A statistician claims that the average score on a standardized test for psychology students is greater that the score for mathematics students. To test this outlandish claim a test was given to 50 students in each group, the results are shown here. Is there enough evidence to support the statistician's claim at $\alpha = 0.01$?

Psychology	Mathematics
$x_1 = 118$	$x_2 = 115$
$s_1 = 15$	$s_2 = 15$
$n_1 = 50$	$n_2 = 50$

Now we have two samples and no population!
What are we going to do?

Which brings us to....

Chapter 10 – Testing the Difference between two samples

1. Testing the difference between two means
2. Testing the difference between two variances
3. Testing the difference between proportions

Testing the difference between two means – Case 1
σ known and σ unknown

Conditions:

1. **Sample sizes ≥ 30**.
2. Samples must be independent.
3. Samples must be drawn from populations that are normally distributed.

Step 1 **State the hypothesis and identify the claim.**

Step 2 **Find the critical value(s).**

Step 3 **Compute the test value. (s can be used in place of σ)**

$$Z=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

Step 4 **Make the decision.**

Step 5 **Summarize the results.**

Note: the denominator is E, the standard error.

The hypotheses

Right Tailed

$H_0: \mu_1 \leq \mu_2$ or $H_0: \mu_1 - \mu_2 \leq 0$

$H_1: \mu_1 > \mu_2$ $H_1: \mu_1 - \mu_2 > 0$

Two Tailed

$H_0: \mu_1 = \mu_2$ or $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 \neq \mu_2$ $H_1: \mu_1 - \mu_2 \neq 0$

Left Tailed

$H_0: \mu_1 \geq \mu_2$ or $H_0: \mu_1 - \mu_2 \geq 0$

$H_1: \mu_1 < \mu_2$ $H_1: \mu_1 - \mu_2 < 0$

Example 10-1

Back in 1995 a survey found that the average hotel room rate in New Orleans is $88.42 with s.d $5.62, and the average room rate in Phoenix is $80.61 with s.d. $4.83. 50 hotels were sampled in each city. assume the data is normal. At α = 0.05 can it be concluded that there is a significant difference in the rates?

Step 1 State the hypothesis and identify the claim.

$H_0: \mu_1 = \mu_2$ $\quad$ $\underline{H_1: \mu_1 \neq \mu_2}$

Step 2 Find the c.v.s. α = 0.05 two tailed so the c.v.s are +1.96 and -1.96

Step 3 Compute the test value.

$$Z = \frac{(88.42 - 80.61) - (0 - 0)}{\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}} = 7.45$$

Step 4 Make the decision. Reject null hypothesis.

Step 5 Summarize the results.

There is enough evidence to support the claim that the means are not equal. There is a difference in the rates.

Example 10-2

A researcher hypothesizes that the average number of sports colleges offer for males is greater than the average number of sports offered for females. A sample of the number of sports offered by 50 colleges was made. These are the results.

Males: $x_1 = 8.6$ and $s_1 = 3.3$ Females: $x_2 = 7.9$ and $s_2 = 3.3$

At $\alpha = 0.10$ is there enough evidence to support the claim?

Step 1 State the hypothesis and identify the claim.

$H_0: \mu_1 \le \mu_2$ $\underline{H_1: \mu_1 > \mu_2}$

Step 2 Find the c.v. $\alpha = 0.10$ right tailed so the c.v. is +1.28

Step 3 Compute the test value.

$$Z = \frac{(8.6 - 7.9) - (0 - 0)}{\sqrt{\frac{3.3^2}{50} + \frac{3.3^2}{50}}} = 1.06$$

Step 4 Make the decision. Do not reject null hypothesis.

Step 5 Summarize the results.

There is not enough evidence to support the claim that colleges offer more sports for males than they offer for females.

A statistician claims that the average score on a standardized test for psychology students is greater that the score for mathematics students. To test this outlandish claim a test was given to 50 students in each group, the results are shown here. Is there enough evidence to support the statistician's claim at $\alpha = 0.01$?

<u>Psychology</u>	<u>Mathematics</u>
$x_1 = 118$	$x_2 = 115$
$s_1 = 15$	$s_2 = 15$
$n_1 = 50$	$n_2 = 50$

Testing the difference between two means – case 2.1

Conditions:

1. **Sample sizes < 30**.
2. Samples must be independent.
3. Samples must be drawn from populations that are normally distributed.
4. **Variances are unequal. i.e. $s_1 \neq s_2$**

Step 1 State the hypothesis and identify the claim.

Step 2 **Find the critical value(s). d.f is the smaller of $n_1 - 1$ or $n_2 - 1$**

Step 3 **Compute the test value.**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Step 4 Make the decision.

Step 5 Summarize the results.

Note: the denominator is E, the standard error.

Testing the difference between two means – case 2.2

Conditions:

1. **Sample sizes < 30**.
2. Samples must be independent.
3. Samples must be drawn from populations that are normally distributed.
4. **Variances are equal. i.e. $s_1 = s_2$**

Step 1 State the hypothesis and identify the claim.

Step 2 **Find the critical value(s). d.f. = $n_1 + n_2 - 2$**

Step 3 **Compute the test value.**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Step 4 Make the decision.

Step 5 Summarize the results.

Note: the whole denominator is E, the standard error.

10-9 The average size of a farm in a one specific county is 191 acres with s.d. 38 acres. (n_1 = 8) The average size of a farm in another county is 199 acres with s.d. 12 acres. (n_2 = 10) Assume the data is normal and the variances are not equal. Can it be concluded at α = 0.05 that the average farm sizes are different?

Step 1 $H_0: \mu_1 = \mu_2$ ~~$H_1: \mu_1 \neq \mu_2$~~

Step 2 α = 0.05 two tailed d.f. is 8-1 = 7 so c.v.s are ±2.365

Step 3

$$t = \frac{(191 - 199) - (0)}{\sqrt{\frac{38^2}{8} + \frac{12^2}{10}}} = -0.57$$

Step 4 Do not reject H_0

Step 5 There is not enough evidence to support the claim that there is a difference between average farm sizes in these two counties.

10-10 A researcher wishes to determine whether the salaries of nurses in private hospitals are higher than those of nurses in public hospitals. She selects 10 nurses from private hospitals and finds the average salary is $26,800 with s.d. $600, and selects 8 nurses from public hospitals and finds the average ssalary is $25,400 with s.d. $450. At α = 0.01 can she conclude private pay more than public? Assume the data is normal and the variances are equal.

Step 1 $H_0: \mu_1 \leq \mu_2$ $H_1: \mu_1 > \mu_2$

Step 2 α = 0.01 right tailed d.f. is 10+8-2 = 16 so c.v. is 2.583

Step 3

$$t = \frac{(26{,}800 - 25{,}400) - (0)}{\sqrt{\frac{(10-1)(600)^2 + (8-1)(450)^2}{10+8-2}}\sqrt{\frac{1}{10} + \frac{1}{8}}} = 5.47$$

Step 4 Reject H_0

Step 5 There is enough evidence to support the claim that private hospitals pay nurses more than public hospitals.

10-46

The VP believes that high school girls miss more days per year than high school boys.
A sample of 16 girls found they missed an average of 3.9 days with s.d. 0.6. A sample
22 of boys found they missed an average of 3.6 days with s.d. of 0.8.
At $\alpha = 0.01$ is there enough evidence to support the VP's claim?
Assume the data is normal and the variances are equal.

A letter to the newspaper claimed that students in private schools score at most 8 points higher on a standardized test than students from public schools. To test this claim a researcher took random samples of 60 students from each type of school. The results are below. At $\alpha = 0.05$ test the claim.

Private school	Public school
$x_1 = 110$	$x_2 = 104$
$s_1 = 15$	$s_2 = 15$
$n_1 = 60$	$n_2 = 60$