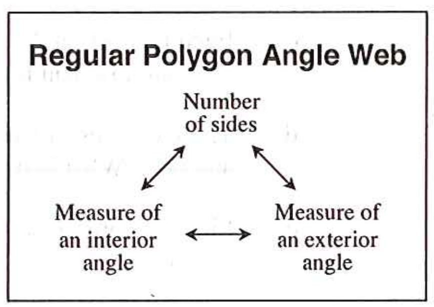
c. SSS ~ d. HL ~

e. None of these

8.1.4 Is there another way?

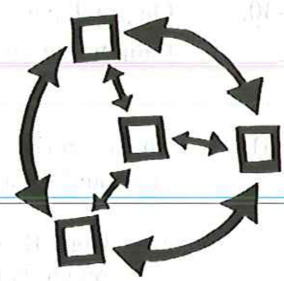
Regular Polygon Angle Connections

During Lessons 8.1.1 through 8.1.3, you have discovered many ways the number of sides of a regular polygon is related to the measures of the interior and exterior angles of the polygon. These relationships can be represented in the diagram at right.



How can these relationships be useful? And what is the most efficient way to go from one measurement to another? This lesson will explore these questions so that you will have a complete set of tools to analyze the angles of a regular polygon.

8-33. Which connections in the Polygon Angle Web do you already have? Which do you still need? Explore this as you answer the questions below.



a. If you know the number of sides of a regular polygon, how can you find the measure of an interior angle directly? Find the measurements of an interior angle of a 15-gon.

b. If you know the number of sides of a regular polygon, how can you find the measure of an exterior angle directly? Find the measurements of an exterior angle of a 10-gon.

c. What if you know that the measure of an interior angle of a regular polygon is 162°? How many sides must the polygon have? Show all work.

d. If the measure of an exterior angle of a regular polygon is 15°, how many sides does it have? What is the measure of an interior angle? Show how you know.

8-34. Suppose a regular polygon has an interior angle measuring 120°. Find the number of sides using *two* different **strategies**. Show all work. Which strategy was most efficient?

8-35. Use your knowledge of polygons to answer the questions below, if possible.

a. How many sides does a polygon have if the sum of the measures of the interior angles is 1980°? 900°?

b. If the exterior angle of a regular polygon is 90°, how many sides does it have? What is another name for this shape?

c. Each interior angle of a regular pentagon has measure $2x + 4°$. What is x? Explain how you found your answer.

d. The measures of four of the exterior angles of a pentagon are 57°, 74°, 56°, and 66°. What is the measure of the remaining angle?

e. Find the sum of the interior angles of an 11-gon. Does it matter if it is regular or not?

8-36. In a Learning Log entry, copy the Regular Polygon Angle Web that your class created. Explain what it represents and give an example of two of the connections. Title this entry "Regular Polygon Angle Web" and include today's date.

METHODS AND MEANINGS

MATH NOTES

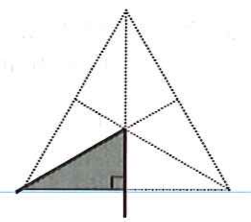
Interior and Exterior Angles of a Polygon

The properties of interior and exterior angles in polygons, where n represents the number of sides in the polygon (n-gon), can be summarized as follows:

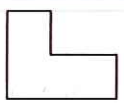
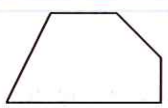
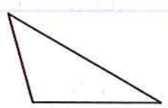
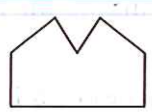
- The sum of the measures of the interior angles of an n-gon is $180(n-2)$.
- The measure of *each* angle in a regular n-gon is $\frac{180(n-2)}{n}$.
- The sum of the exterior angles of an n-gon is always 360°.

Review & Preview

8-37. Esteban used a hinged mirror to create an equilateral triangle, as shown in the diagram at right. If the area of the shaded region is 11.42 square inches, what is the area of the entire equilateral triangle? **Justify** your solution

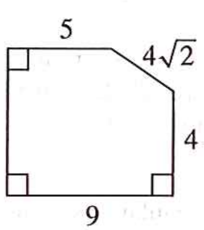


8-38. Copy each shape below on your paper and state if the shape is convex or non-convex. You may want to compare each figure with the examples provided in problem 8-4.

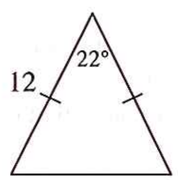
a.  b.  c.  d. 

8-39. Find the area of each figure below. Show all work.

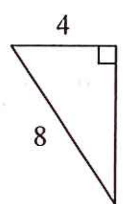
a.



b.



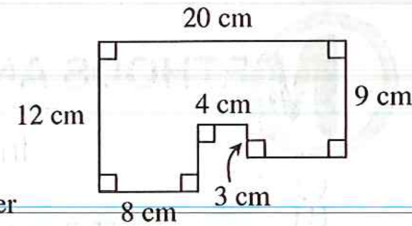
c.



8-40. Find the number of sides in a regular polygon if each interior angle has the following measures.

a. 60° b. 156° c. 90° d. 140°

8-41. At right is a scale drawing of the floor plan for Nzinga's dollhouse. The actual dimensions of the dollhouse are 9 times the measurements provided in the floor plan at right.



a. Use the measurements provided in the diagram to find the area and perimeter of her floor plan.

b. Draw a similar figure on your paper. Label the sides with the actual measurements of Nzinga's dollhouse. What is the perimeter and area of the floor of her actual dollhouse? Show all work.

c. Find the ratio of the perimeters of the two figures. What do you notice?

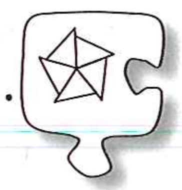
d. Find the ratio of the areas of the two figures. How does the ratio of the areas seem to be related to the zoom factor?

8-42. **Multiple Choice:** A penny, nickel, and dime are all flipped once. What is the probability that at least one coin comes up heads?

a. $\frac{1}{3}$ b. $\frac{3}{8}$ c. 1 d. $\frac{7}{8}$

8.1.5 What's the area?

Finding the Area of Regular Polygons

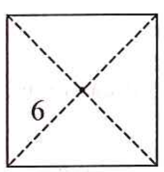


In Lesson 8.1.4, you found the area of a regular hexagon. But what if you want to find the area of a regular pentagon or a regular decagon? Today you will explore these different polygons and generalize how to find the area of any regular polygon with n sides.

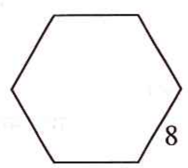
8-43. USING MULTIPLE STRATEGIES

With your team, find the area of each shape below <u>twice</u>, each time using a distinctly different method or **strategy**. Make sure that your results from using different **strategies** are the same. Be sure that each member of your team understands each method.

a. Square



b. regular hexagon



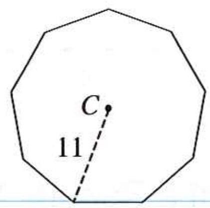
8-44. Create a poster or transparency that shows the two different methods that your team used to find the area of the regular hexagon in part (b) of problem 8-43.

Then, as you listen to other teams present, look for **strategies** that are different than yours. For each one, consider the questions below.

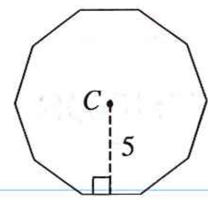
- *Which geometric **tools** does this method use?*
- *Would this method help find the area of other regular polygons (like a pentagon or 100-gon)?*

8-45. Which method presented by teams in problem 8-44 seemed able to help find the area of other regular polygons? Discuss this with your team. Then find the area of the two regular polygons below. If your method does not work, switch to a different method. Assume C is the center of each polygon.

a.



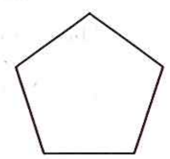
b.



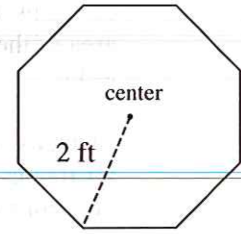
8-46. So far, you have found the area of a regular hexagon, nonagon, and decagon. How can you calculate the area of *any* regular polygon? Write a Learning Log entry describing a general process for finding the area of a polygon with n sides.

8-47. Beth needs to fertilize her flowerbed, which is in the shape of a regular pentagon. A bag of fertilizer states that it can fertilize up to 150 square feet, but Beth is not sure how many bags of fertilizer she should buy.

Beth does know that each side of the pentagon is 15 feet long. Copy the diagram of the regular pentagon below onto your paper. Find the area of the flowerbed and tell Beth how many bags of fertilizer to buy. Explain how you found your answer.



8-48. GO, ROWDY RODENTS!



Recently, your school ordered a stained-glass window with the design of the school's mascot, the rodent. Your student body has decided that the shape of the window will be a regular octagon, shown at right. To fit in the space, the window must have a **radius** of 2 feet. That is, the distance from the center to each vertex must be 2 feet.

a. A major part of the cost of the window is the amount of glass used to make it. The more glass used, the more expensive the window. Your principal has turned to your class to determine how much glass the window will need. Copy the diagram onto your paper and find its area. Explain how you found your answer.

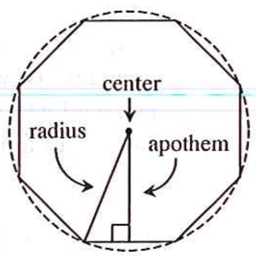
b. The edge of the window will have a polished brass trim. Each foot of trim will cost $48.99. How much will the trim cost? Show all work.

METHODS AND MEANINGS

MATH NOTES

Parts of a Regular Polygon

The **center** of a regular polygon is the center of the smallest circle that completely encloses the polygon.



A line segment that connects the center of a regular polygon with a vertex is called a **radius**.

An **apothem** is the perpendicular line segment from the center of a regular polygon to a side.

Review & Preview

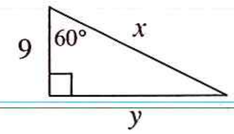
8-49. The exterior angle of a regular polygon is 20°.

a. What is the measure of an interior angle of this polygon? Show how you know.

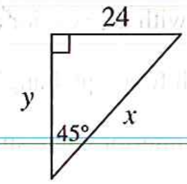
b. How many sides does this polygon have? Show all work.

8-50. Without using your calculator, find the exact values of x and y in each diagram below.

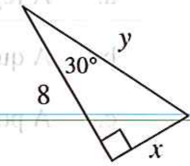
a.



b.



c.



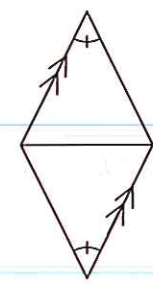
8-51. Find the coordinates of the point at which the diagonals of parallelogram $ABCD$ intersect if $B(-3, -17)$ and $D(15, 59)$. Explain how you found your answer.

8-52. Find the area of an equilateral triangle with side length 20 mm. Draw a diagram and show all work.

8-53. For each equation below, solve for w, if possible. Show all work.

a. $5w^2 = 17$ b. $5w^2 - 3w - 17 = 0$ c. $2w^2 = -3$

8-54. **Multiple Choice:** The triangles at right are congruent because of:

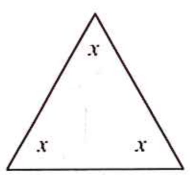


a. SSA ≅ b. HL ≅ c. SAS ≅

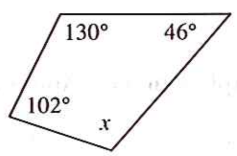
d. SSS ≅ e. None of these

8-55. Solve for x in each diagram below.

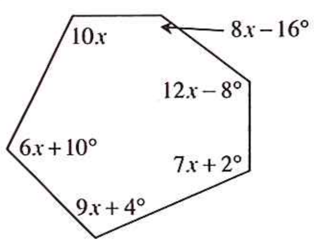
a.



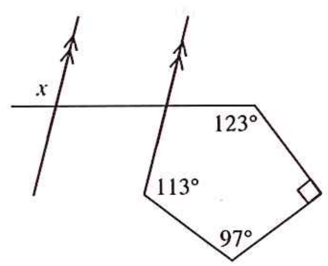
b.



c.



d.



8-56. What is another (more descriptive) name for each polygon described below?

a. A regular polygon with an exterior angle measuring 120°.

b. A quadrilateral with four equal angles.

c. A polygon with an interior angle sum of 1260°.

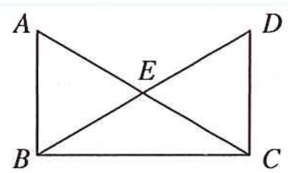
d. A quadrilateral with perpendicular diagonals.

8-57. If ΔABC is equilateral and if $A(0, 0)$ and $B(12, 0)$, then what do you know about the coordinates of vertex C?

8-58. In the figure at right, $\overline{AB} \cong \overline{DC}$ and $\measuredangle ABC \cong \measuredangle DCB$.

a. Is $\overline{AC} \cong \overline{DB}$? Prove your answer.

b. Do the measures of $\measuredangle ABC$ and $\measuredangle DCB$ make any difference in your solution to part (a)? Explain why or why not.

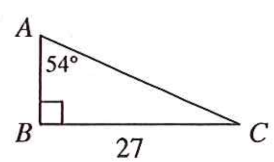


8-59. On graph paper, graph the parabola $y = 2x^2 - x - 15$.

a. What are the roots (x-intercepts) of the parabola? Write your points in (x, y) form.

b. How would the graph of $y = -(2x^2 - x - 15)$ be the same or different? Can you tell without graphing?

8-60. **Multiple Choice:** Approximate the length of $\overline{AB}$.



a. 15.87 b. 21.84 c. 37.16

d. 19.62 e. None of these

8.2.1 How does the area change?

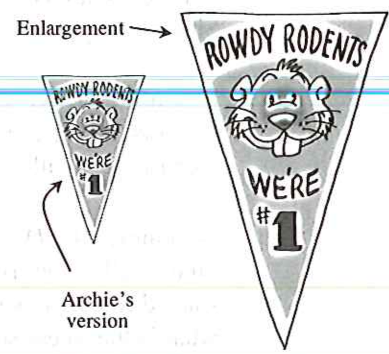
Area Ratios of Similar Figures

Much of this course has focused on similarity. In Chapter 3, you **investigated** how to enlarge and reduce a shape to create a similar figure. You also have studied how to use proportional relationships to find the measures of sides of similar figures. Today you will study how the areas of similar figures are related. That is, as a shape is enlarged or reduced in size, how does the area change?

8-61. MIGHTY MASCOT

To celebrate the victory of your school's championship girls' ice hockey team, the student body has decided to hang a giant flag with your school's mascot on the gym wall.

To help design the flag, your friend Archie has created a scale version of the flag measuring 1 foot wide and 1.5 feet tall.



a. The student body would like the final flag to be 3 feet tall. How wide will the final flag be? **Justify** your solution.

b. If Archie used $2 worth of cloth to create his scale model, then how much will the cloth cost for the full-sized flag? Discuss this with your team. Explain your **reasoning**.

c. Obtain the Lesson 8.2.1A Resource Page and scissors from your teacher. Carefully cut enough copies of Archie's scale version to fit into the large flag. How many did it take? Does this confirm your answer to part (b)? If not, what will the cloth cost for the flag?

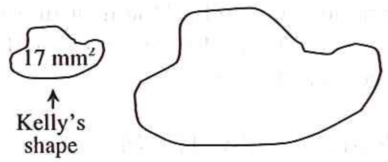
d. 

The student body is reconsidering the size of the flag. It is now considering enlarging the flag so that it is 3 or 4 times the width of Archie's model. How much would the cloth for a similar flag that is 3 times as wide as Archie's model cost? What if the flag is 4 times as wide?

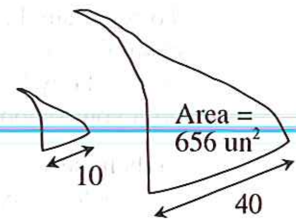
To answer this question, first *estimate* how many of Archie's drawings would fit into each enlarged flag. Then obtain the Lesson 8.2.1B Resource Page (one for you and your team members to share) and confirm each answer by fitting Archie's scale version into the enlarged flags.

8-62. Write down any observations or patterns you found while working on problem 8-61. For example, if the area of one shape is 100 times larger than the area of a similar shape, then what is the ratio of the corresponding sides (also called the **linear scale factor**)? And if the linear scale factor is r, then how many times larger is the area of the new shape?

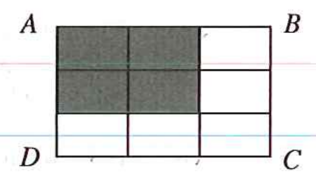
8-63. Use your pattern from problem 8-62 to answer the following questions.



a. Kelly's shape at right has an area of 17 mm^2. If she enlarges the shape with a linear scale (zoom) factor of 5, what will be the area of the enlargement? Show how you got your answer.



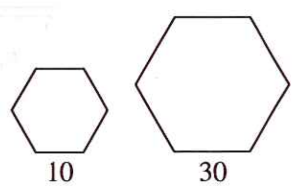
b. **Examine** the two similar shapes at right. What is the linear scale factor? What is the area of the smaller figure?



c. Rectangle *ABCD* at right is divided into nine smaller congruent rectangles. Is the shaded rectangle similar to *ABCD*? If so, what is the linear scale factor? And what is the ratio of the areas? If the shaded rectangle is not similar to *ABCD*, explain how you know.

d. While ordering carpet for his rectangular office, Trinh was told by the salesperson that a 16'-by-24' piece of carpet costs \$200. Trinh then realized that he read his measurements wrong and that his office is actually 8'-by-12'. "Oh, that's no problem," said the salesperson. "That is half the size and will cost \$100 instead." Is that fair? Decide what the price should be.

8-64. If the side length of a hexagon triples, how does the area increase? First make a prediction using your pattern from problem 8-62. Then confirm your prediction by calculating and comparing the areas of the two hexagons shown at right.



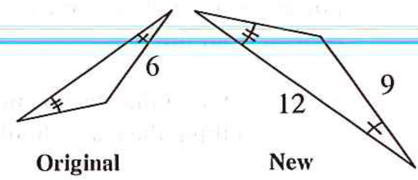
METHODS AND MEANINGS

MATH NOTES

Ratios of Similarity

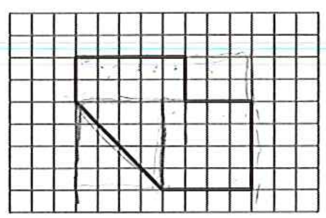
Since Chapter 3, you have used the term **zoom factor** to refer to the ratio of corresponding dimensions of two similar figures. However, now that you will be using other ratios of similar figures (such as the ratio of the areas), this ratio needs a more descriptive name. From now on, this text will refer to the ratio of corresponding sides as the **linear scale factor**. The word "linear" is a reference to the fact that the ratio of the side lengths is a comparison of a single dimension of the shapes. Often, this value is represented with the letter r, for ratio.

For example, notice that the two triangles at right are similar because of AA ~. Since the corresponding sides of the new and original shape are 9 and 6, it can be stated that $r = \frac{9}{6} = \frac{3}{2}$.



Review & Preview

8-65. **Examine** the shape at right.



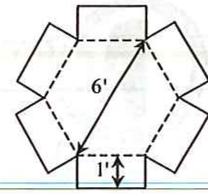
a. Find the area and perimeter of the shape.

b. On graph paper, enlarge the figure so that the linear scale factor is 3. Find the area and perimeter of the new shape.

c. What is the ratio of the perimeters of both shapes? What is the ratio of the areas?

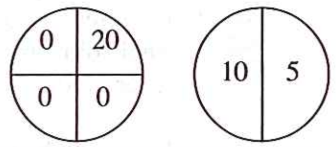
8-66. Sandip noticed that when he looked into a mirror that was lying on the ground 8 feet from him, he could see a clock on the wall. If Sandip's eyes are 64 inches off the ground, and if the mirror is 10 feet from the wall, how high above the floor is the clock? Include a diagram in your solution.

8-67. Mr. Singer has a dining table in the shape of a regular hexagon. While he loves this design, he has trouble finding tablecloths to cover it. He has decided to make his own tablecloth!



In order for his tablecloth to drape over each edge, he will add a rectangular piece along each side of the regular hexagon as shown in the diagram at right. Using the dimensions given in the diagram, find the total area of the cloth Mr. Singer will need.

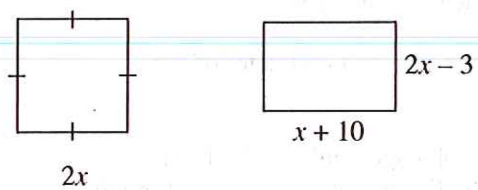
8-68. Your teacher has offered your class extra credit. She has created two spinners, shown at right. Your class gets to spin only one of the spinners. The number that the spinner lands on is the number of extra credit points each member of the class will get. Study both spinners carefully.



a. Assuming that each spinner is divided into equal portions, which spinner do you think the class should choose to spin and why?

b. What if the spot labeled "20" were changed to "100"? Would that make any difference?

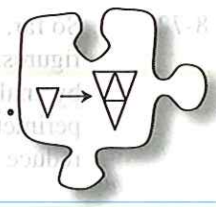
8-69. If the rectangles below have the same area, find x. Is there more than one answer? Show all work.



8-70. **Multiple Choice:** A cable 100 feet long is attached 70 feet up the side of a building. If it is pulled taut (i.e., there is no slack) and staked to the ground as far away from the building as possible, approximately what angle does the cable make with the ground?

a. 39.99° b. 44.43° c. 45.57° d. 12.22°

8.2.2 How does the area change?



Ratios of Similarity

Today you will continue **investigating** the ratios between similar figures. As you solve today's problems, look for connections between the ratios of similar figures and what you already know about area and perimeter.

8-71. TEAM PHOTO

Alice has a 4"-by-5" photo of your school's championship girls' ice hockey team. To celebrate their recent victory, your principal wants Alice to enlarge her photo for a display case near the main office.

a. When Alice went to the print shop, she was confronted with many choices of sizes: 7"-by-9", 8"-by-10", and 12"-by-16". She's afraid that if she picks the wrong size, part of the photo will be cut off. Which size should Alice pick and why?

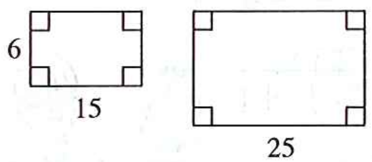
b. The cost of the photo paper to print Alice's 4"-by-5" picture is $0.45. Assuming that the cost per square inch of photo paper remains constant, how much should it cost to print the enlarged photo? Explain how you found your answer.

c. Unbeknownst to her, the Vice-Principal also went out and ordered an enlargement of Alice's photo. However, the photo paper for his enlargement cost $7.20! What are the dimensions of his photo?

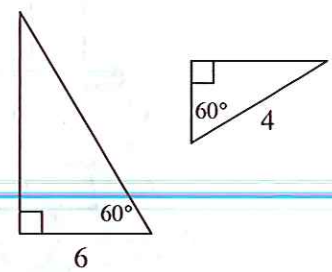
8-72. So far, you have discovered and used the relationship between the areas of similar figures. How are the perimeters of similar figures related? Confirm your intuition by analyzing the pairs of similar shapes below. For each pair, calculate the areas and perimeters and complete a table like the one shown below. To help see patterns, reduce fractions to lowest terms or find the corresponding decimal values.

	Ratio of Sides	Perimeter	Ratio of Perimeters	Area	Ratio of Areas
small figure					
large figure					

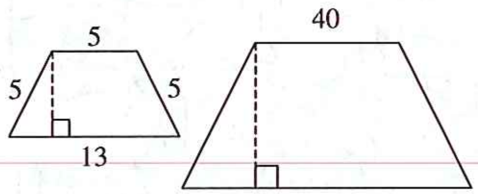
a.



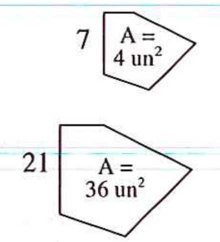
b.



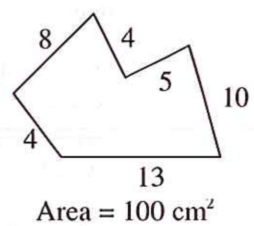
c.



8-73. While Jessie examines the two figures at right, she wonders if they are similar. Decide with your team if there is enough information to determine if the shapes are similar. **Justify** your conclusion.



8-74. Your teacher enlarged the figure at right so that the area of the similar shape is 900 square cm. What is the perimeter of the enlarged figure? Be prepared to explain your method to the class.



8-75. Reflect on what you have learned during Lessons 8.2.1 and 8.2.2. Write a Learning Log entry that explains what you know about the areas and perimeters of similar figures. What connections can you make with other geometric concepts? Be sure to include an example. Title this entry "Area and Perimeter of Similar Figures" and include today's date.

Review & Preview

8-76. Assume Figure *A* and Figure *B*, at right, are similar.

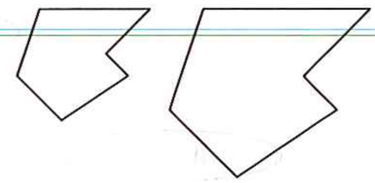


Figure A   Figure B

a. If the ratio of similarity is $\frac{3}{4}$, then what is the ratio of the perimeters of *A* and *B*?

b. If the perimeter of Figure A is *p* and the linear scale factor is *r*, what is the perimeter of Figure B?

c. If the area of Figure A is *a* and the linear scale factor is *r*, what is the area of Figure B?

8-77. Always a romantic, Marris decided to bake his girlfriend a cookie in the shape of a regular dodecagon (12-gon) for Valentine's Day.

a. If the edge of the dodecagon is 6 cm, what is the area of the top of the cookie?

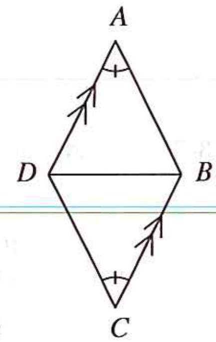
b. His girlfriend decides to divide the cookie into 12 separate but congruent pieces. After 9 of the pieces have been eaten, what area of cookie is left?

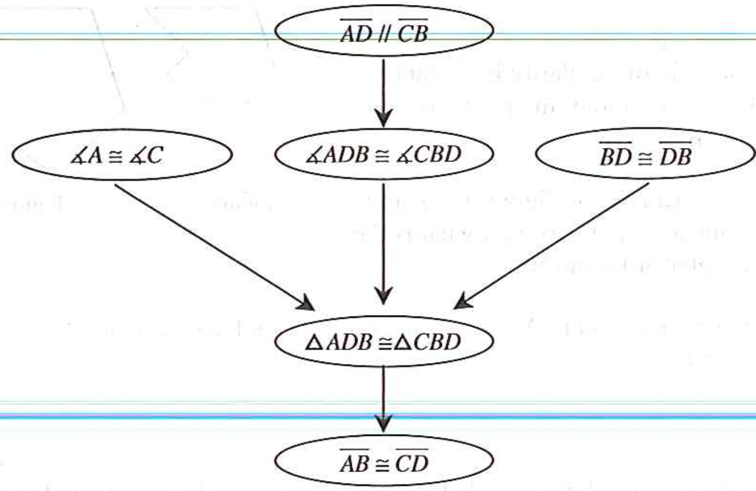
8-78. As her team was building triangles with linguini, Karen asked for help building a triangle with sides 5, 6, and 1. "I don't think that's possible," said her teammate, Kelly.

a. Why is this triangle not possible?

b. Change the lengths of one of the sides so that the triangle is possible.

8-79. Callie started to prove that given the information in the diagram at right, then $\overline{AB} \cong \overline{CD}$. Copy her flowchart below on your paper and help her by **justifying** each statement.

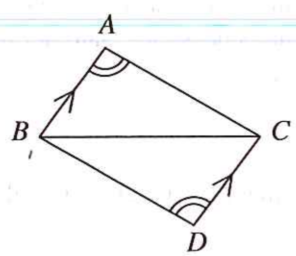




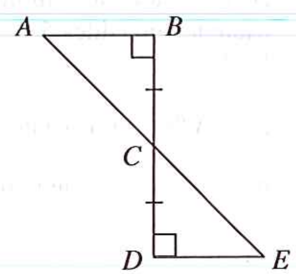
8-80. For each pair of triangles below, decide if the triangles are congruent. If the triangles are congruent:

- State which triangle congruence property proves that the triangles are congruent.
- Write a congruence statement (such as $\triangle ABC \cong \triangle$___).

a.



b.



8-81. **Multiple Choice:** What is the solution to the system of equations at right?

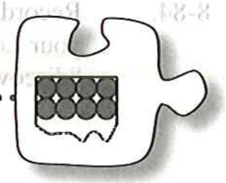
$$y = \frac{1}{2}x - 4$$
$$x - 4y = 12$$

a. (2, 0) b. (16, 4)

c. (–2, –5) d. (4, –2)

e. None of these

8.3.1 What if the polygon has infinite sides?



A Special Ratio

In Section 8.2, you developed a method to find the area and perimeter of a regular polygon with n sides. You carefully calculated the area of regular polygons with 5, 6, 8, and even 10 sides. But what if the regular polygon has an infinite number of sides? How can you predict its area?

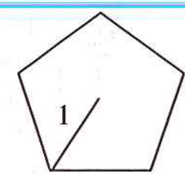
As you **investigate** this question today, keep the following focus questions in mind:

What's the connection?

Do I see any patterns?

How are the shapes related?

8-82. POLYGONS WITH INFINITE SIDES



In order to predict the area and perimeter of a polygon with infinite sides, your team is going to work with other teams to generate data in order to find a pattern.

Your teacher will assign your team three of the regular polygons below. For each polygon, find the area and perimeter if the radius is 1 (as shown in the diagram of the regular pentagon at right). Leave your answer accurate to the nearest 0.01. Place your results into a class chart to help predict the area and perimeter of a polygon with an infinite number of sides.

a.	equilateral triangle	b.	regular octagon	c.	regular 30-gon
d.	square	e.	regular nonagon	f.	regular 60-gon
g.	regular pentagon	h.	regular decagon	i.	regular 90-gon
j.	regular hexagon	k.	regular 15-gon	l.	regular 180-gon

8-83. ANALYSIS OF DATA

With your team, analyze the chart created by the class.

a. What do you predict the area will be for a regular polygon with infinite sides? What do you predict its perimeter will be?

b. What is another name for a regular polygon with infinite sides?

c. Does the number 3.14… look familiar? If so, share what you know with your team. Be ready to share your idea with the class.

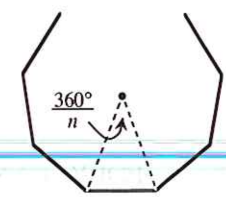
8-84. Record the area and circumference of a circle with radius you're your Learning Log. Then, include a brief description of how you "discovered" π. Title this entry "Pi" and include today's date.

MATH NOTES

METHODS AND MEANINGS

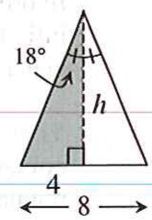
The Area of a Regular Polygon

If a polygon is regular with n sides, it can be subdivided into n congruent isosceles triangles. One way to calculate the area of a regular polygon is to multiply the area of one isosceles triangle by n.



n-gon

To find the area of the isosceles triangle, it is helpful to first find the measure of the polygon's central angle by dividing 360° by n. The height of the isosceles triangle divides the top vertex angle in half.

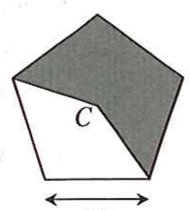


For example, suppose you want to find the area of a regular decagon with side length 4 units. The central angle is $\frac{360°}{10} = 36°$. Then the top angle of the shaded right triangle at right would be $36° \div 2 = 18°$.

Use right triangle trigonometry to find the measurements of the right triangle, then calculate its area. For the shaded triangle above, $\tan 18° = \frac{4}{h}$ and $h \approx 12.311$. Use the height and the base to find the area of the isosceles triangle: $\frac{1}{2}(8)(12.311) \approx 49.242$ sq. units. Then the area of the regular decagon is approximately $10 \cdot 49.242 \approx 492.42$ sq. units. Use a similar approach if you are given a different length of the triangle.

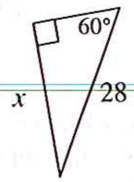
Review & Preview

8-85. Find the area of the shaded region for the regular pentagon at right if the length of each side of the pentagon is 10 units. Assume that point C is the center of the pentagon.

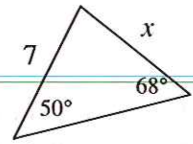


8-86. For each triangle below, find the value of x, if possible. Name which triangle tool you used. If the triangle cannot exist, explain why.

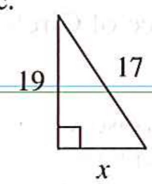
a.



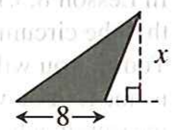
b.



c.

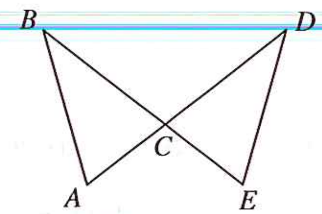


d. Area of the shaded region is 96 un^2.



8-87. Find the measure of each interior angle of a regular 30-gon using **two different methods**.

8-88. **Examine** the diagram at right. Assume that $\overline{BC} \cong \overline{DC}$ and $\measuredangle A \cong \measuredangle E$. Prove that $\overline{AB} \cong \overline{ED}$. Use the form of proof that you prefer (such as the flowchart or two-column proof format). Be sure to copy the diagram onto your paper and add any appropriate markings.



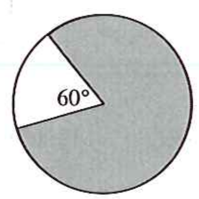
8-89. On graph paper, plot the points $A(-3, -1)$ and $B(6, 11)$.

a. Find the midpoint of $\overline{AB}$.

b. Find the equation of the line that passes through points A and B.

c. Find the distance between points A and B.

8-90. **Multiple Choice:** What fraction of the circle at right is shaded?



a. $\frac{60}{360}$ b. $\frac{300}{360}$ c. $\frac{60}{180}$

d. $\frac{120}{180}$ e. None of these

8.3.2 What's the relationship?

Area and Circumference of Circles

In Lesson 8.3.1, your class discovered that the area of a circle with radius 1 unit is π un^2 and that the circumference is 2π units. But what if the radius of the circle is 5 units or 13.6 units? Today, you will develop a method to find the area and circumference of circles when the radius is not 1. You will also explore parts of circles (called sectors and arcs) and learn about their measurements.

As you and your team work together, remember to ask each other questions such as:

Is there another way to solve it?

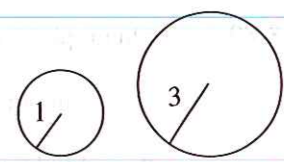
What's the relationship?

What is area? What is circumference?

8-91. AREA AND CIRCUMFERENCE OF A CIRCLE

Now that you know the area and circumference (perimeter) of a circle with radius 1, how can you find the area and circumference of a circle with any radius?

a. First **investigate** how the circles are related. **Examine** the circles at right. Since circles always have the same shape, what is the relationship between any two circles?



b. What is the ratio of the circumferences (perimeters)? What is the ratio of the areas? Explain.

c. If the area of a circle with radius of 1 is π square units, what is the area of a circle with radius 3 units? With radius 10 units? With radius r units?

d. Likewise, if the circumference (perimeter) of a circle is 2π units, what is the circumference of a circle with radius 3? With radius 7? With radius r?

8-92. Read the definitions of radius and diameter in the Math Notes box for this lesson. Then answer the questions below.

a. Find the area of a circle with radius 10 units.

b. Find the circumference of a circle with diameter 7 units.

c. If the area of a circle is 121π square units, what is its diameter?

d. If the circumference of a circle is 20π units, what is its area?

8-93. The giant sequoia trees in California are famous for their immense size and old age. Some of the trees are more than 2500 years old and tourists and naturalists often visit to admire their size and beauty. In some cases, you can even drive a car through the base of a tree!

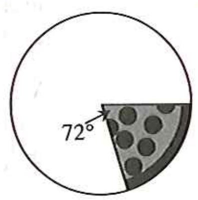
One of these trees, the General Sherman tree in Sequoia National Park, is the largest living thing on the earth. The tree is so gigantic, in fact, that the base has a circumference of 102.6 feet! Assuming that the base of the tree is circular, how wide is the base of the tree? That is, what is its diameter? How does that diameter compare with the length and width of your classroom?

8-94. To celebrate their victory, the girls' ice-hockey team went out for pizza.

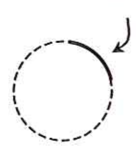
a. The goalie ate half of a pizza that had a diameter of 20 inches! What was the area of pizza that she ate? What was the length of crust that she ate? Leave your answers in exact form. That is, do not convert your answer to decimal form.

b. Sonya chose a slice from another pizza that had a diameter of 16 inches. If her slice had a central angle of 45°, what is the area of this slice? What is the length of its crust? Show how you got your answer.

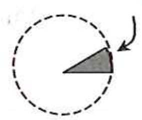
c. As the evening drew to a close, Sonya noticed that there was only one slice of the goalie's pizza remaining. She measured the central angle and found out that it was 72°. What is the area of the remaining slice? What is the length of its crust? Show how you got your answer.



d. A portion of a circle (like the crust of a slice of pizza) is called an **arc**. This is a set of connected points a fixed distance from a central point. The length of an arc is a part of the circle's circumference. If a circle has a radius of 6 cm, find the length of an arc with a central angle of 30°.



e. A region that resembles a slice of pizza is called a **sector**. It is formed by two radii of a central angle and the arc between their endpoints on the circle. If a circle has radius 10 feet, find the area of a sector with a central angle of 20°.



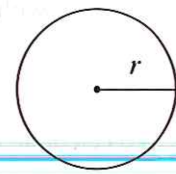
8-95. Reflect on what you have learned today. How did you use similarity to find the areas and circumferences of circles? How are the radius and diameter of a circle related? Write a Learning Log entry about what you learned today. Title this entry "Area and Circumference of a Circle" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Circle Facts

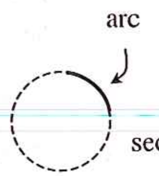
The area of a circle with radius $r = 1$ unit is π un^2. (Remember that $\pi \approx 3.1415926\ldots$)



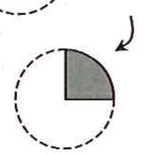
Since all circles are similar, their areas increase by a square of the zoom factor. That is, a circle with radius 6 has an area that is 36 times the area of a circle with radius 1. Thus, a circle with radius 6 has an area of 36π un^2, and a circle with radius r has **area** $A = \pi r^2$ un^2.

Area = πr^2
Circumference = $2\pi r = \pi d$

The **circumference** of a circle is its perimeter. It is the distance around a circle. The circumference of a circle with radius $r = 1$ unit is 2π units. Since the perimeter ratio is equal to the ratio of similarity, a circle with radius r has circumference $C = 2\pi r$ units. Since the diameter of a circle is twice its radius, another way to calculate the circumference is $C = \pi d$ units.



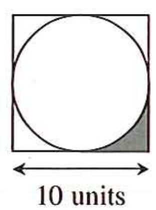
A part of a circle is called an **arc**. This is a set of points a fixed distance from a center and is defined by a central angle. Since a circle does not include its interior region, an arc is like the edge of a crust of a slice of pizza.



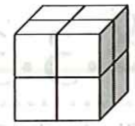
A region that resembles a slice of pizza is called a **sector**. It is formed by two radii of a central angle and the arc between their endpoints on the circle.

Review & Preview

8-96. The diagram at right shows a circle inscribed in a square. Find the area of the shaded region. Show all work.



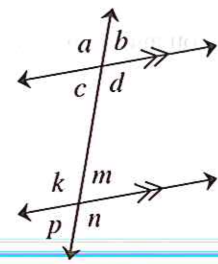
8-97. Reynaldo has a stack of blocks on his desk, as shown below at right.



a. If his stack is 2 blocks wide, 2 blocks long, and 2 blocks tall, how many blocks are in his stack?

b. What if his stack instead is 3 blocks wide, 3 blocks long, and 2 blocks tall? How many blocks are in this stack?

8-98. Find the missing angle(s) in each problem below using the geometric relationships shown in the diagram at right. Be sure to write down the conjecture that **justifies** each calculation. Remember that each part is a separate problem.



a. If $d = 110°$ and $k = 5x - 20°$, write an equation and solve for x.

b. If $b = 4x - 11°$ and $n = x + 26°$, write an equation and solve for x. Then find the measure of $\measuredangle n$.

8-99. An exterior angle of a regular polygon measures 18°.

a. How many sides does the polygon have?

b. If the length of a side of the polygon is 2, what is the area of the polygon?

8-100. A regular hexagon with side length 4 has the same area as a square. What is the length of the side of the square? Explain how you know.

8-101. **Multiple Choice:** Which type of quadrilateral below does not necessarily have diagonals that bisect each other?

a. square b. rectangle c. rhombus d. trapezoid

8.3.3 How can I use it?

Circles in Context

In Lesson 8.3.1, you developed methods to find the area and circumference of a circle with radius r. During this lesson, you will work with your team to solve problems from different contexts involving circles and polygons.

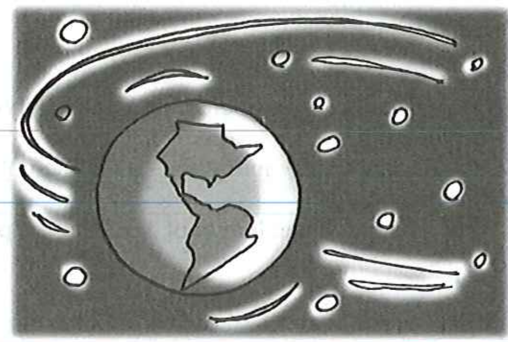
As you and your team work together, remember to ask each other questions such as:

Is there another way to solve it?

What's the connection?

What is area? What is circumference?

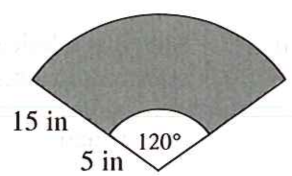
8-102. While the earth's orbit (path) about the sun is slightly elliptical, it can be approximated by a circle with a radius of 93,000,000 miles.



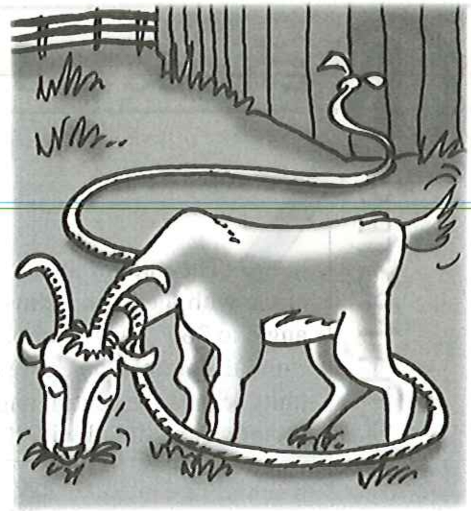
a. How far does the earth travel in one orbit about the sun? That is, what is the approximate circumference of the earth's path?

b. Approximately how fast is the earth traveling in its orbit in space? Calculate your answer in miles per hour.

8-103. A certain car's windshield wiper clears a portion of a sector as shown shaded at right. If the angle the wiper pivots during each swing is 120°, find the area of the windshield that is wiped during each swing.



8-104. THE GRAZING GOAT



Zoe the goat is tied by a rope to one corner of a 15 meter-by-25 meter rectangular barn in the middle of a large, grassy field. Over what area of the field can Zoe graze if the rope is:

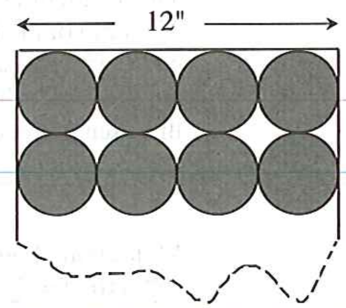
a. 10 meters long?

b. 20 meters long?

c. 30 meters long?

d. Zoe is happiest when she has at least 400 m^2 to graze. What possible lengths of rope could be used?

8-105. THE COOKIE CUTTER



A cookie baker has an automatic mixer that turns out a sheet of dough in the shape of a square 12" wide. His cookie cutter cuts 3" diameter circular cookies as shown at right. The supervisor complained that too much dough was being wasted and ordered the baker to find out what size cookie would have the least amount of waste.

Your Task:

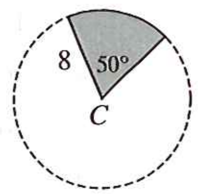
- Analyze this situation and determine how much cookie dough is "wasted" when 3" cookies are cut. Then have each team member find the amount of dough wasted when a cookie of a different diameter is used. Compare your results.

- Write a note to the supervisor explaining your results. **Justify** your conclusion.

MATH NOTES

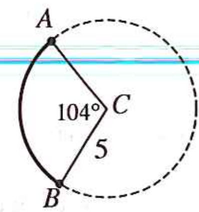
METHODS AND MEANINGS

Arc Length and Area of a Sector



The ratio of the area of a sector to the area of a circle with the same radius equals the ratio of its central angle to 360°. For example, for the sector in circle C at right, the area of the entire circle is $\pi(8)^2 = 64\pi$ square units. Since the central angle is 50°, then the area of the sector can be found with the proportional equation:

$$\frac{50^\circ}{360^\circ} = \frac{\text{area of sector}}{64\pi}$$



To solve, multiply both sides of the equation by 64π. Thus, the area of the sector is $\frac{50^\circ}{360^\circ}(64\pi) = \frac{80\pi}{9} \approx 27.93$ square units.

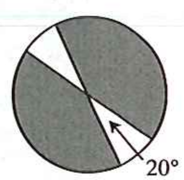
The length of an arc can be found using a similar process. The ratio of the length of an arc to the circumference of a circle with the same radius equals the ratio of its central angle to 360°. To find the length of $\overset{\frown}{AB}$ at right, first find the circumference of the entire circle, which is $2\pi(5) = 10\pi$ units. Then:

$$\frac{104^\circ}{360^\circ} = \frac{\text{arc length}}{10\pi}$$

Multiplying both sides of the equation by 10π, the arc length is $\frac{104^\circ}{360^\circ}(10\pi) = \frac{26\pi}{9} \approx 9.08$ units.

Review & Preview

8-106. Your teacher has constructed a spinner like the one at right. He has informed you that the class gets one spin. If the spinner lands on the shaded region, you will have a quiz tomorrow. What is the probability that you will have a quiz tomorrow? Explain how you know.



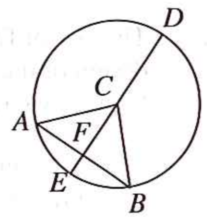
8-107. Use what you know about the area and circumference of circles to answer the questions below. Show all work. Leave answers in terms of π.

a. If the radius of a circle is 14 units, what is its circumference? What is its area?

b. If a circle has diameter 10 units, what is its circumference? What is its area?

c. If a circle has circumference 100π units, what is its diameter? What is its radius?

8-108. Larry started to set up a proof to show that if $\overline{AB} \perp \overline{DE}$ and $\overline{DE}$ is a diameter of $\odot C$, then $\overline{AF} \cong \overline{FB}$. **Examine** his work below. Then complete his missing statements and **reasons**.



Statements	Reasons
1. $\overline{AB} \perp \overline{DE}$ and $\overline{DE}$ is a diameter of $\odot C$.	1.
2. $\measuredangle AFC$ and $\measuredangle BFC$ are right angles.	2.
3. $FC = FC$	3.
4. $\overline{AC} = \overline{BC}$	4. Definition of a Circle (radii must be equal)
5.	5. HL $\cong$
6. $\overline{AF} \cong \overline{FB}$	6.

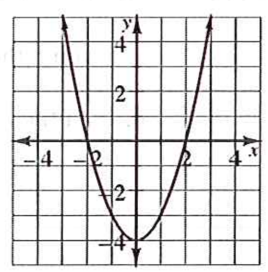
8-109. Match each regular polygon named on the left with a statement about its qualities listed on the right.

a. regular hexagon	(1) Central angle of 36°
b. regular decagon	(2) Exterior angle measure of 90°
c. equilateral triangle	(3) Interior angle measure of 120°
d. square	(4) Exterior angle measure of 120°

8-110. **Examine** the graph of $f(x)$ at right. Use the graph to find the following values.

a. $f(1)$ b. $f(0)$

c. x if $f(x) = 4$ d. x if $f(x) = 0$

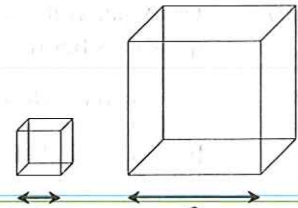


8-111. **Multiple Choice:** How many cubes with edge length 1 unit would fit in a cube with edge length 3 units?

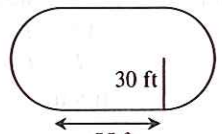
a. 3   b. 9

c. 10   d. 27

e. None of these



8-112. The city of Denver wants you to help build a dog park. The design of the park is a rectangle with two semicircular ends. (Note: A semicircle is half a circle.)



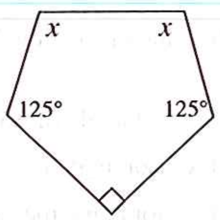
a. The entire park needs to be covered with grass. If grass is sold by the square foot, how much grass should you order?

b. The park also needs a fence for its perimeter. A sturdy chain-linked fence costs about $8 per foot. How much will a fence for the entire park cost?

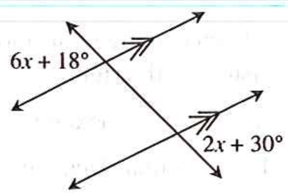
c. The local design board has rejected the plan because it was too small. "Big dogs need lots of room to run," the president of the board said. Therefore, you need to increase the size of the park with a zoom factor of 2. What is the area of the new design? What is the perimeter?

8-113. For each diagram below, write and solve an equation to find x.

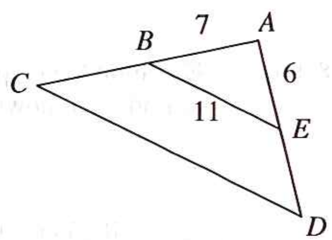
a.



b.



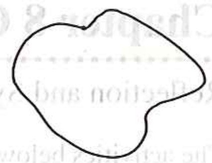
8-114. $\overline{BE}$ is the midsegment of $\triangle ACD$, shown at right.



a. Find the perimeter of $\triangle ACD$.

b. If the area of $\triangle ABE$ is 54 cm^2, what is the area of $\triangle ACD$?

8-115. Christie has tied a string that is 24 cm long into a closed loop, like the one at right.



a. She decided to form an equilateral triangle with her string. What is the area of the triangle?

b. She then forms a square with the same loop of string. What is the area of the square? Is it more or less than the equilateral triangle she created in part (a)?

c. If she forms a regular hexagon with her string, what would be its area? Compare this area with the areas of the square and equilateral triangle from parts (a) and (b).

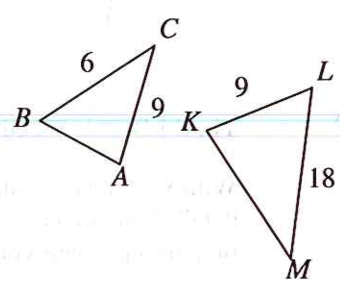
d. What shape should Christie form to enclose the greatest area?

8-116. The **Isoperimetric Theorem** states that of all closed figures on a flat surface with the same perimeter, the circle has the greatest area. Use this fact to answer the questions below.

a. What is the greatest area that can be enclosed by a loop of string that is 24 cm long?

b. What is the greatest area that can be enclosed by a loop of string that is 18π cm long?

8-117. **Multiple Choice:** The diagram at right is not drawn to scale. If $\triangle ABC \sim \triangle KLM$, find KM.



a. 6 b. 12

c. 15 d. 21

e. None of these

Chapter 8 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following three topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Problem Solving: What did you do to solve problems? What different strategies did you use?

Connections: How are the topics, ideas, and words that you learned in previous courses are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

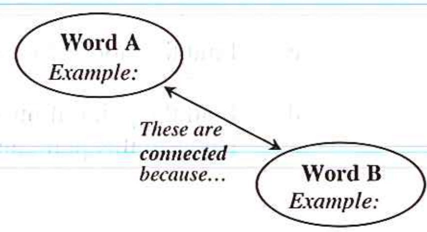
② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

apothem	**arc**	area
central angle	**circumference**	**convex**
diameter	exterior angle	**interior angle**
linear scale factor	**non-convex**	perimeter
pi (π)	polygon	radius
regular polygon	**remote interior angle**	**sector**
similar	zoom factor	

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch of an example.

Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.



While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

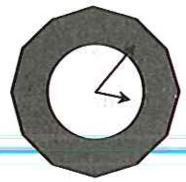
This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 8-118.



Mrs. Frank loves the clock in her classroom because it has the school colors, green and purple. The shape of the clock is a regular dodecagon with a radius of 14 cm. Centered on the clock's face is a green circle of radius 9 cm. If the region outside the circle is purple, which color has more area?

CL 8-119. Graph the quadrilateral *ABCD* if *A*(–2, 6), *B*(2, 3), *C*(2, –2), and *D*(–2, 1).

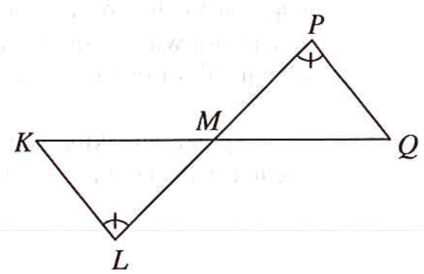
a. What's the best name for this quadrilateral? **Justify** your conclusion.

b. Find the area of *ABCD*.

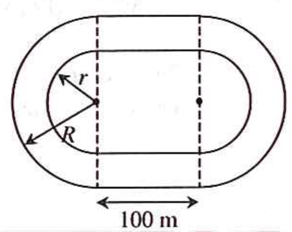
c. Find the slope of the diagonals, $\overline{AC}$ and $\overline{BD}$. How are the slopes related?

d. Find the point of intersection of the diagonals. What is the relationship between this point and diagonal $\overline{AC}$?

CL 8-120. **Examine** the diagram at right. If *M* is the midpoint of $\overline{KQ}$ and if $\measuredangle P \cong \measuredangle L$, prove that $\overline{KL} \cong \overline{QP}$.

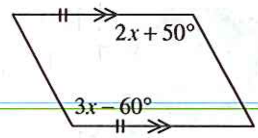


CL 8-121. A running track design is composed of two half circles connected by two straight line segments. Garrett is jogging on the inner lane (with radius *r*) while Devin is jogging on the outer (with radius *R*). If $r = 30$ meters and $R = 33$ meters, how much longer does Devin have to run to complete one lap?

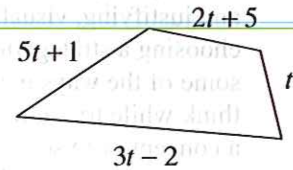


CL 8-122. Use the relationships in the diagrams below to solve for the given variable. **Justify** your solution with a definition or theorem.

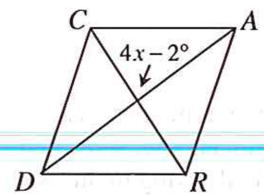
a.



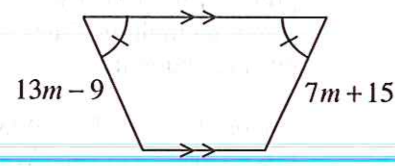
b. The perimeter of the quadrilateral below is 202 units.



c. *CARD* is a rhombus.



d.



CL 8-123. Answer the following questions about polygons. If there is not enough information or the problem is impossible, explain why.

a. Find the sum of the interior angles of a dodecagon.

b. Find the number of sides of a regular polygon if its central angle measures 35°.

c. If the sum of the interior angles of a regular polygon is 900°, how many sides does the polygon have?

d. If the exterior angle of a regular polygon is 15°, find its central angle.

e. Find the exterior angle of a polygon with 10 sides.

CL 8-124. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

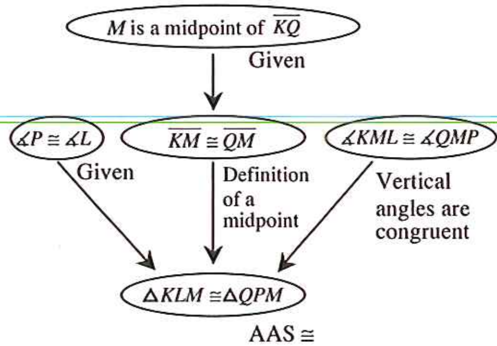
⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: investigating, examining, reasoning and justifying, visualizing, and choosing a strategy/tool. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

Choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 8-118.	Area of green = $81\pi \approx 254.5$ cm^2; area of purple = $588 - 81\pi \approx 333.5$ cm^2, so the area of purple is greater.	Lessons 5.1.2, 8.1.4, 8.1.5, 8.3.1, and 8.3.2 Math Notes boxes	Problems 8-45, 8-47, 8-48, 8-64, 8-67, 8-77, 8-82, 8-85, 8-92, 8-112
CL 8-119.	a. Rhombus. It is a quadrilateral with four equal sides. b. 20 square units c. The slopes are -2 and $\frac{1}{2}$. They are opposite reciprocals. d. The point of intersection is (0, 2). It is the midpoint of the diagonal.	Lessons 2.2.4, 7.2.3, 7.3.2, and 7.3.3 Math Notes boxes	Problems 7-29, 7-32, 7-69, 7-99, 7-107, 7-109, 7-110, 8-51, 8-89

Problem	Solution	Need Help?	More Practice
CL 8-120.		Lessons 3.2.4, 6.1.3, and 7.1.3 Math Notes boxes, problems 7-56 and 7-79	Problems 7-61, 7-78, 7-85, 7-87, 7-96, 7-104, 7-105, 8-20, 8-28, 8-58, 8-79, 8-88
CL 8-121.	Devin must run 6π meters farther than Garrett on each lap.	Lessons 8.3.1 and 8.3.2 Math Notes boxes	Problems 8-92, 8-93, 8-102, 8-107, 8-112
CL 8-122.	a. $x = 110°$ (Opposite angles in a parallelogram are equal.) b. $t = 18$ c. $x = 23°$ (Diagonals of a rhombus are perpendicular.) d. $m = 4$ (Nonparallel sides of an isosceles trapezoid are congruent.)	Lessons 1.1.3, 2.1.4, 7.2.4, and 8.1.2 Math Notes boxes	Problems 7-16, 7-33, 7-40, 7-49, 7-52, 7-70, 8-6, 8-15, 8-21, 8-55, 8-113
CL 8-123.	a. 1800° b. Impossible. In a regular polygon, the central angle must be a factor of 360°. c. 7 sides d. 15° e. 36°	Lessons 7.1.4, 8.1.1, and 8.1.4 Math Notes boxes, problems 8-1, 8-13, and 8-14	Problems 8-15, 8-25, 8-29, 8-33, 8-34, 8-35, 8-40, 8-49, 8-55, 8-56, 8-87, 8-99, 8-109