Finding the confidence interval for the true mean when σ is unknown AND n < 30

When n gets too small, the distribution stops being Normal.

If the data is Normal or approximately Normal then use the t Distribution (aka the Student's Distribution).

The t Distribution is a family of distributions based on n, the sample size.

The distribution is chosen using degrees of freedom

The formula for t Distribution

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{S_x}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{S_x}{\sqrt{n}}\right)$$

degrees of freedom are n-1

e.g. 1

Using the t-table find the $t_{\alpha/2}$ value for a 95% confidence interval when the sample size is 22.

Solution

d.f. = 22 – 1 = 21

From the t-table the value for $t_{\alpha/2}$ is 2.080

e.g. 2 Ten randomly selected automobiles were stopped, and the tread depth of a tire was measured. The mean was 0.32 inches, and the s.d. was 0.08 inches. Find the 95% c.i. of the mean tread depth. Assume the variable is normally distributed.

σ is unknown and n < 30 so use the t-distribution
d.f. = 10 – 1 = 9
From t-table 95% gives a $t_{\alpha/2}$ value of 2.262

$$0.32-(2.262)\left(\frac{0.08}{\sqrt{10}}\right) < \mu < 0.32+(2.262)\left(\frac{0.08}{\sqrt{10}}\right)$$

$$0.26 < \mu < 0.38$$

With 95% confidence the population mean tire tread depth is between 0.26 and 0.38 inches.

e.g.3 The following data represent the number of house fires started by candles for the past twenty years. (From the NFPA) Find the 99% c.i. for the mean number of home fires started by candles each year. Assume the variable is normally distributed.

1980	1982	1984	1986	1990	1992
8200	7300	6700	6700	5500	6100

1994	1996	1998	2000	2002
7200	9900	12500	15700	18000

ü = 9436.36 and Sx = 4181.69

n = 11 so d.f. = 10 and 99% from t-table gives $t_{\alpha/2}$ = 3.169

e.g.3 The following data represent the number of house fires started by candles for the past twenty years. (From the NFPA) Find the 99% c.i. for the mean number of home fires started by candles each year. Assume the variable is normally distributed.

n = 11 so d.f. = 10 and 99% from t-table gives $t_{\alpha/2} = 3.169$

$$9436.36 - (3.169)\left(\frac{4181.69}{\sqrt{11}}\right) < \mu < 9436.36 + (3.169)\left(\frac{4181.69}{\sqrt{11}}\right)$$

$$5440.8 < \mu < 13431.9$$

With 99% confidence the true mean number of house fires started by candles lies between 5440 and 13432.

Source: NFPA Faire Analysis and Research, Quincy, MA

e.g. 4 The BC Postal Times newspaper reported that, with 80% confidence, between 3.79 and 4.21 mail-carriers per day are chased by velociraptors ($S_x = 0.6$). Assume the variable is normally distributed.

1. How large a sample did they use?
2. What feature of the t Distribution allowed you to answer this problem?

Answer:

1. n = 15
2. Symmetry. Both t and Normal are symmetric.

e.g. 5 The heights of 20 – 29 year old males are known to be normally distributed with $\sigma = 2.9$ A random sample of n=15 males 20-29 years old results in the following data. Find the 95% confidence interval about the true population mean μ.

65.5	**72.36**	**68.2**	**65.6**	**68.8**
66.7	**69.6**	**72.6**	**72.9**	**67.5**
71.8	**73.8**	**70.7**	**67.9**	**73.9**

e.g. 6 Fifteen randomly selected female athletes were asked to take a stress test. After three minutes, their pulses were measured and the following data collected. Find the 95% confidence interval about the true population mean μ. Assume the variable is normally distributed.

117	**102**	**98**	**100**	**116**
113	**91**	**92**	**96**	**136**
134	**126**	**104**	**113**	**102**