Chapter 9 ~ Hypothesis Testing

What is a hypothesis?

What might a statistical hypothesis look like?

e.g. 1 A Mar. 2009 Gallup poll finds 34% decline in consumer spending in the first quarter this year as compared to the first quarter last year.

e.g. 2 A record number of Americans believe global warming is exaggerated: 41% and increasing.

e.g. 3 54% of Americans are very worried about the amount of money being added to the federal debt.

e.g. 4 77% of Americans want the govt to spend more on alternate energy sources.

What does hypothesis testing test?

Whether or not the hypothesis is true or merely due to chance.

Why can a result occur by chance?

There are two kinds of hypotheses.

The <u>null hypothesis</u> H_0

"There is no real difference between a parameter and a specific value, or between two parameters."

The <u>alternate hypothesis</u> H_1

"There is a real difference between a parameter and a specific value, or between two parameters."

<u>*parameter*</u> : mean, variance, proportion, s.d., etc.

Need to state a statistical hypothesis in the correct language.

e.g. 5

A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. Will the new drug cause the pulse rate to increase, decrease, or remain unchanged after a patient takes the drug.

e.g. 6

If the mean pulse rate for the population is 82 beats per minute, the hypotheses are:

$H_0: \mu = 82 \quad H_1: \mu \neq 82$

Note: the null hypothesis specifies the mean will remain unchanged and the alternate hypothesis specifies the mean will be changed in some way.

Because the change could be in either direction this is a **two-tailed** test.

e.g. 7

A chemist invents an additive to increase the shelf life of Twinkies. If the mean shelf life of Twinkies is currently 24 months, then the hypotheses are:

$H_0: \mu \leq 24 \qquad H_1: \mu > 24$

Note: the researcher is only interested in increasing shelf life so the alternate hypothesis is that the mean is greater than 24 months. The null will be that the mean remains the same or is less.

Because the researcher is interested in increase only, this is a **right-tailed** test.

e.g. 8 An organic farmer wishes to test whether companion planting with garlic will lower the incidence of coddling moth on her apple trees. She knows that she traps an average of 52 moths per night in his orchard, so the hypotheses are:

$H_0: \mu \geq 52$ $\underline{H_1: \mu < 52}$

Note: the farmer is only interested in decreasing coddling-moth so the alternate hypothesis is that the mean is less then 52 moths. The null will be that the mean remains the same or is more.

Because the farmer is interested in decrease only, this is a **<u>left-tailed</u>** test.

Write the null and alternate hypotheses, state whether the test is right, left, or two tailed, and identify the claim.

e.g. 9

A researcher thinks that if expectant mothers use vitamin supplements, the birth weight of babies will increase. The average birth weight is 8.6 pounds.

$H_0: \mu \leq 8.6$ $\underline{H_1: \mu > 8.6}$

right-tailed

Write the null and alternate hypotheses, state whether the test is right, left, or two tailed, and identify the claim.

e.g. 10

An engineer believes that the mean number of defects in a product can be reduced by replacing humans with robots. The mean number of defects is 18 per 500.

$H_0: \mu \geq 18$ per 500 $H_1: \mu < 18$ per 500

left-tailed

Write the null and alternate hypotheses, state whether the test is right, left, or two tailed, and identify the claim.

e.g. 11

A student believes Red Bull increases concentration and raises test scores. This student's test average score is 67%.

$H_0: \mu \leq 67\%$ $\underline{H_1: \mu > 67\%}$

right-tailed

Write the null and alternate hypotheses, state whether the test is right, left, or two tailed, and identify the claim.

e.g. 12

A student believes listening to music while studying will not effect their test scores. This student's test average score is 97%.

$\underline{H_0: \mu = 97\%}$ $H_1: \mu \neq 97\%$

two-tailed

In Summary

Left-Tailed	Two-Tailed	Right-Tailed
H_0: $\mu \geq$	H_0: $\mu =$	H_0: $\mu \leq$
H_1: $\mu <$	H_1: $\mu \neq$	H_1: $\mu >$

Now you can state a hypothesis, how do you test it?

Testing a hypothesis

Three methods:

Traditional method

P-value method

Confidence Interval method (not covered here)

The two methods we will learn have 5 steps which are essentially the same. They vary in the way values are calculated.

The five steps for all methods are basically the same:

1 State the hypotheses, and identify the claim.

2 Find the critical value(s) from the table.

3 Compute the test value.

4 Make the decision to reject or not reject the null hypothesis.

5 Summarize the results.

Traditional Method - The Steps

1 State the hypotheses, and identify the claim.

State the hypotheses.

Identify the claim.

2 Find the critical value(s) from the table.

a. Identify what level of significance, α, is being used.

Typical values of α are: 0.01, 0.05, and 0.10

b. Determine whether the test is left, two, or right tailed.

If two-tailed then use 0.50 – α/2 to find the critical region.

If left or right tailed use 0.50 – α to find the critical region.

2 Find the critical value(s) from the table (cont).

c. Use the critical region and the z-table to find the critical value, (c.v.).

d. Make sure you get the correct sign on the value.

A left-tailed test the c.v. will be negative.

For a right-tailed test the c.v. will be positive.

For a two-tailed test one c.v. will be positive, the other c.v. negative.

3 Compute the test value.

a. When σ is known use: $z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$

b. When σ is unknown,
but s is known and n ≥ 30 use: $z = \frac{x - \mu}{\frac{s}{\sqrt{n}}}$

c. When σ is unknown,
but s is known and n < 30 use: $t = \frac{x - \mu}{\frac{s}{\sqrt{n}}}$

4 Make the decision to reject or not reject the null hypothesis.

a. Draw the bell curve.

b. Indicate:

the critical region = reject H_0

the non-critical region = do not reject H_0

the critical value

the test value

c. State the decision.

continued on next slide...

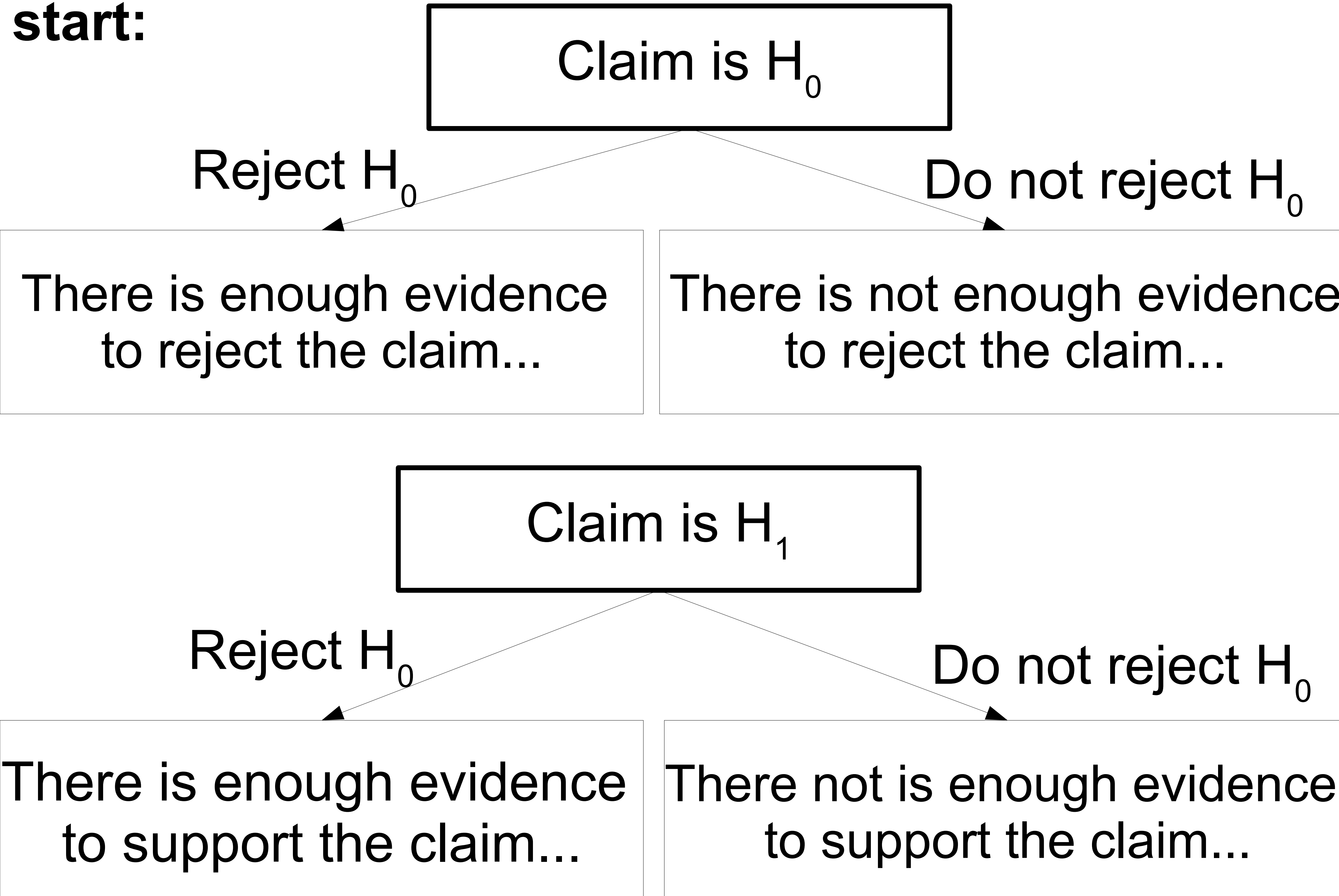
4 Make the decision to reject or not reject the null hypothesis (cont).

If the test value falls in the critical region then reject H_0: the null hypothesis.

If the test value falls outside the critical region then do not reject H_0: the null hypothesis.

Depending on which hypothesis is the claim, decide whether there is enough evidence to support the claim or not.

5 Summarize the results. Your summary should start:



One more time, the five steps are:

1 State the hypotheses, and identify the claim.

2 Find the critical value(s) from the table.

3 Compute the test value.

4 Make the decision to reject or not reject the null hypothesis.

5 Summarize the results.

Let's practice Step 1

e.g. 10 A student is interested in whether listening to music during a test makes no difference to test performance. The average test score in the student's class is 70%.

State the hypotheses:

Null Hypothesis: $H_0: \mu = 70\%$

Alternative Hypothesis: $H_1: \mu \neq 70\%$

Indicate the claim.

Tails? Two-tailed

e.g. 11 A physics student is researching the best mix of flammable gases for a potato cannon. The student is interested only in whether each mixture causes the potato to be propelled further than the 150ft obtained by using the standard propellant.

State the hypotheses:

Null Hypothesis: H_0: $\mu \leq 150$ feet

Alternative Hypothesis: H_1: $\mu > 150$ feet

Indicate the claim.

Tails? One, right-tailed

e.g. 12 A cook wants to know if adding salt to water will really make the boiling temperature lower.

State the hypotheses:

Null Hypothesis: $H_0: \mu \geq 100^{o}C$

Alternative Hypothesis: $H_1: \mu < 100^{o}C$

Indicate the claim.

Tails? One, left-tailed

Let's practice Step 2

Drawing a bell-curve at this point HELPS A LOT.

Find these critical values.

Left-tailed

$\alpha = 0.10$ c.v. = -1.28

$\alpha = 0.05$ c.v. = -1.65

$\alpha = 0.01$ c.v. = -2.33

Drawing a bell-curve at this point HELPS A LOT.
Find these critical values.

Right-tailed

$\alpha = 0.10$ c.v. = 1.28

$\alpha = 0.05$ c.v. = 1.65

$\alpha = 0.01$ c.v. = 2.33

Drawing a bell-curve at this point HELPS A LOT.
Find these critical values.

Two-tailed

$\alpha = 0.10$ c.v. = -1.65 and +1.65

$\alpha = 0.05$ c.v. = -1.96 and +1.96

$\alpha = 0.01$ c.v. = -2.58 and +2.58

Do you want more practice finding critical values?

Let's put it all together.

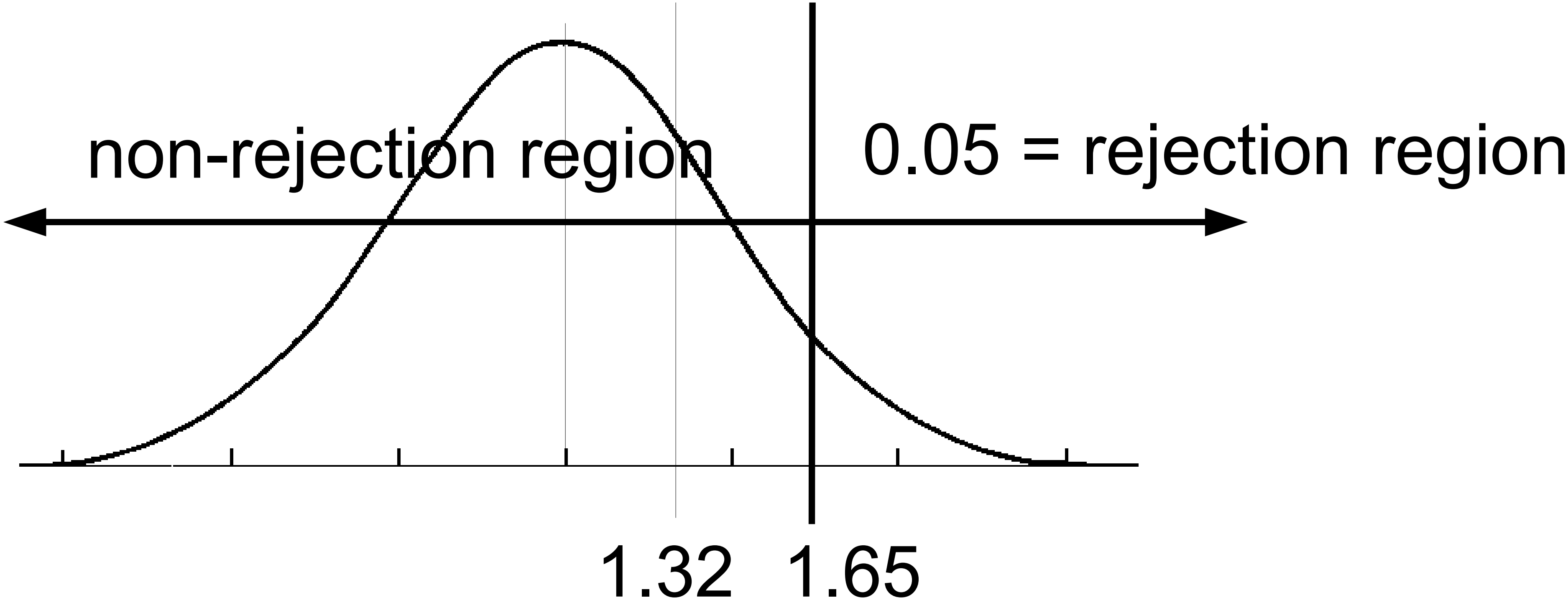
A researcher for the Oregonian reports that teachers in PPS have an average salary of \$42,000 per year. A sample of 30 teachers has a mean salary of \$43,260. At $\alpha = 0.05$, test the claim that teachers earn more than \$42,000 per year. The standard deviation of teachers' salaries in PPS is \$5230.

1. $H_0: \mu \leq \$42,000$ $\quad$ $H_1: \mu > \$42,000$

2. $\alpha = 0.05$ right-tailed test (rtt) c.v. = +1.65

3. $z = (43260 - 42000) / (5230/\sqrt{(30)}) = 1.32$

4.



Inside the non-rejection region: Do not reject H_0

5. There is not enough evidence to support the claim that teachers in PPS earn more than $42,000 per year.

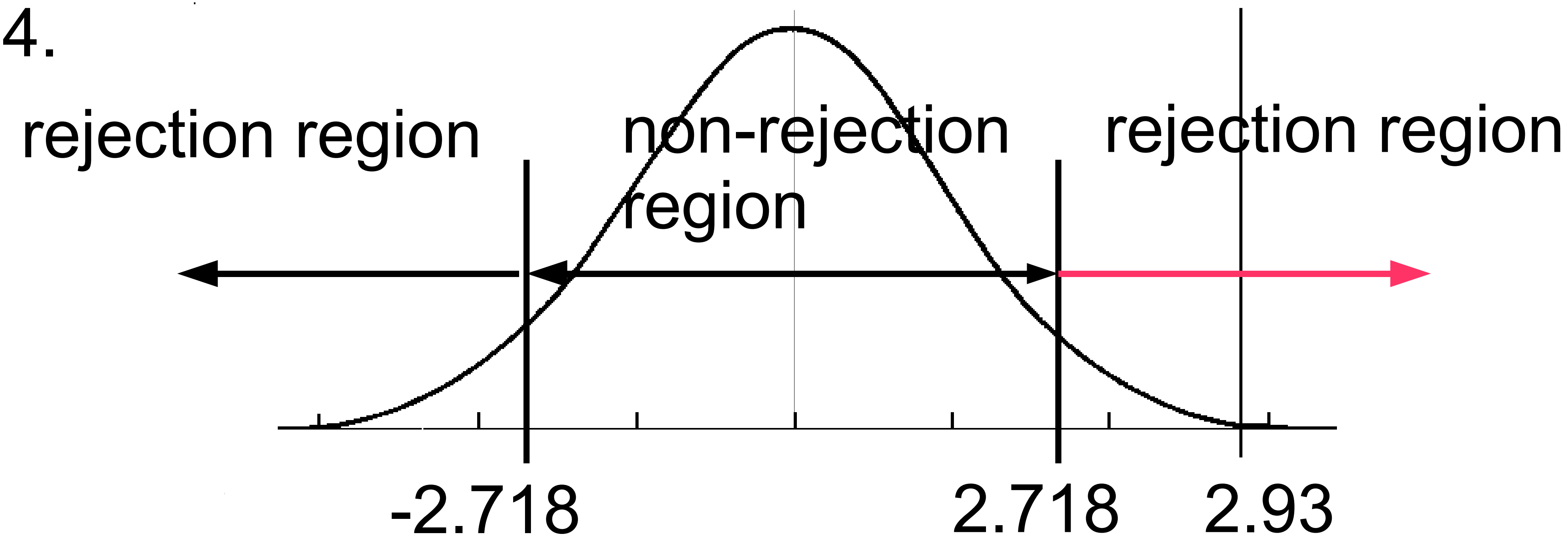
A maker of frozen meals claims that the average caloric content of its meals is 280 cal, with a standard deviation 13 cal. A dietician tested 12 meals and found that the average number of calories was 291 cal. Is the maker's claim true? Use $\alpha = 0.02$.

1. $\underline{H_0: \mu = 280}$ $\quad H_1: \mu \neq 280$

2. $\alpha = 0.02$ two-tailed test c.v. = ±

3. $z = (291 - 280) / (13/\sqrt{(12)}) =$

4.



Inside the rejection region: Reject H_0

5. There is enough evidence to reject the claim that caloric content of frozen dinners is 280 calories.

You should now be able to state hypotheses, calculate critical values, calculate test values, and perform a traditional hypothesis test.

We will go on to learn:

1. the p-value method;
2. how to test for small sample sizes;
3. how to test hypotheses about proportions; and
4. how to test hypotheses about variance.


"THAT'S ALL FOLKS"
MEL BLANC
MAN OF 1000 VOICES
BELOVED HUSBAND AND FATHER
1908 — 1989