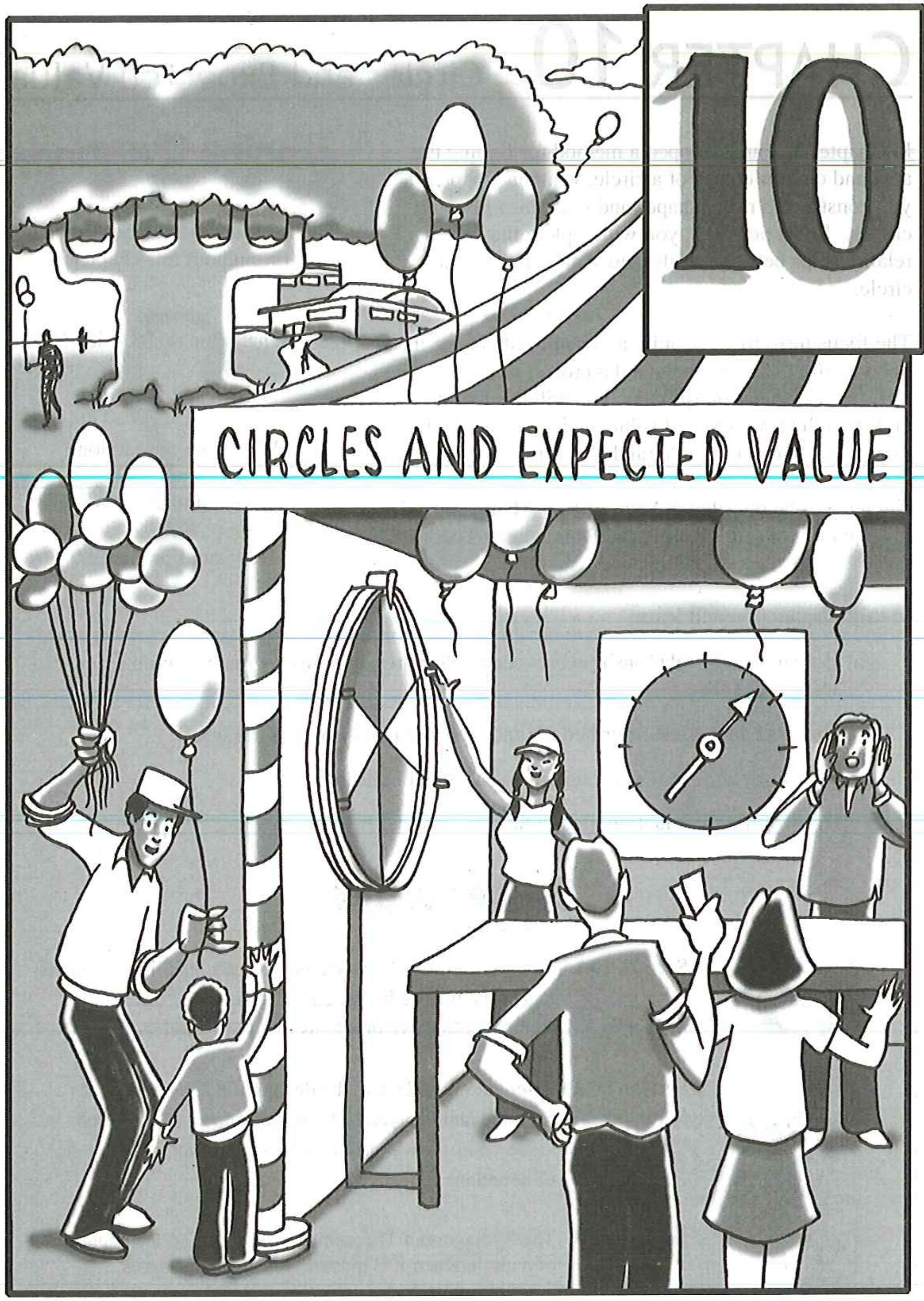

10
CIRCLES AND EXPECTED VALUE

CHAPTER 10 Circles and Expected Value

In Chapter 8, you developed a method for finding the area and circumference of a circle, while in Chapter 9 you constructed many shapes and relationships using circles. In Section 10.1, you will explore the relationships between angles, arcs, and chords in a circle.

The focus turns to probability and games of chance in Section 10.2. As you analyze the probabilities of different outcomes on spinners, you will develop an understanding of expected value, which is a method to predict the outcome of a random event.

Circles will be revisited in Section 10.3 when you find the equation of a circle using the Pythagorean Theorem.

Guiding Questions

Think about these questions throughout this chapter:

What's the relationship?

How can I solve it?

What's the connection?

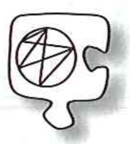
Is there another method?

How can I predict it?

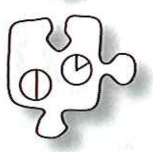
In this chapter, you will learn:

- How to use the relationships between angles, arcs, and line segments within a circle to solve problems.
- How to find a circle inscribed in (and circumscribed about) a triangle.
- How to find the expected outcome of a game of chance.
- How to find the equation of a circle.

Chapter Outline



Section 10.1 The relationships between angles, arcs, and line segments in a circle will be **investigated** to develop "circle tools" that can help solve problems involving circles.



Section 10.2 After data is collected, the design of a spinner will be revealed and analyzed. Then the concept of expected value will be developed as a tool to predict the outcome of a random generator.

Section 10.3 The Pythagorean Theorem will help to find the equation of a circle when it is graphed on coordinate axes.

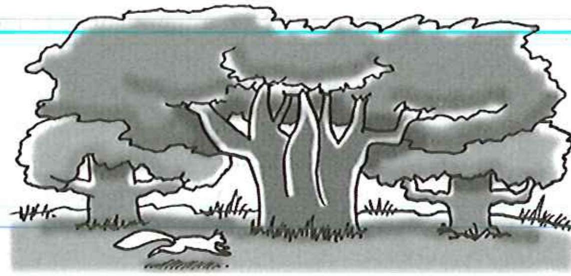
10.1.1 What's the diameter?

Introduction to Chords

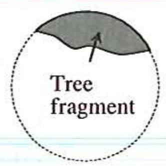
In Chapter 8, you learned that the diameter of a circle is the distance across the center of the circle. This length can be easily measured if the entire circle is in front of you and the center is marked, or if you know the radius of the circle. However, what if you only have part of a circle (called an **arc**)? Or what if the circle is so large that it is not practical to measure its diameter using standard measurement tools (such as finding the diameter of the Earth's equator)?

Today you will consider a situation that demonstrates the need to learn more about the parts of a circle and the relationships between them.

10-1. THE WORLD'S WIDEST TREE



The baobab tree is a species of tree found in Africa and Australia. It is often referred to as the "world's widest tree" because it has been known to be up to 45 feet in diameter!



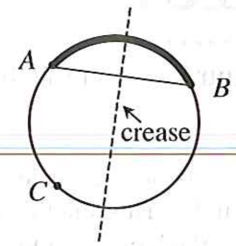
While digging at an archeological site, Rafi found a fragment of a fossilized baobab tree that appears to be wider than any tree on record! However, since he does not have the remains of the entire tree, he cannot simply measure across the tree to find its diameter. He needs your help to determine the radius of this ancient tree. Assume that the shape of the tree's cross-section is a circle.

a. Obtain the Lesson 10.1.1 Resource Page from your teacher. On it, locate $\overset{\frown}{AB}$, which represents the curvature of the tree fragment. Trace this arc as neatly as possible on tracing paper. Then decide with your team how to fold the tracing paper to find the center of the tree. (Note: This will take more than one fold.) Be ready to share with the class how you found the center.

b. In part (a), you located the center of a circle. Use a ruler to measure the radius of that circle. If 1 cm represents 10 feet of tree, find the approximate radius and diameter of the tree. Does the tree appear to be larger than 45 feet in diameter?

10-2. PARTS OF A CIRCLE, Part One

A line segment that connects the endpoints of an arc is called a **chord**. Thus, $\overline{AB}$ in the diagram at right is an example of a chord.



a. One way to find the center of a circle when given an arc is to fold it so that the two parts of the arc coincide (lie on top of each other).

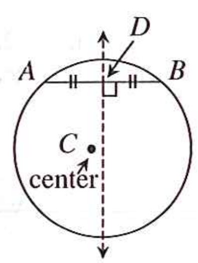
If you fold $\overset{\frown}{AB}$ so that A lies on B, what is the relationship between the resulting crease and the chord $\overline{AB}$? Explain how you know.

b. The tree fragment in problem 10-1 was an arc between points A and B. However, the missing part of the tree formed another larger arc of the tree. With your team, find the larger arc formed by the circle and points A and B above. Then propose a way to name the larger arc to distinguish it from $\overset{\frown}{AB}$.

c. In problem 10-1, the tree fragment formed the shorter arc between two endpoints. The shorter arc between points A and B is called the **minor arc** and is written $\overset{\frown}{AB}$. The larger arc is called a **major arc** and is usually written using three points, such as $\overset{\frown}{ACB}$. What do you know about $\overline{AB}$ if the minor and major arcs are the same length? Explain how you know.

10-3. In problem 10-1, folding the arc several times resulted in a point that seemed to be the center of the circle. But how can we prove that the line bisecting an arc (or chord) will pass through the center? To consider this, first assume that the perpendicular bisector does *not* pass through the center. (This is an example of a proof by contradiction.)

a. According to our assumption, if the perpendicular bisector does not pass through the center, then the center, C, will be off the line in the circle, as shown at right. Copy this diagram onto your paper.



b. Now consider ΔACD and ΔBCD. Are these two triangles congruent? Why or why not?

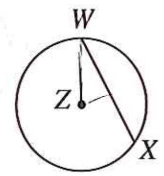
c. Explain why your result from part (b) contradicts the original assumption. That is, explain why the center must lie on the perpendicular bisector of $\overline{AB}$.

10-4. What if you know the lengths of two chords in a circle? How can you use the chords to find the center of the circle?

a. On the Lesson 10.1.1 Resource Page, locate the chords provided for $\odot P$ and $\odot Q$. Work with your team to determine how to find the center of each circle. Then use a compass to draw the circles that contain the given chords. Tracing paper may be helpful.

b. Describe how to find the center of a circle without tracing paper. That is, how would you find the center of $\odot P$ with only a compass and a straightedge? Be prepared to share your description with the rest of the class.

10-5. **Examine** the chord $\overline{WX}$ in $\odot Z$ at right. If $WX = 8$ and the radius of $\odot Z$ is 5, how far from the center is the chord? Draw the diagram on your paper and show all work.



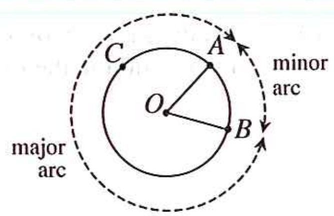
METHODS AND MEANINGS

MATH NOTES

Circle Vocabulary

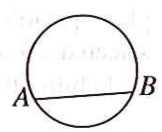
An **arc** is a part of a circle. Remember that a circle does not contain its interior. A bicycle tire is an example of a circle. The piece of tire between any two spokes of the bicycle wheel is an example of an arc.

Any two points on a circle create two arcs. When these arcs are not the same length, the larger arc is referred to as the **major arc**, while the smaller arc is referred to as the **minor arc.**



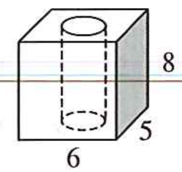
To name an arc, an arc symbol is drawn over the endpoints, such as $\overset{\frown}{AB}$. To refer to a major arc, a third point on the arc should be used to identify the arc clearly, such as $\overset{\frown}{ACB}$.

A **chord** is a line segment that has both endpoints on a circle. $\overline{AB}$ in the diagram at right is an example of a chord. When a chord passes through the center of the circle, it is called a **diameter.**

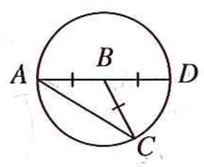


Review & Preview

10-6. A rectangular prism has a cylindrical hole removed, as shown at right. If the radius of the cylindrical hole is 2 inches, find the volume and total surface area of the solid.



10-7. In the diagram below, $\overline{AD}$ is a diameter of $\odot B$.

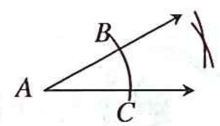


a. If $m\measuredangle A = 35°$, what is $m\measuredangle CBD$?

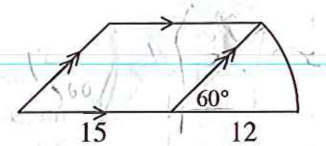
b. If $m\measuredangle CBD = 100°$, what is $m\measuredangle A$?

c. If $m\measuredangle A = x$, what is $m\measuredangle CBD$?

10-8. Lavinia started a construction at right. Explain what she is constructing. Then copy her diagram and finish her construction.



10-9. A sector is attached to the side of a parallelogram, as shown in the diagram at right. Find the area and perimeter of the figure.



10-10. On the same set of axes, graph both equations listed below. Then name all points of intersection in the form (x, y). How many times do the graphs intersect?

$$y = 4x - 7$$

$$y = x^2 - 2x + 2$$

10-11. **Multiple Choice:** Dillon starts to randomly select cards out of a normal deck of 52 playing cards. After selecting a card, he does not return it to the deck. So far, he has selected a 3 of clubs, an ace of spades, a 4 of clubs, and a 10 of diamonds. Find the probability that his fifth card is an ace.

a. $\frac{1}{16}$ b. $\frac{3}{52}$ c. $\frac{1}{13}$ d. $\frac{1}{52}$

10.1.2 What's the relationship?

Angles and Arcs

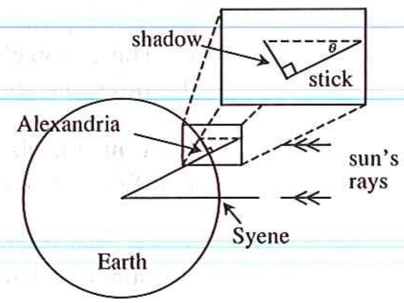
In order to learn more about circles, we need to **investigate** the different types of angles and chords that are found in circles. In Lesson 10.1.1, you studied an application with a tree to learn about the chords of a circle. Today you will study a different application that will demonstrate the importance of knowing how to measure the angles and arcs within a circle.

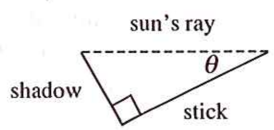
10-12. 

ERATOSTHENES' REMARKABLE DISCOVERY

Eratosthenes (who lived in the 3rd century B.C.) was able to determine the circumference of the Earth at a time when most people thought the world was flat! Since he knew that the Earth was round, he discovered that he could use a shadow to help calculate the Earth's radius.

Eratosthenes knew that Alexandria was located about 500 miles north of a town near the equator, called Syene. When the sun was directly overhead at Syene, a meter stick had no shadow. However, at the same time in Alexandria, a meter stick had a shadow due to the curvature of the Earth. Since the sun is so far away from the Earth, Eratosthenes assumed that the sun's rays were essentially parallel once they entered the Earth's atmosphere and realized that he could therefore use the stick's shadow to help calculate the Earth's radius.



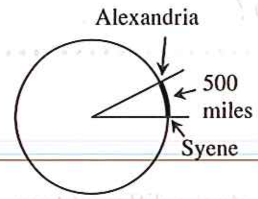


a. Unfortunately, the precise data used by Eratosthenes was lost long ago. However, if Eratosthenes used a meter stick for his experiment today, then the stick's shadow in Alexandria would be 127 mm long. Determine the angle θ that the sunrays made with the meter stick. Remember that a meter stick is 1000 millimeters long.

b. Assuming that the sun's rays are essentially parallel, determine the central angle of the circle if the angle passes through Alexandria and Syene. How did you find your answer?

Problem continues on next page →

10-12. *Problem continued from previous page.*

c. 

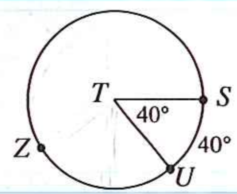
Since the distance along the Earth's surface from Alexandria to Syene is about 500 miles, that is the length of the arc between Alexandria and Syene. Use this information to approximate the circumference of the Earth.

d. Use your result from part (c) to approximate the radius of the Earth.

10-13. PARTS OF A CIRCLE, Part Two

In order to find the circumference of the Earth, Eratosthenes used an angle that had its vertex at the center of the circle. Like the angles in polygons that you studied in Chapter 8, this angle is called a **central angle.**

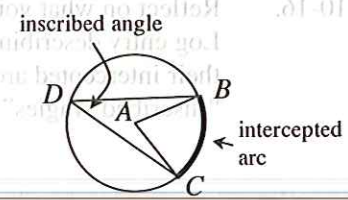
a. An **arc** is a part of a circle. Every central angle has a corresponding arc. For example, in $\odot T$ at right, $\measuredangle STU$ is a central angle and corresponds to $\overset{\frown}{SU}$. Since the measure of an angle helps us know what part of 360° the angle is, an arc can also be measured in degrees, representing what fraction of an entire circle it is. Thus, an **arc's measure** is equal to the measure of its corresponding central angle.



Examine the circle above. What is the measure of $\overset{\frown}{SU}$ (written $m\overset{\frown}{SU}$)? What is $m\overset{\frown}{SZU}$? Show how you got your answer.

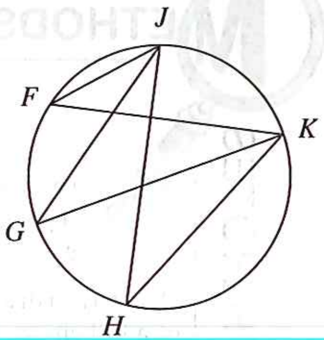
b. When Eratosthenes measured the distance from Syene to Alexandria, he measured the length of an arc. This distance is called **arc length** and is measured with units like centimeters or feet. One way to find arc length is to wrap a string about a part of a circle and then to straighten it out and measure its length. Calculate the arc length of $\overset{\frown}{SU}$ above if the radius of $\odot T$ is 12 inches.

10-14. INSCRIBED ANGLES

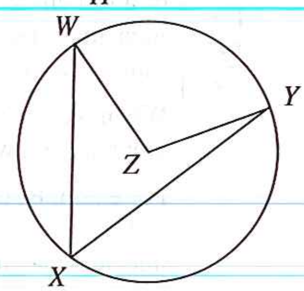


In the diagram at right, $\measuredangle BDC$ is an example of an **inscribed angle**, because it lies in $\odot A$ and its vertex lies on the circle. It corresponds to central angle $\measuredangle BAC$ because they both intercept the same arc, $\overset{\frown}{BC}$. (An **intercepted arc** is an arc with endpoints on each side of the angle.)

Investigate the measure of inscribed angles as you answer the questions below.



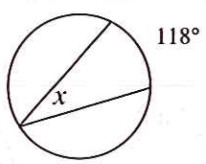
a. In the circle at right, $\measuredangle F$, $\measuredangle G$, and $\measuredangle H$ are examples of inscribed angles. Notice that all three angles intercept the same arc ($\overset{\frown}{JK}$). Use tracing paper to compare their measures. What do you notice?



b. Now compare the measurements of the central angle (such as $\measuredangle WZY$ in $\odot Z$ at right) and an inscribed angle (such as $\measuredangle WXY$). What is the relationship of an inscribed angle and its corresponding central angle? Use tracing paper to test your idea.

10-15. In problem 10-14, you found that the measure of an inscribed angle is always half of the measure of its corresponding central angle. Since the measure of the central angle always equals the measure of its intercepted arc, then the measure of the inscribed angle must be half of the measure of its intercepted arc.

Examine the diagrams below. Find the measures of the indicated angles. If a point is labeled C, assume it is the center of the circle.

a.


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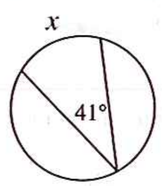
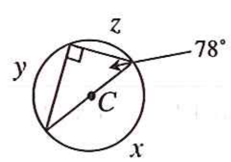


c.


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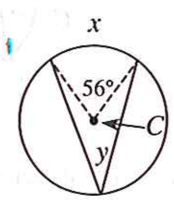
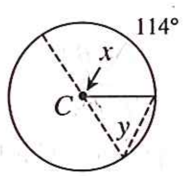
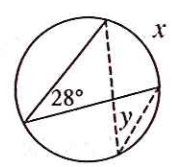


e.


f.


10-16. Reflect on what you have learned during this lesson. Write a Learning Log entry describing the relationships between inscribed angles and their intercepted arcs. Be sure to include an example. Title this entry "Inscribed Angles" and include today's date.

METHODS AND MEANINGS

MATH NOTES

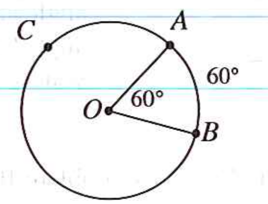
More Circle Vocabulary

The vertex of a **central angle** is at the center of a circle. An **inscribed angle** has its vertex on the circle with each side intersecting the circle at a different point.

One way to discuss an arc is to consider it as a fraction of 360°, that is, as a part of a full circle. When speaking about an arc using degrees, we call this the **arc measure**. The arc between the endpoints of the sides of a central angle has the same measure (in degrees) as its corresponding central angle.

When we want to know how *far* it is from one point to another as we travel along an arc, we call this the **arc length** and measure it in feet, miles, etc.

For example, point O is the **center** of $\odot O$ at right, and $\measuredangle AOB$ is a **central angle**. The sides of the angle intersect the circle at points A and B, so $\measuredangle AOB$ **intercepts** $\overset{\frown}{AB}$. In this case, the **measure** of $\overset{\frown}{AB}$ is 60°, while the measure of the major arc, $m\overset{\frown}{ACB}$, is 300° because the sum of the major and minor arcs is 360°. The **length** of $\overset{\frown}{AB}$ is $\frac{60}{360} = \frac{1}{6}$ of the circumference.

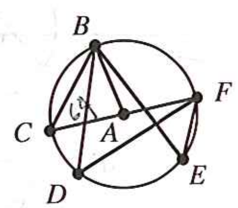


Review & Preview

10-17. In $\odot A$ at right, $\overline{CF}$ is a diameter and $m\measuredangle C = 64°$. Find:

a. $m\measuredangle D$ b. $m\overset{\frown}{BF}$ c. $m\measuredangle E$

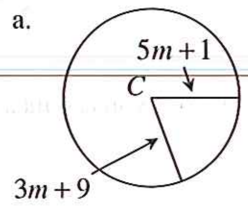
d. $m\overset{\frown}{CBF}$ e. $m\measuredangle BAF$ f. $m\measuredangle BAC$



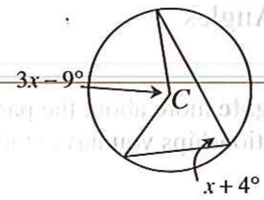
10-18. Find the area of a regular polygon with 100 sides and with a perimeter of 100 units.

10-19. For each of the geometric relationships represented below, write and solve an equation for the given variable. For parts (a) and (b), assume that C is the center of the circle. Show all work.

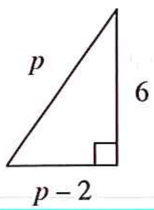
a.



b.



c.



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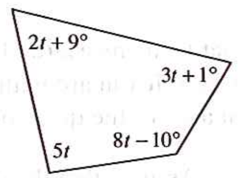


10-20. On graph paper, plot $\triangle ABC$ if $A(-1, -1)$, $B(1, 9)$ and $C(7, 5)$.

a. Find the midpoint of $\overline{AB}$ and label it D. Also find the midpoint of $\overline{BC}$ and label it E.

b. Find the length of the midsegment, $\overline{DE}$. Use it to predict the length of $\overline{AC}$.

c. Now find the length of $\overline{AC}$ and compare it to your prediction from (b).

10-21. $ABCDE$ is a regular pentagon inscribed in $\odot O$.

a. Draw a diagram of $ABCDE$ and $\odot O$ on your paper.

b. Find $m\measuredangle EDC$. How did you find your answer?

c. Find $m\measuredangle BOC$. What relationship did you use?

d. Find $m\overset{\frown}{EBC}$. Is there more than one way to do this?

10-22. **Multiple Choice:** Jill's car tires are spinning at a rate of 120 revolutions per minute. If her car tires' radii are each 14 inches, how far does she travel in 5 minutes?

a. 140π b. 8400π in c. 3360π in d. 16800π in

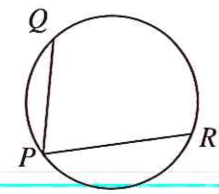
10.1.3 What more can I learn about circles?

Chords and Angles

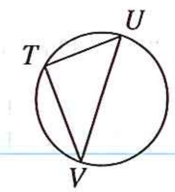
As you **investigate** more about the parts of a circle, look for connections you can make to other shapes and relationships you have studied so far.

10-23. WHAT IF IT'S A SEMICIRCLE?

What is the measure of an angle when it is inscribed in a **semicircle** (an arc with measure 180°)? Consider this as you answer the questions below.

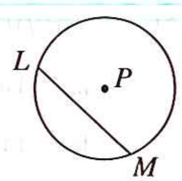


a. Assume that the diagram at right is not drawn to scale. If $m\overset{\frown}{QR} = 180°$, then what is $m\measuredangle P$? Why?



b. Since you have several tools to use with right triangles, the special relationship you found in part (a) can be useful. For example, $\overline{UV}$ is a diameter of the circle at right. If $TU = 6$ and $TV = 8$, what is the radius of the circle? What is its area?

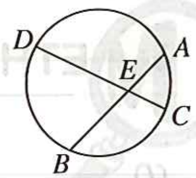
10-24. In Lesson 10.1.1, you learned that a chord is a line segment that has its endpoints on a circle. What geometric tools do you have that can help find the length of a chord?



a. **Examine** the diagram of chord $\overline{LM}$ in $\odot P$ at right. If the radius of $\odot P$ is 6 units and if $m\overset{\frown}{LM} = 150°$, find LM. Be ready to share your method with the class.

b. What if you know the length of a chord? How can you use it to reverse the process? Draw a diagram of a circle with radius 5 units and chord $\overline{AB}$ with length 6 units. Find $m\overset{\frown}{AB}$.

10-25. Timothy asks, "What if two chords intersect inside a circle? Can triangles help me learn something about these chords?" Copy his diagram at right in which chords $\overline{AB}$ and $\overline{CD}$ intersect at point E.



a. Timothy decided to create two triangles ($\triangle BED$ and $\triangle ACE$). Add line segments $\overline{BD}$ and $\overline{AC}$ to your diagram.

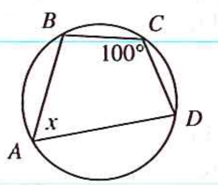
b. Compare $\measuredangle B$ and $\measuredangle C$. Which is bigger? How can you tell? Likewise, compare $\measuredangle D$ and $\measuredangle A$. Write down your observations.

c. How are $\triangle BED$ and $\triangle ACE$ related? **Justify** your answer.

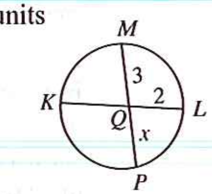
d. If $DE = 8$, $AE = 4$, and $EB = 6$, then what is EC? Show your work.

10-26. Use the relationships in the diagrams below to solve for the variable. **Justify** your solution.

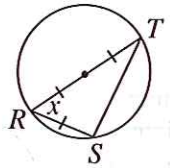
a.



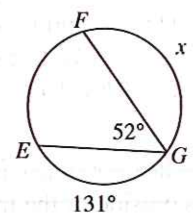
b. $\overline{KL}$ and $\overline{MP}$ intersect at Q and $KL = 8$ units



c. $\overline{RT}$ is a diameter



d.



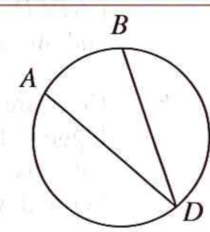
10-27. Look over your work from today. Consider all the geometric tools you applied to learn more about angles and chords of circles. In a Learning Log entry, describe which connections you made today. Title this entry "Connections with Circles" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Inscribed Angle Theorem

The measure of any inscribed angle is half of the measure of its intercepted arc. Likewise, any intercepted arc is twice the measure of any inscribed angles whose sides pass through the endpoints of the arc.

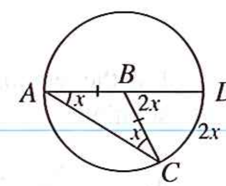


For example, in the diagram at right:

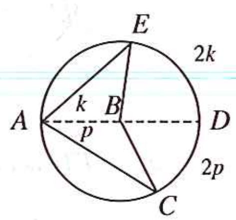
$$m\measuredangle ADB = \tfrac{1}{2} m\overset{\frown}{AB} \text{ and } m\overset{\frown}{AB} = 2m\measuredangle ADB.$$

Proof:

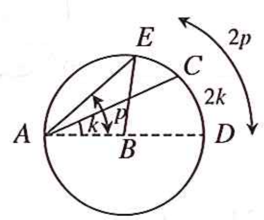
To prove this relationship, consider the relationship between an inscribed angle and its corresponding central angle. In problem 10-7, you used the isosceles triangle ΔABC to demonstrate that if one of the sides of the inscribed angle is a diameter of the circle, then the inscribed angle must be half of the measure of the corresponding central angle. Therefore, in the diagram at right, $m\measuredangle DAC = \frac{1}{2} m\overset{\frown}{DC}$.



But what if the center of the circle instead lies in the interior of an inscribed angle, such as $\measuredangle EAC$ shown at right? By constructing the diameter $\overline{AD}$, the work above shows that if $m\measuredangle EAD = k$ then $m\overset{\frown}{ED} = 2k$ and if $m\measuredangle DAC = p$, then $m\overset{\frown}{DC} = 2p$. Since $m\measuredangle EAC = k + p$, then $m\overset{\frown}{EC} = 2k + 2p = 2(k + p) = 2m\measuredangle EAC$.



The last possible case to consider is when the center lies outside of the inscribed angle, as shown at right. Again, constructing a diameter $\overline{AD}$ helps show that if $m\measuredangle CAD = k$ then $m\overset{\frown}{CD} = 2k$ and if $m\measuredangle EAD = p$, then $m\overset{\frown}{ED} = 2p$. Since $m\measuredangle EAC = p - k$, then $m\overset{\frown}{EC} = 2p - 2k = 2(p - k) = 2m\measuredangle EAC$.

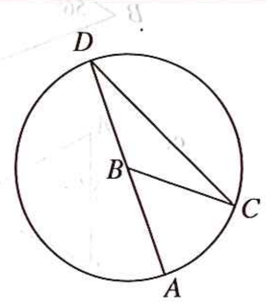


Therefore, an arc is always twice the measure of any inscribed angle that intercepts it.

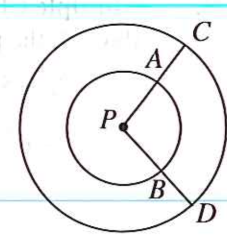
Review & Preview

10-28. Assume point B is the center of the circle below. Match each item in the left column with the best description for it in the right column.

a.	$\overline{AB}$	1.	inscribed angle
b.	$\overline{CD}$	2.	semicircle
c.	$\overset{\frown}{AD}$	3.	radius
d.	$\measuredangle CDA$	4.	minor arc
e.	$\overset{\frown}{AC}$	5.	central angle
f.	$\measuredangle ABC$	6.	chord



10-29. The figure at right shows two concentric circles.

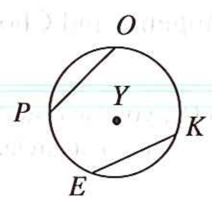


a. Which arc has greater **measure**: $\overset{\frown}{AB}$ or $\overset{\frown}{CD}$? Explain.

b. Which arc has greater **length**? Explain how you know.

c. If $m\measuredangle P = 60°$ and $PD = 14$, find the length of $\overset{\frown}{CD}$. Show all work.

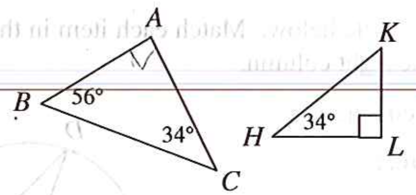
10-30. In $\odot Y$ at right, assume that $m\overset{\frown}{PO} = m\overset{\frown}{EK}$. Prove that $\overline{PO} \cong \overline{EK}$. Use the format of your choice.



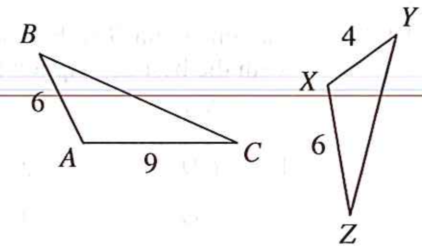
10-31. While working on the quadrilateral hotline, Jo Beth got this call: "I need help identifying the shape of the quadrilateral flowerbed in front of my apartment. Because a shrub covers one side, I can only see three sides of the flowerbed. However, of the three sides I can see, two are parallel and all three are congruent. What are the possible shapes of my flowerbed?" Help Jo Beth answer the caller's question.

10-32. For each pair of triangles below, decide if the triangles are similar or not and explain how you know. If the triangles are similar, complete the similarity statement $\triangle ABC \sim \triangle$_________.

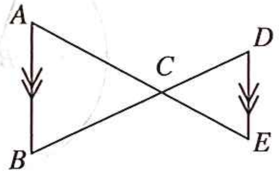
a.



b.



c.



10-33. **Multiple Choice:** Which equation below is perpendicular to $y = \frac{-2}{5}x - 7$ and passes through the point (4, –1)?

a. $2x - 5y = 13$ b. $2x + 5y = 3$ c. $5x - 2y = 22$

d. $5x + 2y = 18$ e. None of these

10.1.4 What's the relationship?

Tangents and Chords

So far, you have studied about the relationships that exist between angles and chords (line segments) in a circle. Today you will extend these ideas to include the study of lines and circles.

10-34. Consider all the ways a circle and a line can intersect. Can you **visualize** a line and a circle that intersect at exactly one point? What about a line that intersects a circle twice? On your paper, draw a diagram for each of the situations below, if possible. If it is not possible, explain why.

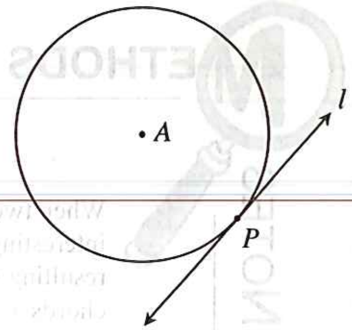
a. Draw a line and a circle that do not intersect.

b. Draw a line and a circle that intersect at exactly one point. When this happens, the line is called a **tangent**.

c. Draw a line and a circle that intersect at exactly two points. A line that intersects a circle twice is called a **secant**.

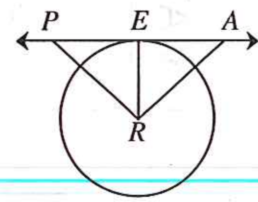
d. Draw a line and a circle that intersect three times.

10-35. A line that intersects a circle exactly once is called a **tangent**. What is the relationship of a tangent to a circle?



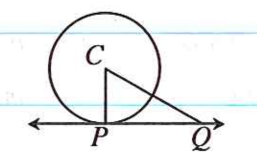
To **investigate**, carefully copy the diagram showing line l tangent to $\odot A$ at right onto tracing paper. Fold the tracing paper so that the crease is perpendicular to line l through point P. Your crease should pass through point A. What does this tell you about the tangent line?

10-36. In the figure at right, $\overleftrightarrow{PA}$ is tangent to $\odot R$ at E and $PE = EA$. Is $\Delta PER \cong \Delta AER$? If so, prove it. If not, show why not.

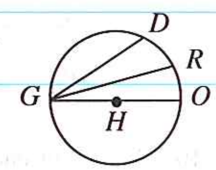


10-37. Use the relationships in the diagrams below to answer the following questions. Be sure to name what relationship(s) you used.

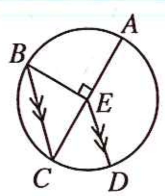
a. $\overleftrightarrow{PQ}$ is tangent to $\odot C$ at P. If $PQ = 5$ and $CQ = 6$, find CP and $m\measuredangle C$.



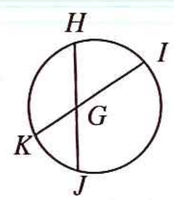
b. In $\odot H$, $m\overset{\frown}{DR} = 40°$ and $m\overset{\frown}{GOR} = 210°$. Find $m\overset{\frown}{GD}$, $m\overset{\frown}{OR}$, and $m\measuredangle RGO$.



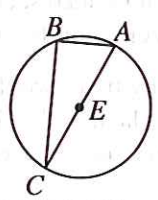
c. $\overline{AC}$ is a diameter of $\odot E$ and $\overline{BC} \parallel \overline{ED}$. Find the measure of $\overset{\frown}{CD}$.



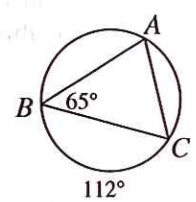
d. $\overline{HJ}$ and $\overline{IK}$ intersect at G. If $HG = 9$, $GJ = 8$, and $GK = 6$, find IG.



e. $\overline{AC}$ is a diameter of $\odot E$, the area of the circle is 289π un^2, and $AB = 16$ units. Find BC and $m\overset{\frown}{BC}$.



f. ΔABC is inscribed in the circle at right. Using the measurements provided in the diagram, find $m\overset{\frown}{AB}$.

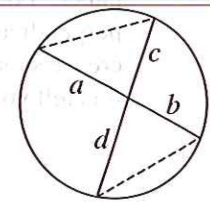


METHODS AND MEANINGS

MATH NOTES

Intersecting Chords

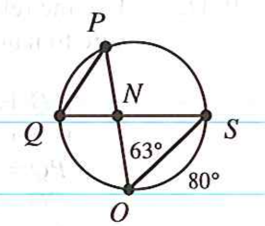
When two chords in a circle intersect, an interesting relationship between the lengths of the resulting segments occurs. If the ends of the chords are connected as shown in the diagram, similar triangles are formed (see problem 10-25). Then, since corresponding sides of similar triangles have a common ratio, $\frac{a}{d} = \frac{c}{b}$, and



$$ab = cd$$

Review & Preview

10-38. If $\overline{QS}$ is a diameter and $\overline{PO}$ is a chord of the circle at right, find the measure of the geometric parts listed below.

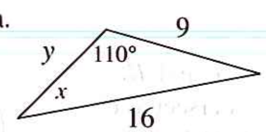


a. $m\measuredangle QSO$ b. $m\measuredangle QPO$ c. $m\measuredangle ONS$

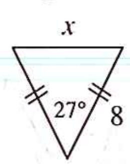
d. $m\overset{\frown}{PS}$ e. $m\overset{\frown}{PQ}$ f. $m\measuredangle PQN$

10-39. For each triangle below, solve for the given variables.

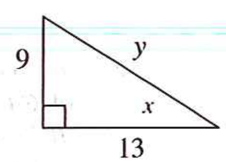
a.



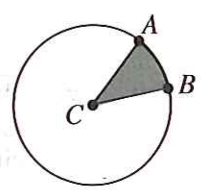
b.



c.



10-40. The spinner at right is designed so that if you randomly spin the spinner and land in the shaded sector, you win \$1,000,000. Unfortunately, if you land in the unshaded sector, you win nothing. Assume point C is the center of the spinner.

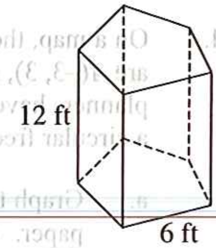


a. If $m\measuredangle ACB = 90°$, how many times would you have to spin to reasonably expect to land in the shaded sector at least once? How did you get your answer?

b. What if $m\measuredangle ACB = 1°$? How many times would you have to spin to reasonably expect to land in the shaded sector at least once?

c. Suppose P(winning \$1,000,000) $= \frac{1}{5}$ for each spin. What must $m\measuredangle ACB$ equal? Show how you got your answer.

10-41. Calculate the total surface area and volume of the prism at right. Assume that the base is a regular pentagon.



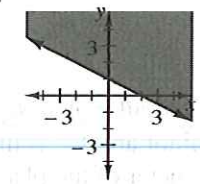
10-42. Quadrilateral $ABCD$ is graphed so that $A(3, 2)$, $B(1, 6)$, $C(5, 8)$, and $D(7, 4)$.

a. Graph $ABCD$ on graph paper. What shape is $ABCD$? **Justify** your answer.

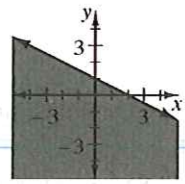
b. $ABCD$ is rotated 180° about the origin to create $A'B'C'D'$. Then $A'B'C'D'$ is reflected across the x-axis to form $A''B''C''D''$. Name the coordinates of C' and D''.

10-43. **Multiple Choice:** Which graph below represents $y > -\frac{1}{2}x + 1$?

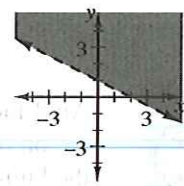
a.



b.



c.



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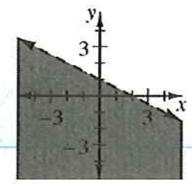


10.1.5 How can I solve it?

Problem Solving with Circles

Your work today is focused on consolidating your understanding of the relationships between angles, arcs, chords, and tangents in circles. As you work today, ask yourself the following focus questions:

Is there another way?

What's the relationship?

10-44. On a map, the coordinates of towns A, B, and C are $A(-3, 3)$, $B(5, 7)$, and $C(6, 0)$. City planners have decided to connect the towns with a circular freeway.

a. Graph the map of the towns on graph paper. Once the freeway is built, $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ will be chords of the circle. Use this information to find the center of the circle (called the **circumcenter** of the triangle because it is the center of the circle that circumscribes the triangle).

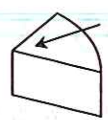
b. Use a compass to draw the circle connecting all three towns on your graph paper. Then find the radius of the circular freeway.

c. The city planners also intend to locate a new restaurant at the point that is an equal distance from all three towns. Where on the map should that restaurant be located? **Justify** your conclusion.

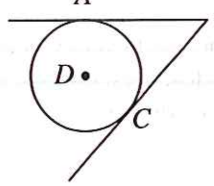
10-45. 

An 8-inch dinner knife is sitting on a circular plate so that its ends are on the edge of the plate. If the minor arc that is intercepted by the knife measures 120°, find the diameter of the plate. Show all work.

10-46. A cylindrical block of cheese has a 6-inch diameter and is 2 inches thick. After a party, only a sector remains that has a central angle of 45°. Find the volume of the cheese that remains. Show all work.



10-47. Dennis plans to place a circular hot tub in the corner of his backyard so that it is tangent to a fence on two sides, as shown in the diagram at right.



a. Prove that $\overline{AB} \cong \overline{CB}$.

b. The switch to turn on the air jets is located at point B. If the diameter of the hot tub is 6 feet and $AB = 4$ feet, how long does his arm need to be for him to reach the switch from the edge of the tub? (Assume that Dennis will be in the tub when he turns the air jets on and that the switch is level with the edge of the hot tub.)

MATH NOTES

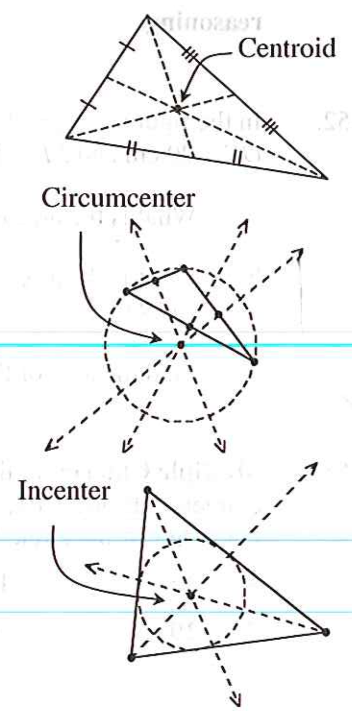
METHODS AND MEANINGS

Points of Concurrency

In Chapter 9, you learned that the **centroid** of a triangle is the intersection of the three medians of the triangle, as shown at right. When three lines intersect at a single point, that point is called a **point of concurrency**.

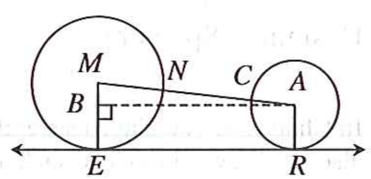
Another point of concurrency, located where the perpendicular bisectors of each side of a triangle meet, is called the **circumcenter**. This point is the center of the circle that circumscribes the triangle. See the example at right. Note that the point that represents the location of the restaurant in problem 10-44 is a circumcenter.

The point where the three angle bisectors of a triangle meet is called the **incenter**. It is the center of the circle that is inscribed in a triangle. See the example at right.

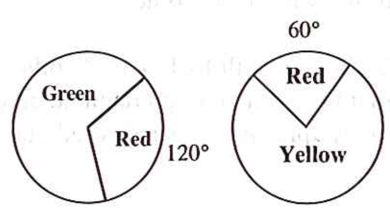


Review & Preview

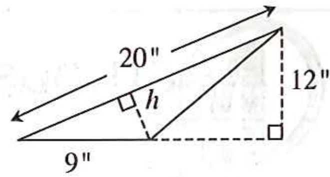
10-48. In the diagram at right, $\odot M$ has radius 14 feet and $\odot A$ has radius 8 feet. $\overleftrightarrow{ER}$ is tangent to both $\odot M$ and $\odot A$. If $NC = 17$ feet, find ER.



10-49. Phinneus is going to spin both spinners at right once each. If he lands on the same color twice, he will go to tonight's dance. Otherwise, he will stay home. What is the probability that Phinneus will attend the dance?

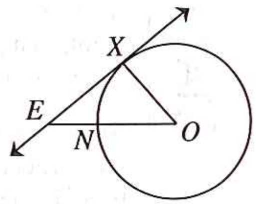


10-50. In the figure at left, find the interior height (h) of the obtuse triangle. Show all work.



10-51. A cylinder with volume 500π cm^3 is similar to a smaller cylinder. If the scale factor is $\frac{1}{5}$, what is the volume of the smaller cylinder? Explain your **reasoning**.

10-52. In the figure at right, $\overrightarrow{EX}$ is tangent to $\odot O$ at point X. $OE = 20$ cm and $XE = 15$ cm.

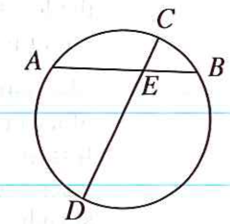


a. What is the area of the circle?

b. What is the area of the sector bounded by $\overline{OX}$ and $\overline{ON}$?

c. Find the area of the region bounded by $\overline{XE}$, $\overline{NE}$, and $\overset{\frown}{NX}$.

10-53. **Multiple Choice:** In the circle at right, $\overline{CD}$ is a diameter. If $AE = 10$, $CE = 4$, and $AB = 16$, what is the radius of the circle?



a. 15 b. 16 c. 18

d. 19 e. None of these

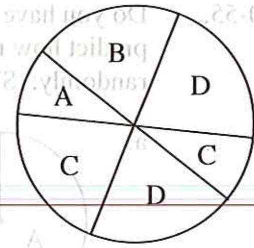
10.2.1 What's the probability?

Designing Spinners

In Chapter 4, you played several games and learned how to determine if a game is fair. You also used several models, such as tree diagrams and area models, to represent the probabilities of the various outcomes.

Today you will look at probability from a new perspective. What if you want to design a game that has particular, predictable outcomes? How can you design a spinner so that the result of many spins matches a desired outcome?

10-54. To review your understanding of probability, play the game as described below 50 times to determine a winner. You will need a paperclip and a Lesson 10.2.1 Resource Page. Then answer the questions that follow.



- Choose a different member of your team to be responsible for the following tasks:
    - Keeping track of time
    - Spinning a paperclip
    - Recording the result of each spin
    - Tallying the number of spins

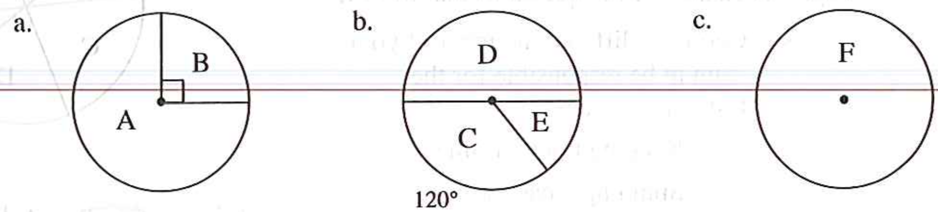
- Assign each team member a region (or pair of regions) alphabetically by first name. That is, the team member whose name is first alphabetically will be assigned region A, the next person region B, and so on. You may want to color the spinner so that the four regions are different colors.
- Place the resource page on a flat, level surface. Then place a paperclip at the center of the spinner, hold it in place with the point of a pen or pencil, and spin it 50 times. Each time the spinner lands on a player's letter, that player gets a point. The person with the most points wins.

a. Which player (A, B, C, or D) won the game? Is the result what you expected? Why or why not?

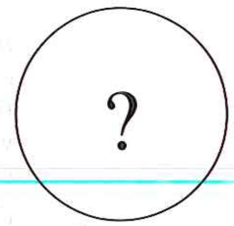
b. Use a protractor to measure the central angles for each region. What is the probability that the spinner will land in each region?

c. Calculate the percentage of the points scored for each region based on your results from playing the game and compare them to the probabilities you calculated in part (b). How closely did the results from spinning match the actual probabilities? Explain any large differences.

10-55. Do you have to collect data to predict the outcomes? For each spinner below, predict how many times a spinner would land in each region if you spun it 60 times randomly. Show all work.



10-56. Your teacher will now spin a hidden spinner. Your team's task is to use the results to predict what the spinner looks like. Then, using the blank spinner on the resource page, use a protractor to design the spinner. As your teacher gives you the result of each spin, take careful notes! Your accuracy depends on it.



10-57. For a school fair, Donny is going to design a spinner with red, white, and blue regions. Since he has a certain proportion of three types of prizes, he wants the P(red) = 40% and P(white) = 10%.

a. If the spinner only has red, white, and blue regions, then what is P(blue)? Explain how you know.

b. Find the central angles of this spinner if it has only three sections. Then draw a sketch of the spinner. Be sure to label the regions accurately.

c. Is there a different spinner that has the same probabilities? If so, sketch another spinner that has the same probabilities. If not, explain why there is no other spinner with the same probabilities.

METHODS AND MEANINGS

MATH NOTES

Probability

While the information below was provided in Chapter 1, it is reprinted here for your reference during this section.

Probability is a measure of the likelihood that an event will occur at random. It is expressed using numbers with values that range from 0 to 1, or from 0% to 100%. For example, an event that has no chance of happening is said to have a probability of 0 or 0%. An event that is certain to happen is said to have a probability of 1 or 100%. Events that "might happen" have values somewhere between 0 and 1 or between 0% and 100%.

The probability of an event happening is written as the ratio of the number of ways that the desired outcome can occur to the total number of possible outcomes (assuming that each possible outcome is equally likely).

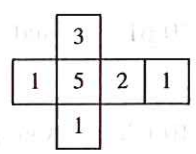
$$P(\text{event}) = \frac{\text{Number of Desired Outcomes}}{\text{Total Possible Outcomes}}$$

For example, on a standard die, P(5) means the probability of rolling a 5. To calculate the probability, first determine how many possible outcomes exist. Since a die has six different numbered sides, the number of possible outcomes is 6. Of the six sides, only one of the sides has a 5 on it. Since the die has an equal chance of landing on any of its six sides, the probability is written:

$$P(5) = \frac{\text{1 side with the number five}}{\text{6 total sides}} = \frac{1}{6} \text{ or } 0.1\overline{6} \text{ or approximately } 16.7\%$$

Review & Preview

10-58. When the net at right is folded, it creates a die with values as shown.

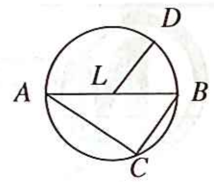


a. If the die is rolled randomly, what is P(even)? P(1)?

b. If the die is rolled randomly 60 times, how many times would you expect an odd number to surface? Explain how you know.

c. Now create your own net so that the resulting die has P(even) = $\frac{1}{3}$, P(3) = 0, and P(a number less than 5) = 1.

10-59. In the diagram at right, $\overline{AB}$ is a diameter of $\odot L$. If $BC = 5$ and $AC = 12$, use the relationships shown in the diagram to solve for the quantities listed below.



a. AB

b. radius of $\odot L$

c. $m\measuredangle ABC$

d. $m\overset{\frown}{AC}$

10-60. When Erica and Ken explored a cave, they each found a gold nugget. Erica's nugget is similar to Ken's nugget. They measured the length of two matching parts of the nuggets and found that Erica's nugget is five times longer than Ken's. When they took their nuggets to the metallurgist to be analyzed, they learned that it would cost \$30 to have the surface area and weight of the smaller nugget calculated, and \$150 to have the same analysis done on the larger nugget.

"I won't have that kind of money until I sell my nugget, and then I won't need it analyzed!" Erica says.

"Wait, Erica. Don't worry. I'm pretty sure we can get all the information we need for only \$30."

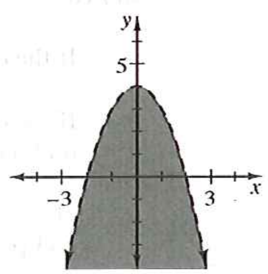
a. Explain how they can get all the information they need for \$30.

b. If Ken's nugget has a surface area of 110 cm^2, what is the surface area of Erica's nugget?

c. If Ken's nugget weighs 56 g (about 1.8 oz), what is the weight of Erica's nugget?

10-61. Find x if the angles of a quadrilateral are $2x$, $3x$, $4x$, and $5x$.

10-62. A graph of an inequality is shown at right. Decide if each of the points (x, y) listed below would make the inequality true or not. For each point, explain how you know.



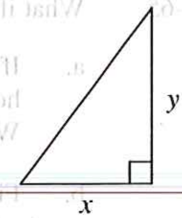
a. (1, 1)

b. (–3, 2)

c. (–2, 0)

d. (0, –2)

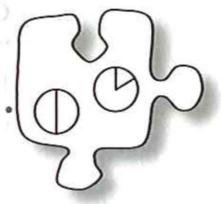
10-63. **Multiple Choice:** Which expression below represents the length of the hypotenuse of the triangle at right?



a. $\frac{y}{x}$ b. $\sqrt{x^2 + y^2}$ c. $x + y$

d. $\sqrt{y^2 - x^2}$ e. None of these

10.2.2 What should I expect?



Expected Value

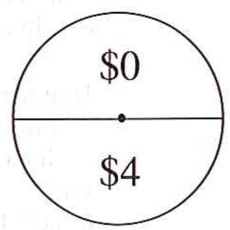
Around the world, different cultures have developed creative forms of games of chance. For example, native Hawaiians play a game called Konane, which uses markers and a board and is similar to checkers. Native Americans play a game called To-pe-di, in which tossed sticks determine how many points a player receives.

When designing a game of chance, much attention must be given to make sure the game is fair. If the game is not fair, or if there is not a reasonable chance that someone can win, no one will play the game. In addition, if the game has prizes involved, care needs to be taken so that prizes will be distributed based on availability. In other words, if you only want to give away one grand prize, you want to make sure the game is not set up so that 10 people win the grand prize!

Today your team will analyze different games to learn about **expected value**, which helps to predict the result of a game of chance.

10-64. TAKE A SPIN

Consider the following game: After you spin the wheel at right, you win the amount spun.

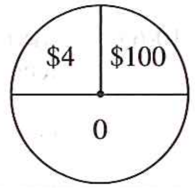


a. If you play the game 10 times, how much money would you expect to win? What if you played the game 30 times? 100 times? Explain your process.

b. What if you played the game n times? Write a rule that governs how much money one can expect to win after playing the game n times.

c. If you were to play only once, what should you expect to earn according to your rule in part (b)? Is it actually possible to win that amount? Explain why or why not.

10-65. What if the spinner looks like the one at right instead?

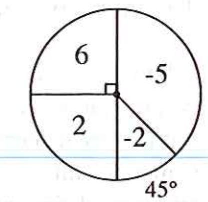


a. If you win the amount that comes up on each spin, how much would you expect to win after 4 spins? What about after 100 spins?

b. Find this spinner's **expected value**. That is, what is the expected amount you will win for each spin? Be ready to **justify** your answer.

c. Gustavo describes his thinking this way: "Half the time, I'll earn nothing. One-fourth the time, I'll earn $4 and the other one-fourth of the time I'll earn $100. So, for one spin, I can calculate $\frac{1}{2}(0)+\frac{1}{4}(\$4)+\frac{1}{4}(\$100)$. Calculate Gustavo's expression. Does his result match your result from part (b)?

10-66. Jesse has created the spinner at right. This time, if you land on a positive number, you win that amount of money. However, if you land on a negative number, you lose that amount of money! Want to try it?

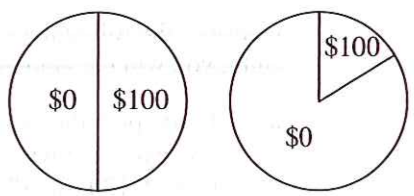


a. Before analyzing the spinner, predict whether a person would win money or lose money after many spins.

b. Now calculate the actual expected value. How does the result compare to your estimate from part (a)?

10-67. Finding an expected value is similar to finding a **weighted average** because it takes into account the different probabilities for each possible outcome. For example, in problem 10-66, –5 is expected to result three times as often as –2. Therefore, in averaging these values, –5 must be weighted three times for every –2. However, the 2 and the 6 have equal probabilities, so they must be averaged using the same weighting.

To understand the effect of weighted averaging, consider the two spinners at right. Each has two sections, labeled $100 and $0. Which spinner has the greater expected value? How can you tell?

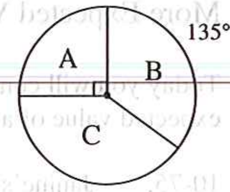


Spinner A Spinner B

10-68. In your Learning Log, explain what "expected value" means. What does it find? When is it useful? Be sure to include an example. Title this entry "Expected Value" and include today's date.

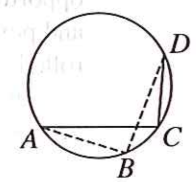
Review & Preview

10-69. The spinner at right has three regions: A, B, and C. If it is spun 80 times, how many times would you expect each region to result? Show your work.

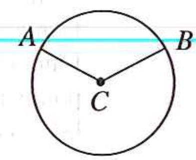


10-70. Review what you know about the angles and arc of circles below.

a. A circle is divided into nine congruent sectors. What is the measure of each central angle?

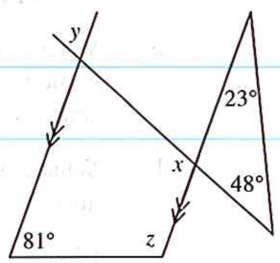


b. In the diagram at right, find $m\overset{\frown}{AD}$ and $m\measuredangle C$ if $m\measuredangle B = 97°$.



c. In $\odot C$ at right, $m\measuredangle ACB = 125°$ and $r = 8$ inches. Find $m\overset{\frown}{AB}$ and the length of $\overset{\frown}{AB}$. Then find the area of the smaller sector.

10-71. **Examine** the diagram at right. Use the given geometric relationships to solve for x, y, and z. Be sure to **justify** your work by stating the geometric relationship and applicable theorem.

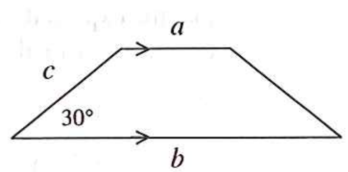


10-72. Solve each equation below for x. Check your work.

a. $\frac{x}{2} = 17$ b. $\frac{x}{4} = \frac{1}{3}$ c. $\frac{x+6}{2} + 2 = \frac{5}{2}$ d. $\frac{4}{x} = \frac{5}{8}$

10-73. Mrs. Cassidy solved the problem $(w-3)(w+5) = 9$ and got $w = 3$ and $w = -5$. Is she correct? If so, show how you know. If not, show how you know and find the correct solution.

10-74. **Multiple Choice:** Which expression represents the area of the trapezoid at right?



a. $\frac{c(a+b)}{4}$ b. $\frac{c(a+b)}{2}$

c. $\frac{bc}{2}$ d. $\frac{a+b+c}{2}$

e. None of these

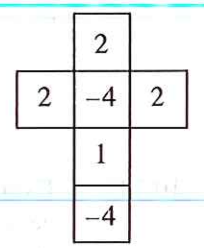
10.2.3 What can I expect?

More Expected Value

Today you will continue to focus on mathematical expectation to look for new ways to find the expected value of a game of chance.

10-75. Janine's teacher has presented her with this opportunity to raise her grade: She can roll a die and possibly gain points. If a positive number is rolled, Janine gains the number of points indicated on the die. However, if a negative roll occurs, then Janine loses that many points.

Janine does not know what to do! The die, formed when the net at right is folded, offers four sides that will increase her number of points and only two sides that will decrease her grade. She needs your help to determine if this die is **fair**.



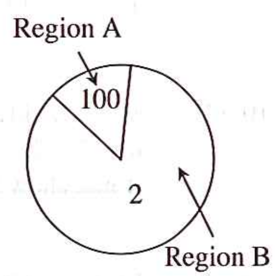
a. What are the qualities of a fair game? How can you tell if a game is fair? Discuss this with your team and be ready to share your ideas with the class.

b. What is the expected value of one roll of this die? Show how you got your answer. Is this die fair?

c. Change only one side of the die in order to make the expected value 0.

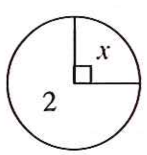
d. What does it mean if a die or spinner has an expected value of 0? Elaborate.

10-76. **Examine** the spinner at right. If the central angle of Region A is 7°, find the expected value of one spin **two different ways**. Be ready to share your methods with the class.

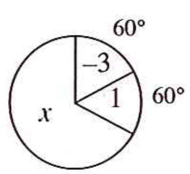


10-77. Now reverse the process. For each spinner below, find x so that the expected value of the spinner is 3. Be prepared to explain your method to the class.

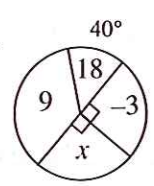
a.



b.



c.



10-78. Revisit your work from part (c) of problem 10-77.

a. To solve for x, Julia wrote the equation:

$$\frac{140}{360}(9)+\frac{40}{360}(18)+\frac{90}{360}(-3)+\frac{90}{360}x=3$$

Explain how her equation works.

b. She's not sure how to solve her equation. She'd like to rewrite the equation so that it does not have any fractions. What could she do to both sides of the equation to eliminate the fractions? Rewrite her expression and solve for x.

c. Review the equation-rewriting techniques you learned in algebra by solving the equations below. You may benefit by reading the Math Notes box for this lesson.

(1) $\frac{4}{3}+\frac{x}{7}=5$ (2) $\frac{1}{2}(5x-3)+\frac{7}{4}=\frac{x}{2}$

10-79. If you have not done so already, write an equation and solve for x for parts (a) and (b) of problem 10-75. Did your answers match those you found in problem 10-75?

10-80. During this lesson, you examined two ways to find the expected value of a game of chance. What do these methods have in common? How are they different? Explain any connections you can find. Title this entry "Method for Finding Expected Value" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Solving Equations by Rewriting (Fraction Busters)

Two equations are **equivalent** if they have the same solution(s). There are many ways to change one equation into a different, equivalent equation. If an equation contains a fraction, it may be easier to solve if it is first rewritten so that it has no fractions. This process is sometimes referred to as **fraction busters.**

Example: Solve for x: $\frac{x}{3}+\frac{x}{5}=2$

$$\frac{x}{3}+\frac{x}{5}=2$$

The complicating issue in this problem is dealing with the fractions. We could add them by first writing them in terms of a common denominator, but there is an easier way.

The lowest common denominator of $\frac{x}{3}$ and $\frac{x}{5}$ is 15.

$$15\cdot\left(\frac{x}{3}+\frac{x}{5}\right)=15\cdot 2$$

There is no need to use the time-consuming process of adding the fractions if we can "eliminate" the denominators. To do this, we will need to find a common denominator of all fractions and multiply both sides of the equation by that common denominator. In this case, the lowest common denominator is 15, so we multiply both sides of the equation by 15. Be sure to multiply every term on each side of the equation!

$$15\cdot\frac{x}{3}+15\cdot\frac{x}{5}=15\cdot 2$$

$$5x+3x=30$$

$$8x=30$$

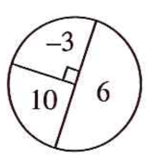
$$x=\frac{30}{8}=\frac{15}{4}=3.75$$

The result is an equivalent equation without fractions! Now the equation looks like many you have seen before, and it can be solved using standard methods, as shown above. Note: If you cannot determine the common denominator, then multiply both sides by the product of the denominators.

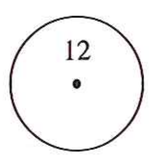
Review & Preview

10-81. For each spinner below, find the expected value of one spin.

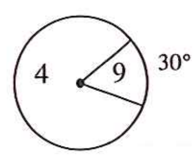
a.



b.



c.

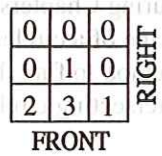


10-82. For each equation below, write an equivalent equation that contains no fractions. Then solve your equation for x and check your answer.

a. $\frac{2}{3}x - \frac{1}{4} = \frac{x}{2}$

b. $\frac{7x}{1000} + \frac{2}{500} = \frac{11}{100}$

10-83. The mat plan for a three-dimensional solid is shown at right.

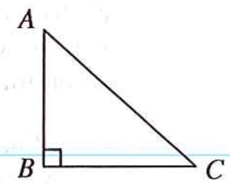


a. On graph paper, draw all of views of this solid. (There are six views.) Compare the views. Are any the same?

b. Find the volume and surface area of the solid. Explain your method.

c. Do the views you drew in part (a) help calculate volume or surface area? Explain.

10-84. For the triangle at right, find each trigonometric ratio below. The first one is done for you.



a. $\tan C = \frac{AB}{BC}$ b. $\sin C$ c. $\tan A$

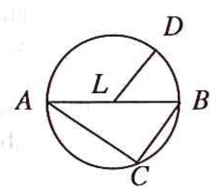
d. $\cos C$ e. $\cos A$ f. $\sin A$

10-85. Review circle relationships as you answer the questions below.

a. On your paper, draw a diagram of $\odot B$ with arc $\overset{\frown}{AC}$. If $m\overset{\frown}{AC} = 80°$ and the radius of $\odot B$ is 10, find the length of chord $\overline{AC}$.

b. Now draw a diagram of a circle with two chords, $\overline{EF}$ and $\overline{GH}$, that intersect at point K. If $EF = 15$, $EK = 6$, and $HK = 3$, what is GK?

10-86. **Multiple Choice: Examine $\odot L$ at right.** Which of the mathematical statements below is not necessarily true?



a. $LD = AL$ b. $m\measuredangle DLB = m\overset{\frown}{DB}$

c. $\overline{LD} \parallel \overline{CB}$ d. $m\overset{\frown}{BC} = 2m\measuredangle BAC$

e. $2AL = AB$

10.3.1 What's the equation?

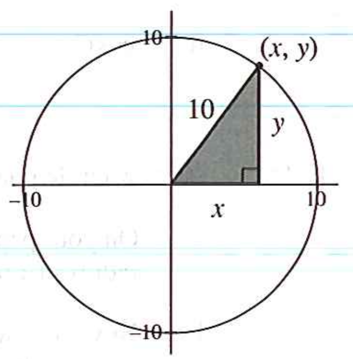
The Equation of a Circle

During Chapters 7 through 10, you studied circles *geometrically*, that is, based on the geometric shape of a circle. For example, the relationship of circles and polygons helped you develop a method to find the area and circumference of a circle, while geometric relationships of intersecting circles helped you develop constructions of shapes such as a rhombus and a kite.

However, how can circles be represented *algebraically* or *graphically*? And how can you use these representations to learn more about circles? Today your team will develop the equation of a circle.

10-87. EQUATION OF A CIRCLE

We have equations for lines and parabolas, but what type of equation could represent a circle? On a piece of graph paper, draw a set of $x \rightarrow y$ axes. Then use a compass to construct a circle with radius 10 units centered at the origin (0, 0).



a. What do all of the points on this circle have in common? That is, what is true about each point on the circle?

b. Find all of the points on the circle where $x = 6$. For each point, what is the y-value? Use a right triangle (like the one shown at right) to **justify** your answer.

c. What if $x = 3$? For each point on the circle where $x = 3$, find the corresponding y-value. Use a right triangle to **justify** your answer.

d. Mia picked a random point on the circle and labeled it (x, y). Unfortunately, she does not know the value of x or y! Help her write an equation that relates x, y, and 10 based on her diagram above.

e. Does your equation from part (d) work for the points (10, 0) and (0, 10)? What about (–8, –6)? Explain.

10-88. In problem 10-87, you wrote an equation of a circle with radius 10 and center at (0, 0).

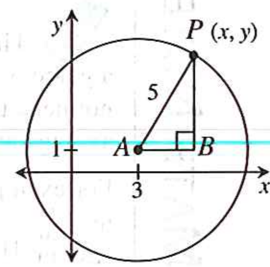
a. What if the radius were instead 4 units long? Discuss this with your team and write an equation of a circle with center (0, 0) and radius 4.

b. Write the equation of a circle centered at (0, 0) with radius r.

c. On graph paper, sketch the graph of $x^2 + y^2 = 36$. Can you graph it without a table? Explain your method.

d. Describe the graph of the circle $x^2 + y^2 = 0$.

10-89. What if the center of the circle is not at (0, 0)? On graph paper, construct a circle with a center $A(3, 1)$ and radius 5 units.



a. On the diagram at right, point P represents a point on the circle with no special characteristics. Add a point P to your diagram and then draw a right triangle like ΔABC in the circle at right.

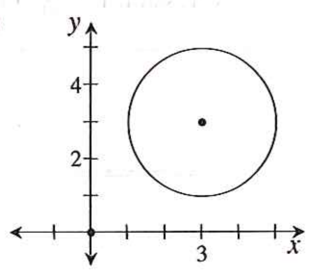
b. What is the length of $\overline{PB}$? Write an expression to represent this length. Likewise, what is the length of $\overline{AB}$?

c. Use your expressions for AB and BP, along with the fact that the radius of the circle is 5, to write an equation for this circle. (Note: You do not need to worry about multiplying any binomials.)

d. Find the equation of each circle represented below.

(1) The circle with center (2, 7) and radius 1.

(2)



(3) The circle for which (6, 0) and (–6, 0) are the endpoints of a diameter.

10-90. On graph paper, graph and shade the solutions for the inequalities below. Then find the area of each shaded region.

a. $x^2 + y^2 \le 49$

b. $(x-3)^2 + (y-2)^2 \le 4$

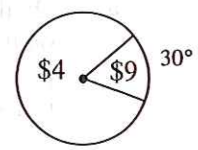
10-91. In a Learning Log entry, describe what you learned in this lesson about the equation of a circle. What connections did you make to other areas of algebra or geometry? Be sure to include an example of how to find the equation of a circle given its center and radius. Title this entry "Equation of a Circle" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Expected Value

The amount you would expect to win (or lose) per game after playing a game of chance many times is called the **expected value.** This value does not need to be a possible outcome of a single game, but instead reflects an average amount that will be won or lost per game.

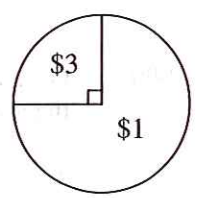


For example, the "$9" portion of the spinner at right makes up $\frac{30°}{360°} = \frac{1}{12}$ of the spinner, while the "$4" portion is the rest, or $\frac{11}{12}$, of the spinner. If the spinner was spun 12 times, probability predicts that it would land on "$9" once and "$4" eleven times. Therefore, someone spinning 12 times would expect to receive $1(\$9) + 11(\$4) = \$53$. On average, each spin would earn an expected value of $\frac{\$53}{12 \text{ spins}} \approx \$4.42$ per spin. You could use this value to predict the result for any number of spins. For example, if you play 30 times, you would expect to win 30($4.42) = $132.50.

Another way to calculate expected value involves the probability of each possible outcome. Since "$9" is expected $\frac{1}{12}$ of the time, and "$4" is expected $\frac{11}{12}$ of the time, then the expected value can be calculated with the expression $(\$9)(\frac{1}{12}) + (\$4)(\frac{11}{12}) = \frac{\$53}{12} \approx \$4.42$.

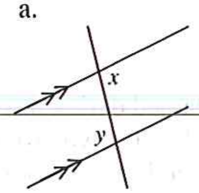
Review & Preview

10-92. Jamika designed a game that allows some people to win money and others to lose money, but overall Jamika will neither win nor lose money. Each player will spin the spinner at right and will win the amount of money shown in the result. How much should each player pay to spin the spinner? Explain your **reasoning**.

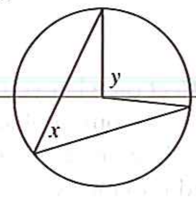


10-93. For each diagram below, write an equation to represent the relationship between x and y.

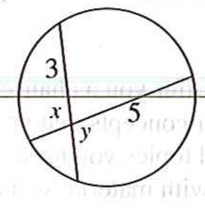
a.



b.



c.



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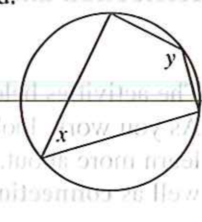
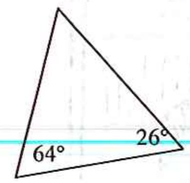
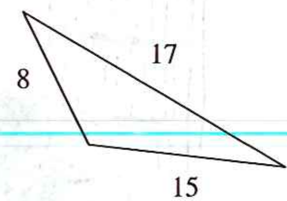


10-94. For each triangle below, use the information in the diagram to decide if it is a right triangle. **Justify** each conclusion. Assume the diagrams are not drawn to scale.

a.



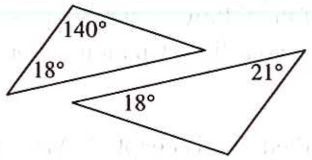
b.



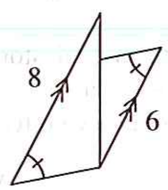
10-95. A cube (a rectangular prism in which the length, width, and depth are equal) has an edge length of 16 units. Draw a diagram of the cube and find its volume and surface area.

10-96. For each pair of triangles below, decide if the pair is similar, congruent or neither. **Justify** your conclusion (such as with a similarity or congruence property like AA ~ or SAS ~ or the reasons why the triangles cannot be similar or congruent). Assume that the diagrams are not drawn to scale.

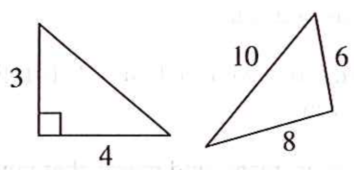
a.



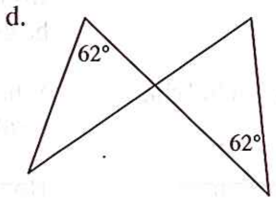
b.



c.



d.



10-97. **Multiple Choice:** $\triangle ABC$ is a right triangle and is graphed on coordinate axes. If $m\measuredangle B = 90°$ and if the slope of $\overline{AB}$ is $-\frac{4}{5}$, what is the slope of $\overline{BC}$?

a. $\frac{4}{5}$ b. $\frac{5}{4}$ c. $-\frac{5}{4}$ d. $-\frac{4}{5}$

e. Cannot be determined

Chapter 10 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following three topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Problem Solving: What did you do to solve problems? What different **strategies** did you use?

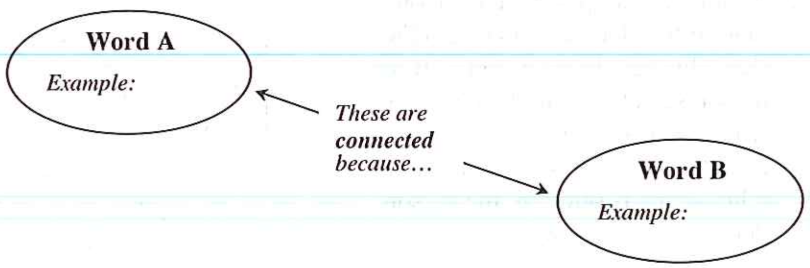
Connections: How are the topics, ideas, and words that you learned in previous courses are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

arc length	**arc measure**	center
central angle	**chord**	circle
circumference	**circumscribed**	diameter
expected value	**fair**	inscribed
major arc	measure	**minor arc**
perpendicular	probability	radius
secant	**semicircle**	similar
tangent	$x^2 + y^2 = r^2$	**weighted average**

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

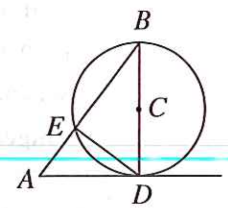
This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 10-98. Copy the diagram at right onto your paper. Assume $\overrightarrow{AD}$ is tangent to $\odot C$ at D.

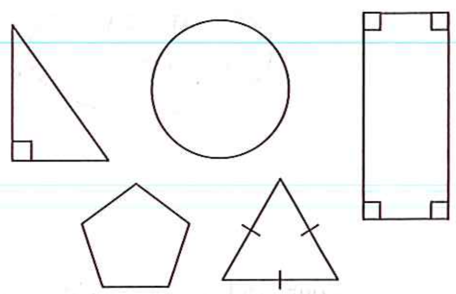


a. If $AD = 9$ and $AB = 15$, what is the area of $\odot C$?

b. If the radius of $\odot C$ is 10 and the $m\overparen{ED} = 30°$, what is $m\overparen{EB}$? AD?

c. If $m\overparen{EB} = 86°$ and if $BC = 7$, find EB.

CL 10-99. A game is set up so that a person randomly selects a shape from the shape bucket shown at right. If the person selects a triangle, he or she wins \$5. If the person selects a circle, he or she loses \$3. If any other shape is selected, the person does not win or lose money.



a. If a person plays 100 times, how much money should the person expect to win or lose?

b. What is the expected value of this game?

CL 10-100. Consider the solid represented by the mat plan at right.

3	1	0
0	1	1
0	2	3

Front (bottom edge of mat plan); Right (right edge)

a. Draw the front, right, and top view of this solid on graph paper.

b. Find the volume and surface area of this solid.

c. If this solid is enlarged by a linear scale factor of 3.5, what will be its new volume and surface area?

CL 10-101. Consider the descriptions of the different shapes below. Which shapes <u>must</u> be a parallelogram? If a shape does not have to be a parallelogram, what other shapes could it be?

a. A quadrilateral with two pairs of parallel sides.

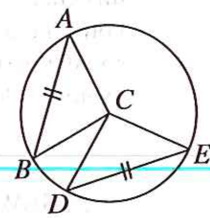
b. A quadrilateral with two pairs of congruent sides.

c. A quadrilateral with one pair of sides that is both congruent and parallel.

d. A quadrilateral with two diagonals that are perpendicular.

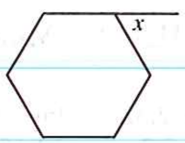
e. A quadrilateral with four congruent sides.

CL 10-102. In $\odot C$ at right, $\overline{AB} \cong \overline{DE}$. Prove that $\measuredangle ACB \cong \measuredangle DCE$.

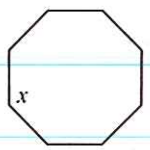


CL 10-103. Find the measure of x in each diagram below. Assume each polygon is regular.

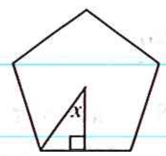
a.



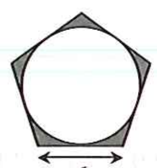
b.



c.



CL 10-104. The circle at right is inscribed in a regular pentagon. Find the area of the shaded region.



CL 10-105. On graph paper, graph the equation $x^2 + y^2 = 100$.

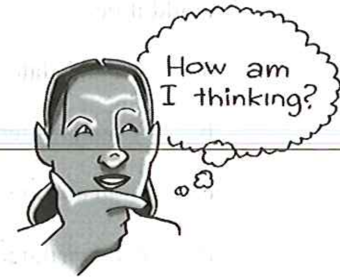
a. What are the values of x when $y = 8$? Show how you know.

b. What are the values of y when $x = 11$? Show how you know.

CL 10-106. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

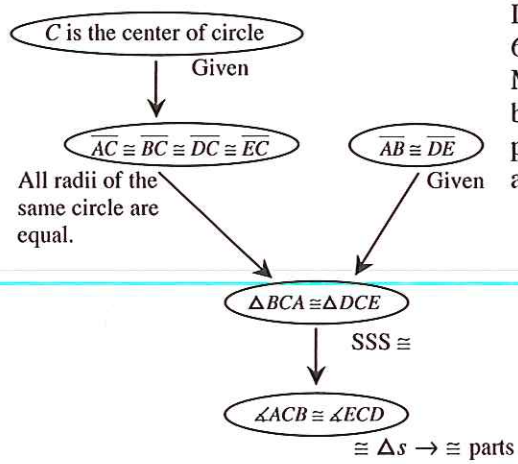
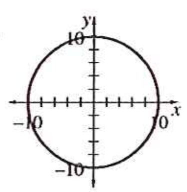
This course focuses on five different **Ways of Thinking**: investigating, examining, choosing a strategy/tool, visualizing, and reasoning and justifying. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!



Choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 10-98.	a. 36π b. $m\overset{\frown}{EB} = 150°$, $AD = 20\tan 15° \approx 5.36$ c. ≈ 9.55	Lessons 2.3.3, 5.1.2, 8.3.2, 10.1.1, 10.1.2, 10.1.3 Math Notes boxes	Problems 10-7, 10-15, 10-17, 10-23, 10-24, 10-26, 10-37, 10-38, 10-52, 10-59, 10-70, 10-86, 10-93
CL 10-99.	a. $140 should be won after 100 games b. $1.40 should be won per game	Lesson 10.3.1 Math Notes box	Problems 10-55, 10-64, 10-66, 10-67, 10-69, 10-75, 10-76, 10-77, 10-81, 10-92
CL 10-100.	a.  Front Right Top b. V = 11 un^3, SA = 42 un^2 c. V = $11(3.5)^3 = 471.625$ un^3, $42(3.5)^2 = 514.5$ un^2	Lessons 9.1.3 an 9.1.5 Math Notes boxes, problems 9-1, 9-2, and 9-14	Problems 9-3, 9-4, 9-5, 9-7, 9-13, 9-15, 9-16, 9-20, 9-25, 9-35, 9-37, 9-39, 9-40, 9-45, 9-46, 9-47, 9-48, 9-63, 9-73, 9-79

Problem	Solution	Need Help?	More Practice
CL 10-101.	Must be a parallelogram: (a), (c), and (e) (b) could be a kite or an isosceles trapezoid (d) could be a kite	Lessons 7.2.3, 8.1.2, and 9.2.2 Math Notes boxes, problem 7-47	Problems 7-101, 7-106, 7-116, 7-117, 7-121, 8-11, 8-56, 8-101, 9-50, 10-31
CL 10-102.		Lessons 3.2.4, 6.1.3, and 7.1.2 Math Notes boxes, problems 7-56 and 7-79	Problems 7-61, 7-78, 7-85, 7-87, 7-96, 7-104, 7-105, 8-20, 8-28, 8-58, 8-79, 8-88, 9-12, 9-31, 9-71, 9-80, 10-30, 10-36, 10-47
CL 10-103.	a. 60° b. 135° c. 36°	Lessons 7.1.4, 8.1.1, and 8.1.4 Math Notes boxes	Problems 8-25, 8-29, 8-33, 8-34, 8-35, 8-40, 8-49, 8-55, 8-56, 8-87, 8-99, 8-109, 9-21, 9-50, 9-72, 9-83, 9-91, 10-61
CL 10-104.	Area of shaded region ≈ 8.37 un^2	Lessons 8.1.4, 8.3.1, and 8.3.2 Math Notes boxes	Problems 8-45, 8-47, 8-48, 8-67, 8-85, 8-103, 9-10
CL 10-105.	See graph at right.  a. $x = 6$ or -6 because $6^2 + 8^2 = 100$ b. y does not exist when $x = 10$ because it is off the graph.	Problem 10-87	Problems 10-88, 10-89, 10-90