

Algebra 3-4 Summer Review Packet. This packet covers skills AA1 through AA7, AA9, AA10, AA15. Answers will be posted later in the summer on [sites.google.com/site/mrtinlingsmathpage](https://sites.google.com/site/mrtinlingsmathpage)

**AA1\_1: Simplify or Solve:**

a.  $\frac{\sqrt{96}}{\sqrt{6}} =$

b.  $3\sqrt{98} =$

c.  $\sqrt{18} + \sqrt{50} - \sqrt{72} =$

d.  $\frac{1}{x-3} \div \frac{1}{x^2-11x+24} =$

e.  $\left(\frac{10d^7e^3}{5d^5e^7}\right)^2 =$

f.  $\frac{ab^5c^3}{a^3b^3c^4} =$

g.  $\left(\frac{3x^3y^5}{6xy^7}\right)^{-2} =$

h.  $\frac{4}{x+4} - \frac{3}{x-5} =$

i.  $(x+5)^2 =$

j.  $\frac{x^2-4x-12}{x^2-4}$

k.  $x^3 - x^2 - 2x = 0$

l.  $\frac{x-3}{2} = \frac{x}{5}$

m.  $\frac{\frac{a}{b}-2+\frac{b}{a}}{\frac{1}{b}-\frac{1}{a}} =$

n.  $\frac{t+2}{2t-6} - \frac{3}{t-3} =$

o.  $\frac{x-3}{2x+4} = 7$

p.  $\frac{x-2}{3} = \frac{4}{x}$

q.  $\frac{3}{5}x - 17 = 28$

r.  $7 - 4x > 15$

s.  $A(3,5)$  and  $B(-4,6)$

Find the slope of the line  $AB$

t. Solve:  $|2x-3|=5$

u. Solve the system :  $y = x - 3$   
 $2x + 4y = 8$

v. Solve the system :  $6x - 7n = 6$   
 $7x - 8n = 15$

w. Solve:  $|x-6| < 6$

x. Solve:  $|2z+3| > 5$

y. Factor:  $8x^2 - 2x - 15 =$

z. Solve for x:  
 $ax - 4 = bx + 6$

AA2: Functions and Relations

AA2\_1: For  $f(x) = x^2 + x - 1$  and  $g(x) = 4x + 7$ , evaluate the following:

- a)  $f(1)$  b)  $g\left(\frac{1}{3}\right)$  c)  $g(0)$  d) Find the value(s) of  $x$  for which  $g(x) = -5$

AA2\_2: For  $f(x) = \frac{1}{2-x}$  and  $g(x) = 1 - 2x$ , evaluate the following:

- a)  $f(2)$  b)  $g\left(\frac{1}{2}\right)$  c)  $g\left(-\frac{1}{2}\right)$

- d) Find the value of  $x$  for which  $g(x) = -3$

AA2\_3: Use your graphing calculator to investigate the function  $f(x) = \frac{x^2 - 3x + 2}{(x - 2)^2}$

Find the domain, range, max, min, intercepts, asymptotes etc. (if they exist).

Draw a complete graph.

AA2\_4: Given the following two functions:

$$f(x) = x^2 - 2x + 4$$

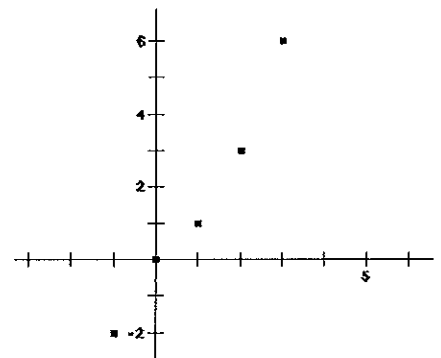
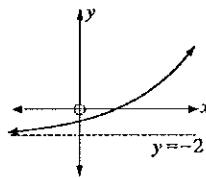
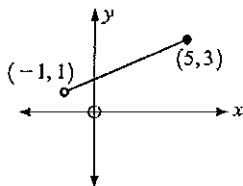
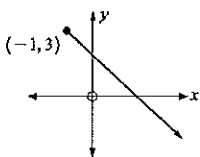
$$g(x) = 3x - 10$$

- a) Find the function  $h(x) = f(x) - g(x)$   
 b) For what values of  $x$  does  $h(x) = 0$ ?

AA2\_5: Give an example of a function that is not a relation.

AA2\_6: Describe the domain and range of each of the following graphs.

- a) D,R:      b) D,R:      c) D,R:      d)D,R



AA3\_1: Determine whether the following sequences are arithmetic, geometric or neither.

- a. 25, 5, 1, 0.2, ...      b. 0, 2, 4, 6, ...      c. 0, 1, 2, 4, 8, ...

AA3\_2: Find an equation to represent each table below as a sequence and **complete** the table.

a)  $t(n) =$

n	t(n)
0	36
1	27
2	18
3	

b)  $t(n) =$

n	t(n)
0	-4
1	2
2	0
3	

AA3\_3: Ryan was doing his homework when a dog bit off a piece of the paper. All he had left was a part of the sequence: 4, 12, ... and the fact that 972 was a term in the sequence.

- a. Could the sequence be arithmetic? If so, what would be the equation, and what term number is 972?  
b. Could the sequence be geometric? If so, what would be the equation, and what term number is 972?

AA3\_4: . For an **arithmetic** sequence, circle **ALL** the statements that are true:

- a. The constant multiplier between cells can be any number.  
b. The sequence is a discrete function.  
c. The first term (start value) must be a non-negative integer.  
d. The graph of the sequence will be a connected straight line of points.  
e. The value of  $n$  must be a member of the integers and not a real (decimal) number.  
f. The constant difference between cells may be a negative number.  
g. The constant difference between cells may not be a decimal number.

AA3\_5: For a **geometric** sequence, circle **ALL** the statements that are true:

- a. The constant multiplier between cells can be any real number.

- b. The sequence is a continuous function because the exponent can generate real numbers.
- c. The start value can be a negative number.
- d. The graph of the sequence will never go below the x-axis (or n-axis if that is the way you think of it).
- e. The value of n must be a member of the integers and not a real (decimal) number.

AA3\_6: Determine whether the following sequences are arithmetic, geometric or neither.

- a. 1, 3, 27, 81...
- b.  $-12, -4, \frac{-4}{3}, \dots$
- c. -12, -10, -8, ...

AA3\_7: Ryan was doing his homework when a dog bit off a piece of the paper. All he had left was a part of the sequence: X, 8, 12 ... . He also discovered another slip of paper with the number 91.125 on it. He was pretty certain that this number was also part of the sequence, but he doesn't know where in the sequence.

- a. Could the sequence be arithmetic? If so, show the equation.
- b. Could the sequence be geometric? If so, show the equation.

AA3\_8: Find values for x and y if the following sequence is geometric

$$2y, 2xy, 2, \frac{xy}{2}, \dots$$

AA3\_9: Find x and y if the following sequence is arithmetic.

$$2y + x, 5y + x, 38, 2y + 7x, \dots$$

**Hint: Remember that in an arithmetic sequence, the difference between cells is a constant difference (or adder).**

AA4\_1: . Give the compounded interest equation for an investment at 3.5% compounded every 6 months, with a starting value of \$3000.

AA4\_2: Cobalt-60 is a radioactive isotope used for medical purposes, mainly in radiotherapy. It has a half-life of 5.25 years. If 5 kg. (5000 g) of cobalt-60 was purchased by a hospital in 2000, and forgotten in the mail room for 20 years:

- a) how much cobalt-60 was remaining?
- b) After how many years will you have less than 1 gram of Cobalt-60 left?

AA4\_3: . Give the equation for the exponential curve that goes through the points (2, 6.75) and (5, 22.781).

AA4\_4: Simplify the following expressions without negative or fractional exponents:

a.  $(3x^{-3})(2x)^{-2}$

b.  $(6x^4y^7)(2x^2y^5)$

c.  $\left(\frac{x^3-6y}{z^2}+4\right)^3 * \left(\frac{x^3-6y}{z^2}+4\right)^{-2}$

AA4\_5: Solve the following equations for x. (You should check your answer before you turn your test in.) (Level 5)

a.  $2^{(x+3)} = 128$

b.  $x^{\frac{1}{4}} = 8$

c.  $27^{\frac{2}{3}x} = 243$

AA4\_6: Suppose you invest \$750.00 in an account that earns 9% interest compounded annually.

a) Write an equation to model the growth of your money.

b) How much money will be in the account after seven years?

c) How much money will be in the account after seven years if it earns the same annual interest but is now compounded quarterly? Show your equations and work.

AA4\_7: ) A certain medication is eliminated from the bloodstream at the rate of 21% per hour. The initial dosage of the medication was 45 mg (milligrams).

a) Write a function to model the decay.

b) How much medication remains in the bloodstream after 5 hours. Answer in a complete sentence.

AA4\_8: Simplify using the properties of exponents. Be sure all of your answers are written with positive exponents and none of the exponents are fractions.

a.  $4^{-3}$

b.  $27^{-\frac{2}{3}}$

c.  $\left(64^{\frac{2}{3}}\right) * \left(64^{\frac{2}{3}}\right)$

d.  $\left(\frac{(x^3)(y^4)^3}{y^4x^3}\right)$

e.  $(32w^{412})^{35} (32w^{412})^{-35}$

AA4\_9: . Solve the following equations for x. (You should check your answer before you turn your test in.)

a.  $2^{(x+2)} = 256$

b.  $x^{\frac{1}{2}} = 8$

AA5\_1: Write the equation of a parabola whose vertex is (4, -3). The parabola opens downward. It has stretch equal to 2.

AA5\_2: Write an equation for each function described below:

a.A parabola just like  $y = 2(x-1)^2$  but shifted down 3 units and to the right 5 units.

b.A square root with a stretch factor of 4 and with the locator point) at (-3, 2) (note: the "locator point is that point at which the parent graph would start for (0,0).

AA5\_3: . In the statements below, circle all true statements.

a. A square root function has the asymptote  $y = 0$ ;

b. A parabola is never symmetrical around the y-axis.

- c. The domain of an absolute function can include negative, real values.
- d. Changing the sign of the expansion coefficient (i.e. "a") in any of our equations will flip the graph over the x-axis.
- e. The range of the absolute value parent function is always positive
- f. A transformed square-root function may have negative values for the domain.

**AA5\_4:** Write the equation of a parabola whose vertex is (-3,-4). The parabola opens downward. It has no stretch or compression

**AA5\_5:** Write an equation for the following:

A parabola just like  $y = 2x^2$  but shifted down 3 units and to the right 5 units.

**AA5\_6:** In the statements below, circle all **true statements**.

- a. A square root function has no asymptotes.
- b. A parabola is always symmetrical around the y-axis.
- c. The domain of an absolute function can include negative, real values.
- d. Changing the sign of the expansion coefficient (i.e. "a") in any of our equations will flip the graph over.
- e. The range of the absolute value parent function is always positive, but the range of its transformed functions may be negative.
- f. A transformed square-root function may have negative values for the domain.
- g. The locator point of the following function has been moved two units to the **right** of the parent function.

$$f(x) = 3(x - 2)^2 + 2$$

1. **AA6\_1:** For each of the quadratic equations below, complete the following:

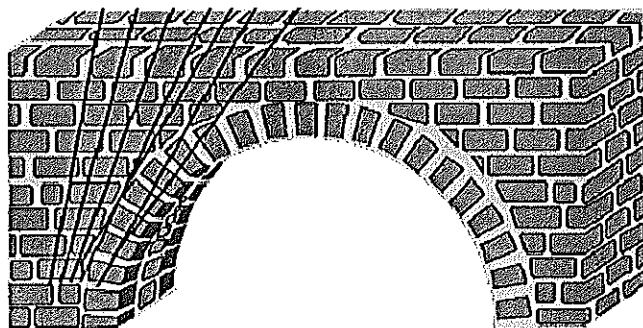
Write the coordinates of the x and y- intercepts. Write the coordinates of the vertex. Write the equation in graphing (vertex) form. Sketch the graph of each parabola on its own set of axes. Place values of important points in your sketch.

1)  $y = .5(x + 1)(x - 4)$

2)  $y = x^2 + -9x + 11$

**AA6\_2:** A new parabolic bridge is being constructed. The space between the supports needs to be 1000 feet; the height at the center of the parabolic arch needs to be 250 feet. **Level 6/7**

- a) Find the equation of a parabola with these characteristics. (Level 6)



- b) A sailboat with a mast that sticks up 100 feet above the water needs to pass as close to the right support of the bridge as possible. If the mast is in the center of the boat and the boat is twenty feet wide, how close can the boat sail to the right side of the arch?

AA6\_3: Find the  $x$ -intercepts of each of the quadratics. Use an algebraic method to solve these problems.

a.  $y = 10x - 25x^2$

b.  $y = x^2 - x - 12$

AA6\_4: Find the  $x$ -intercepts of each of the quadratics. Use an algebraic method to solve these problems

a.  $f(x) = -25 + 16x^2$

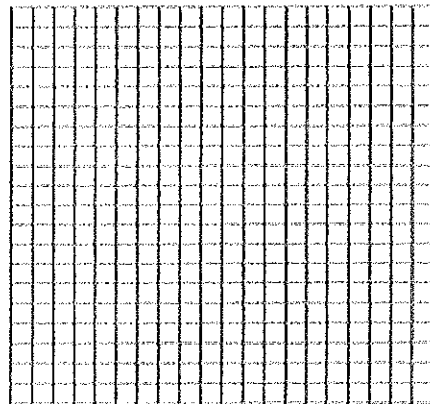
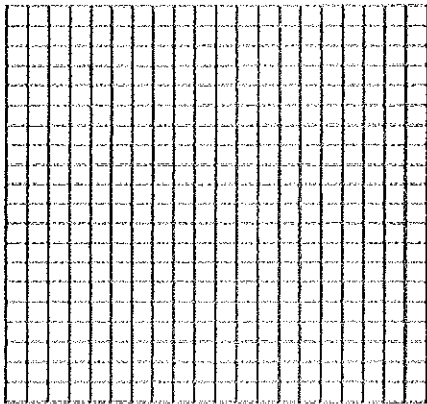
b.  $y = x^2 + x - 12$

2. AA6\_5: For each of the quadratic equations below, complete the following:

Write the coordinates of the  $x$  and  $y$  intercepts. Write the coordinates of the vertex. Write the equation in graphing (vertex) form. Sketch the graph of each parabola on its own set of axes. Place values of important points in your sketch.

1)  $y = 3(x + 2)(x - 1)$

2)  $y = x^2 - 2x + 1$



AA6\_6: Find the  $x$ -intercepts of each of the quadratics. Using an algebraic method in at least two of the problems. LEAVE ANSWERS IN FRACTIONAL FORM, DO NOT CONVERT TO DECIMAL!

a.  $y = 33x^2 - 3x$

b.  $y = -x^2 + x - 6$

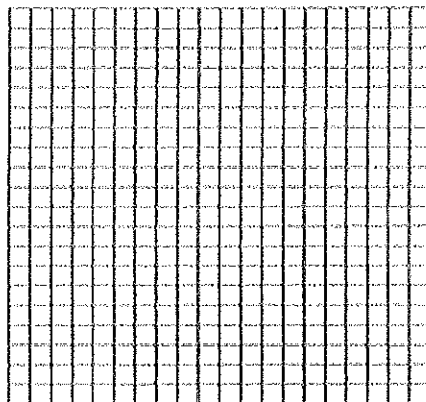
AA7\_1: Look inside, rewrite, or undo to solve each equation.

$\sqrt{1 - 2x} = 10$

$|2x + 1| = 5$

AA7\_2: Graph the following inequalities:

$y \geq |x + 2| - 3$  LEVEL 4



AA7\_3: The system  $y \leq -2(x - 3)^2 + 4$   
 $y > (x - 1)^2 - 2$

Show the final solutions for  $x$  on the number line below.

AA7\_4: Solve the inequality and graph the solution on the number line.

$$x^2 - 2x - 15 < 0$$

AA9\_1: Give the inverse of each equation. Leave in y form.

a)  $y = 3x - 2$

b)  $y = \frac{x+1}{4}$

c)  $y = x^3 + 1$

d)  $y = 1 + \sqrt{x+5}$

AA10\_1: Using your calculator, solve the problems below:

a.  $\log x^6 = 125$

b.  $x^{-4} = \log_2 100$

c.  $(x+2)^3 = 125$

d.  $(x-2)^{12} = 4096$

e.  $x^6 = 125$

f.  $x^{-4} = 100$

g.  $(x+2)^3 = 65$

h.  $4(x-2)^{12.5} = 2486$

AA10\_2: Solve the following problems for the appropriate variable:

a.  $5(1.31)^{2x} + 6 = 73$

b.  $2 \log_b(x) = \log_b(4) + \log_b(x-1)$

c.  $\log_2(x+3) - \log_2(2x-1) = 3$

g.  $5(1.31)^{3x} + 6 = 73$

d.  $\log_2(4a^2) + \log_2\left(\frac{a}{2}\right) = 10$

h.  $\log_2(x+3) - \log_2(2x-1) = 2$

e.  $\log_b x^2 = \log_b(2x-1)$

i.  $\log(3a) + \log\left(\frac{a}{2}\right) = 7$

f.  $\log_b(x^2 - 30) = \log_b(x)$

AA10\_3: Earthquake intensity is measured by the Richter scale. The formula for the Richter rating of a given quake is given by



$R = \log [ I \div I_0 ]$  where  $I_0$  is the "threshold quake", or movement that can barely be detected, and the intensity  $I$  is given in terms of multiples of that threshold intensity. You have a seismograph set up at home, and see that there was an event while you were out that had an intensity of

$$I = 989I_0.$$

a) Given that a heavy truck rumbling by can cause a microquake with a Richter rating of 3 or 3.5, and "moderate" quakes have a Richter rating of 4 or more, what was likely the event that occurred while you were out? (What was the Richter value of the event and was it an earthquake or possibly a heavy truck?)

b) If you recorded a Richter number of 7.4, how many more times intense would it be than the  $989I_0$  event?

**AA10\_4:** Suppose a colony of bacteria double in number every hour. Suppose the initial number of micrograms of bacteria is 0.3 micrograms. How many hours must pass before the colony has 2.1 milligrams? Note: a microgram is  $10^{-6}$  grams. A milligram is  $10^{-3}$  grams. Identify any variables in your equation. Answer should be to 3 decimal places.

**AA10\_5:** "Loudness" is measured in decibels. The formula for the loudness of a sound is given by

$dB = 10 \log [ I \div I_0 ]$  where  $I_0$  is the intensity of "threshold sound" or sound that can barely be perceived. ("dB" is one variable meaning decibels. It is **not** the quantity (d times b). Other sounds are defined in terms of how many times more intense they are than the threshold sound. For instance, a cat's purr is about 316 times as intense as threshold sound, for a decibel rating of:

$$\begin{aligned} db &= 10 \log \left[ \frac{I}{I_0} \right] \\ &= 10 \log \left[ \frac{(316I_0)}{I_0} \right] \\ &= 10 \log [316] \\ &= 24.997 \end{aligned}$$

or, about 25 decibels.

Prolonged exposure to sounds above 85 decibels can damage your hearing. So, if a rifle shot from a .22 caliber weapon has an intensity  $I$  equal to  $(2.5 * 10^{13}) * I_0$ , then should you wear ear protection on the rifle range?

To answer this question, first determine the decibel level of a rifle shot. Show all steps and be very clear about what you are doing. Justify your answer.

**AA15\_1:** A bag has 3 red balls, two green balls, and 4 yellow balls. Give the probability of each of the following outcomes: (Red = R, Green = G, Y= Yellow)

- a.  $P(R) =$
- b.  $P(Y) =$
- c.  $P(G \text{ or } R) =$
- d.  $P(G \text{ and } R)$  (pulling 2 balls out, NO REPLACEMENT) (Level 5) =

2. A family wants to have 3 children.

a. Show the sample space for all the outcomes of the three children. Use correct notation.

Level 4

b. What is the probability they will have 3 boys?

c. Assume they already have one boy (and therefore still need two more children). What is the probability they will end up with 3 boys?

**AA15\_2:** . Take a typical card deck of 52 cards, 4 suits (spades, diamonds, clubs and hearts). Thirteen cards for each suit.

a) What is the probability of pulling a heart out of the deck?

b) What is the probability of pulling a heart and then a club from a complete deck?

c) Suppose you are dealt 2 cards. What is the probability that you have a pair of twos?

d) Suppose you are dealt 2 cards from the deck. What is the probability that you have a pair (of any number ... e.g. aces, kings, tens etc). This is a kind of hard problem and is meant more as a general thinking problem.

**AA15\_3:** If a bag contains only two colors of chips, blue and red, and the probability of pulling a blue chip out of the bag is  $x$ : (note: after the chip is pulled out, it is put back in the bag).

a. What is the probability of pulling a red chip out?

b. Suppose two chips are pulled out. What is the probability the chips are different colors. Express as a function of  $x$ . **EXPLAIN AND SUPPORT YOUR REASONING!**

**AA15\_4:** Given the following table:

Area of Study	Class				
	Freshman	Soph	Junior	Senior	Totals
Natural Sciences	50	35	33	29	147
Social Sciences	20	25	28	24	97
Humanities	40	40	39	37	156
Totals	110	100	100	90	400

a. If a student is selected at random, what is the probability that:

1: the student's area of study is the natural sciences?

2: the student is a freshman in the social sciences?

b. If a sophomore is selected at random, what is the probability that his or her area of study is the humanities?

**AA15\_5:** Level 4 Given a die with 8 sides (numbered 1 to 8):

a. What is the sample space for this dice? Be EXACT in your definition of the sample space

b. Define the event EVEN that includes all even numbers of the sample space.

c. Define the event ODD that includes all the odd numbers in the sample space.

- d. Define the event `DivisibleByThree` that includes all numbers divisible by 3 with no remainder.
- e. What value is  $P(\text{ODD or DivisibleByThree})$ ?

**AA15\_6:** Machine A produces 60% of the ball bearings manufactured by a factory and Machine B produces the rest. Five percent of Machine A's bearings fail the required precision test. Two percent of Machine B's bearings fail the required precision test.

- a. Incorporate the above facts into a tree diagram.
- b. What percent of the bearings fail to pass the precision test?
- c. If a bearing is inspected and fails to have the required precision, what is the probability it was produced by Machine A?

**AA15\_7:** A certain element "decays". That is, it gives off subatomic particles as radiation. The radiation comes in three forms, alpha particles, beta particles, and gamma particles. A researcher studies the decay of this element over time and discovers that twice as many alpha particles are produced compared to beta particles. She also finds that twice as many gamma particles are produced as alpha particles.

- a. Convert the above information into a sample space and list the probability of each outcome (hint:  $P(S) = 1$ . USE CORRECT NOTATION!
- b. What is the probability that the first three particles will **NOT** be gamma particles?

**(That is: NONE of the 3 particles are gamma)**