Ch5 – Probability

A Measure of Likelihood

Probability is a method for turning short term uncertainty into long term prediction.

- e.g. a coin toss
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- Can you accurately predict the outcome of these events?
- a coin toss?
- dice roll?
- gender of foetus?
- height of the next person you see?
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- Can you predict the long term trend of those events?

Calculator simulation of long term trend of binary event.

Vocab

Event(**s**): outcome(s) of a probability experiment.

Probability Experiment: a chance process that leads to well defined outcomes.

Outcome: result of a probability experiment.

Sample Space: the set of all possible outcomes.

The probability of an event E occurring is given a value between 0 and 1.

$$0 \leq P(E) \leq 1$$

Use proportions where appropriate so you don't lose information.

Round decimals to three decimal places.

- Some things to know
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- If P(E) = 0 then the event E will not happen.
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- If P(E) = 1 then the event E is a certainty, it will happen.
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- If S is a complete set of events where
  - S = $\{e_1, e_2, e_3, ..., e_n\}$ then
  - $P(e_1) + P(e_2) + P(e_3) + ... + P(e_n) = 1$
  -
  - sometimes written as $\sum_{1}^{n} P(e_n) = 1$
  -
- P(E not happening) = 1 – P(E)
-
  - sometimes written as: P(not E) = 1 – P(E)
  -
  - or $P(\bar{E}) = 1 - P(E)$ or $P(\neg E) = 1 - P(E)$
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The Main Types of Probability

Classical

Empirical

Subjective

1. Classical "*simple*"

All outcomes are equally likely to occur.

eg in a coin toss heads and tails have equal likelihood of occurring.

P(H) = ½ and P(T) = ½

P(H) + P(T) = 1

2. Empirical “*experimental*”

Relies on experience to determine the likelihood of outcomes.

eg a survey if 7 out of every 10 people interviewed prefer Starbucks to Seattle's Best, what's the probability that the next person interviewed will prefer Seattle's Best?

3. Subjective “*expert opinion*”

Based on opinion or an educated guess.

eg Weather reports, earthquake predictions, sports team performance, stock market.

1. Classical Probability

Given an event E and sample space S

$$P(E) = \frac{\text{total number of ways E can occur}}{\text{total number of elements in S}}$$

eg1 single coin toss

$S = \{H, T\}$

$P(H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$

1. Classical Probability

eg2 two coin toss

$S = \{HH, HT, TH, TT\}$

$P(HT) = 1/4$ $P(TT) = 1/4$

NB probability cares that HT is different than TH

1. Classical Probability

eg3 2-child family

S = { BB, BG, GB, GG }

P(BB) = ¼

If the first child is born a boy, is it appropriate to predict the gender of the next child?

eg4 The candy jar contains a mixture of Jolly Rancher Candies. There are five red, two yellow, and three green. If you pick a candy without looking find these probabilities.

$$= \frac{5}{5+2+3} = P(\text{it is red}) = \frac{1}{2}$$

$$= \frac{3}{5+2+3} = P(\text{it is green}) = \frac{3}{10}$$

$$= P(\text{if is red or green}) = \frac{8}{10}$$

$$= 1 - P(\text{it is not green}) = \frac{7}{10}$$

$$= 1 - P(\text{it is not red}) = \frac{5}{10}$$

eg5 A roulette wheel in a casino has 38 spaces numbered 1 through 36, 0, and 00. In a spin of the roulette wheel a ball will eventually land on one of the 38 spaces. There are many bets you place, including guessing which number the ball will land on, though you cannot bet on the 0 or 00.

In a spin of the wheel, find these probabilities.

P(An odd number) = 18/38 ≈ 0.474

P(A number greater than 12) = 13/38 ≈ 0.342

P(A number less than 15, not counting 0 or 00) = 14/38 ≈ 0.368

eg6 In a family with 4 children, what is:

P(2 girls)	1	GGGG
	2	GGGB
	3	GGBG
P(4 girls)	4	GGBB
	5	GBGG
	6	GBGB
P(No girls)	7	GBBG
	8	GBBB
	9	BGGG
P(All of one gender)	10	BGGB
	11	BGBG
	12	BGBB
P(2 girls and 2 boys)	13	BBGG
	14	BBGB
	15	BBBG
P(3 of one gender)	16	BBBG

2. Empirical Probability

- The likelihood or probability of outcomes is determined by experience or experiment.
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- For example
- Surveys
- Lab experiments
- Observational studies, usually long term

Given event E

$$P(E) = \frac{\text{frequency of E occurring}}{\text{number of trials conducted}}$$

A random sample of 100 GHS students were asked the question:
"What mode of transport did you use to get to school?"

It was found that: 33 came by car
28 walked
18 bussed
10 biked
11 other

If a student is chosen at random find :

P(they came by **bus**) = 18/100 = 0.18

P(they came by **car**) = 33/100 = 0.33

P(they came by **other**) = 89/100 = 0.89

P(they came by **walking or bus**) = 46/100 = 0.46

Who prefers Pepsi products to Coke products?

If a person from this class is chosen at random find these probabilities:

P(the person prefers Pepsi products)

P(the person prefers Coke products)

P(the person does not prefer coke products)

2. Subjective Probability

Expert opinion
Educated guess

eg1 There is a 50% chance of rain.

eg2

> "The ability to anticipate the supply and demand of currency has a success rate no better than that of forecasting the outcome of a coin toss."

Allan Greenspan
ex-Chairman of the Federal Reserve
11/19/2004