Testing the Difference – What we've covered so far.

Mean	Variance	Proportion
Large independent Samples	P-Value Method	
Small Independent Samples and $\sigma_1^2 \neq \sigma_2^2$		
Small Independent Samples and $\sigma_1^2 = \sigma_2^2$		
Small Dependent Samples		

Do you remember how?

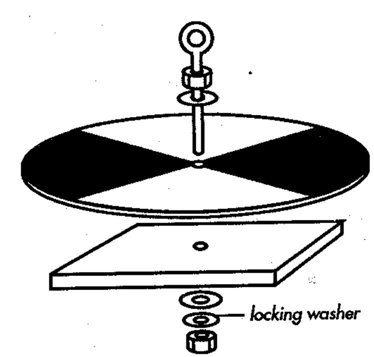
Try this:

Secchi Disk

A Secchi Disk is an eight-inch diameter weighted disk that's painted black and white and attached to a rope. The disk is lowered into water and the depth(in inches) at which the disk is no longer visible is recorded. The measurement is an indication of water clarity.

These monthly measurements were taken at the same site in a local lake five years apart. Test the claim that the clarity of the lake is improving at the $\alpha = 0.05$ level of significance. Assume the data are normally distributed.

Observation	1	2	3	4	5	6	7	8
Initial Depth	38	58	65	74	56	36	56	52
Depth 5 Years Later	52	60	72	72	54	48	58	60



Testing the Difference – What's next?

Mean	Variance	Proportion
Large independent Samples	P-Value Method	**Proportions**
Small Independent Samples and $\sigma_1^2 \neq \sigma_2^2$		
Small Independent Samples and $\sigma_1^2 = \sigma_2^2$		
Small Dependent Samples		

Testing the Difference between Proportions

Requirements

1. Samples must be independent.

2. n_1p_1, n_1q_1, n_2p_2, and n_2q_2 must all be 5 or more.

The Steps

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical values.

Step 3 Compute the test value.

Step 4 Make the decision.

Step 5 Summarize the result.

Step 3 Compute the test value.

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Note: the denominator is the standard error.

Where:

Proportion of sample 1: $\hat{p}_1 = \frac{X_1}{n_1}$

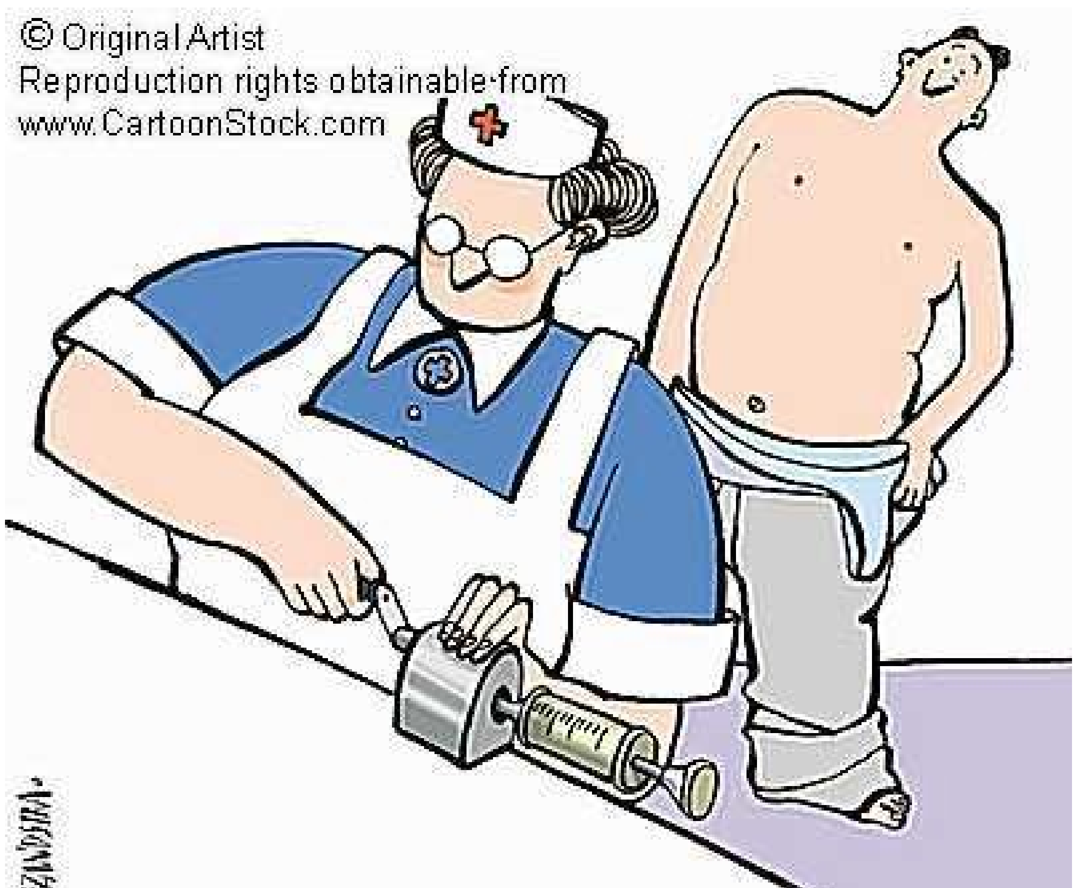
Proportion of sample 2: $\hat{p}_2 = \frac{X_2}{n_2}$

Weighted estimate of population proportion: $\bar{p} = \frac{X_1 + X_2}{n_1 + n_2}$ and $\bar{q} = 1 - \bar{p}$

Let's do an example!

10-15

In a nursing home study researchers found that 12 out of 34 small nursing homes had a vaccination rate of less than 80%, while 17 out of 24 large nursing homes had a vaccination rate of less than 80%. At $\alpha = 0.05$ test the claim that there is no difference in the proportions of the small and large nursing homes with a vaccination rate of less than 80%.



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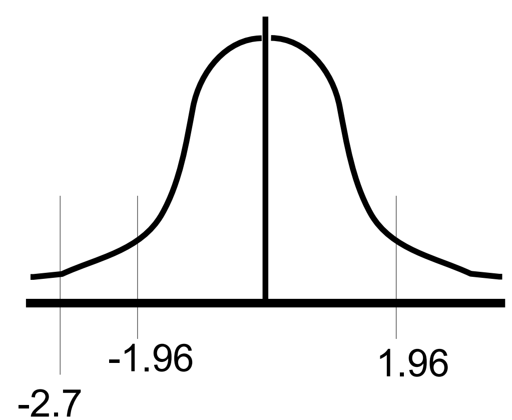
1. $H_0: p_1 = p_2$ $H_1: p_1 \neq p_2$

2. $\alpha = 0.05$, two tails, z-table -> ± 1.96

3. Small Nursing Homes $\hat{p}_1 = \frac{X_1}{n_1} = \frac{12}{34} = 0.35$ Large Nursing Homes $\hat{p}_2 = \frac{X_2}{n_2} = \frac{17}{24} = 0.71$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{29}{58} = 0.50 \qquad \bar{q} = 1 - \hat{p} = 1 - 0.50 = 0.50$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.35 - 0.71) - 0}{\sqrt{(0.5)(0.5)\left(\frac{1}{34} + \frac{1}{24}\right)}} = -2.7$$



4. Reject H_0

5. There is enough evidence to reject the claim that there is no difference in the proportions of small and large nursing homes with a vaccination rate of less than 80%.