Confidence Intervals for Proportions

E.g. If 20% of a sample of students have used a tobacco product, what is the true proportion of the student population to have used a tobacco product?

Symbols used

p = population proportion
$\hat{p}$ = sample proportion

$q = 1 - p$ and $\hat{q} = 1 - \hat{p}$

Formula for finding true proportion

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\,\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

maximum error is

$$E = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

sample size is

$$n = \hat{p}\,\hat{q}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2$$

when np and *nq* are each greater than 5

e.g. 1 A sample of 500 nursing applications contained 60 from men. Find the 90% ci of the true proportion of men who applied to the nursing program.

90% ci means $Z_{\frac{\alpha}{2}} = 1.65$

$$\hat{p} = \frac{60}{500} = 0.12 \quad \text{and} \quad \hat{q} = 1 - 0.12 = 0.88$$

$$0.12 - (1.65)\sqrt{\frac{(0.12)(0.88)}{500}} < p < 0.12 + (1.65)\sqrt{\frac{(0.12)(0.88)}{500}}$$

$$0.096 < p < 0.144$$

With 90% confidence the proportion of men who applied lies between 9.6% and 14.4%

e.g. 2 A researcher wishes to estimate with 95% confidence the proportion of people who own a home computer. The last study shows 56.6% of those interviewed had a computer at home. The researcher wants to be accurate within 2% of the true proportion. Find the minimum sample size necessary.

95% ci means $z_{\frac{\alpha}{2}} = 1.96$

$$n = \hat{p}\hat{q}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = (0.566)(0.434)\left(\frac{1.96}{0.02}\right)^2 = 2359.1$$

The minimum sample size needed is 2360 people.

When finding E or n and $\hat{p}$ or $\hat{q}$ is not known use

$\hat{p} = 0.50$ and $\hat{q} = 0.50$

A survey of 240 freshmen showed that 42 did not wish to work after marriage. Find the 95% c.i. of the true proportion of freshmen who do not wish to work after marriage.

$$\hat{p} = \frac{42}{240} = 0.175 \quad \text{and} \quad \hat{q} = 1 - 0.175 = 0.825$$

$$0.175 - (1.96)\sqrt{\frac{(0.175)(0.825)}{240}} < p < 0.175 + (1.96)\sqrt{\frac{(0.175)(0.825)}{240}}$$

$$0.1269 < p < 0.2231$$

With 95% confidence the true proportion of freshmen who do not wish to work after marriage lies between 12.69% and 22.31%.

A survey of 80 fatal traffic accidents showed that 46 were alcohol related. Find the 90% c.i. of the true proportion of fatal alcohol-related traffic accidents.

$$\hat{p} = \frac{46}{80} = 0.575 \text{ and } \hat{q} = 1 - 0.575 = 0.425$$

$$0.575 - (1.65)\sqrt{\frac{(0.575)(0.425)}{80}} < p < 0.575 + (1.65)\sqrt{\frac{(0.575)(0.425)}{80}}$$

$$0.4838 < p < 0.6662$$

With 90% confidence the true proportion of fatal alcohol-related traffic accidents lies between 48.38% and 66.62%.

Your teacher wants to estimate, with 98% confidence and within 3%, the true proportion of statistics students who study at least 1 hour per night.

a. How large a sample will need to be used?

$$n=(0.5)(0.5)\left(\frac{2.33}{0.03}\right)^2$$

$$n=1508.028$$

The sample size will need to be at least 1509 students.

b. If this class is used how large is the maximum error of the estimate?

$$E=(2.33)\sqrt{\frac{(0.5)(0.5)}{n}}$$

Find the true proportion of Grant students who believe the president is currently doing a good job.

USA Today/Gallup reported that in Feb. 2008 32% believed the president was doing a good job.
Source http://www.pollingreport.com/BushJob.htm

How does this compare to your class?
Why the difference/similarity?

Thaaaaaats all folks.