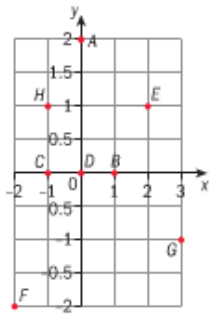
Summer Packet for IB Math SL/HL (1-2) Name: _____

This is your first HW for next year. Please complete prior to the first day of school. You are expected to be proficient with this material. **GDC** means **Graphing Display Calculator.** There is no space in this worksheet to work on the problems. Keep your work organized and labeled on separate pieces of paper. **I am sorry about the numbering** ☺

Basic

1 **a** Plot these points on a coordinate plane. $A(1,3)$, $B(5,-3)$, $C(4,4)$, $D(-3,2)$, $E(2,-3)$, $F(0,3)$.

b Write down the coordinates of points A to H.



2 Given that $x = 4$, $y = 6$ and $z = -10$, find

a $4x + 3y$ **b** $z^2 - 3y$ **c** $y - z$ **d** $\frac{2x+5}{yz}$

3 Solve

a $3x - 6 = 6$ **b** $5x + 7 = -3$ **c** $\frac{x}{2} + 6 = 11$

4 Graph these functions on your GDC within the given domain. Then sketch the functions on paper.

a $y = 2x - 3$, $-4 \le x \le 7$

b $y = 10 - 2x$, $-2 \le x \le 5$

c $y = x^2 - 3$, $-3 \le x \le 3$.

5 Expand

a $(x + 4)(x + 5)$ **b** $(x - 1)(x - 3)$

c $(x + 5)(x - 4)$

Functions: In IB we use this Notation $f(g(x)) = (f \circ g)(x)$

3 Use your GDC to sketch these graphs. Write down the domain and range of each.

a $y = 2x - 3$ **b** $y = x^2$

c $y = x^2 + 5x + 6$ **d** $y = x^3 - 4$

e $y = \sqrt{x}$ **f** $y = \sqrt{4 - x}$

g $y = \frac{1}{x}$ **h** $y = e^x$

i $y = \frac{1}{x+2}$ **j** $y = \frac{x+4}{x-2}$

k $y = \frac{x^2-9}{x+3}$ **l** $y = \frac{2}{x^2+1}$

1 If $f(x) = \frac{x+4}{2}$ and $g(x) = 2x - 4$, find

a **i** $g(1)$ and $(f \circ g)(1)$ **ii** $f(-3)$ and $(g \circ f)(-3)$

iii $(f \circ g)(x)$ **iv** $(g \circ f)(x)$

b What does this tell you about functions f and g?

2 Find the inverse for each of these functions.

a $f(x) = 3x - 1$ **b** $g(x) = x^3 - 2$ **c** $h(x) = \frac{1}{4}x + 5$

d $f(x) = \sqrt[3]{x} - 3$ **e** $g(x) = \frac{1}{x} - 2$ **f** $h(x) = 2x^3 + 3$

g $f(x) = \frac{x}{3+x}, x \ne -3$ **h** $g(x) = \frac{2x}{5-x}, x \ne 5$

3 If $g(x) = 4x - 5$ and $h(x) = 7 - 2x$

a find x when $g(x) = 3$

b find x when $h(x) = -15$

c find x when $g(x) = h(x)$.

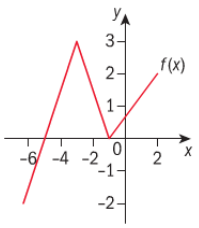
4 **a** If $h(x) = \frac{1}{x-6}$ find $h(-3)$.

b Is there a value where $h(x)$ does not exist? Explain.

4 Copy the graph of $f(x)$. Sketch the graph of each of these functions, and state the domain and range for each.

a $2f(x-5)$

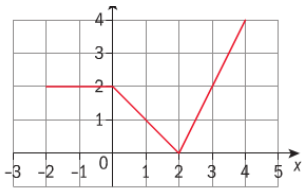
b $-f(2x)+3$



11 The graph shows the function $f(x)$, for $-2 \le x \le 4$.

a Let $h(x) = f(-x)$. Sketch the graph of $h(x)$.

b Let $g(x) = \frac{1}{2} f(x-1)$. The point $A(3, 2)$ on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P.



1 If $f(x) = \frac{x+4}{2}$ and $g(x) = 2x-4$, find

a **i** $g(1)$ and $(f \circ g)(1)$ **ii** $f(-3)$ and $(g \circ f)(-3)$

iii $(f \circ g)(x)$ **iv** $(g \circ f)(x)$

b What does this tell you about functions f and g?

Quadratics

2 Factorize each expression.

a $2k^2 - 10k$

b $14a^3 + 21a^2 - 49a$

c $2x^2 + 4xy + 3x + 6y$

d $5a^2 - 10a - ab + 2b$

e $n^2 + 4n + 3$

f $2x^2 - x - 3$

g $m^2 - 36$

h $25x^2 - 81y^2$

1 Solve each equation.

a $3a - 5 = a + 7$

b $4x^2 + 1 = 21$

c $3(n-4) = 5(n+2)$

1. Factor the expressions below completely

a. $3x^2+11x+10$ b. $6x^3-31x^2+5x$ c. $6ab^2+15ab-21a$

d. $x^2+5x-24$ e. x^3-3x^2-18x f. $5x^3-125x$

3 The two perpendicular sides of a right-angled triangle have lengths $x + 2$ and $5x - 3$.
The hypotenuse has length $4x + 1$. Find x.

2 Find the values of p such that the equation has two different real roots.

a $x^2 + 4x + p = 0$ **b** $px^2 + 5x + 2 = 0$

3 Find the values of k such that the equation has two equal real roots.

a $x^2 + 10x + k = 0$ **b** $2x^2 - 3x + k = 0$

5 Find the values of q for which the quadratic equation $qx^2 - 4qx + 5 - q = 0$ will have no real roots.

2 The height, h metres above the water, of a stone thrown off a bridge is modeled by the function $h(t) = 15t + 20 - 4.9t^2$, where t is the time in seconds after the stone is thrown.

a What is the initial height from which the stone is thrown?

b What is the maximum height reached by the stone?

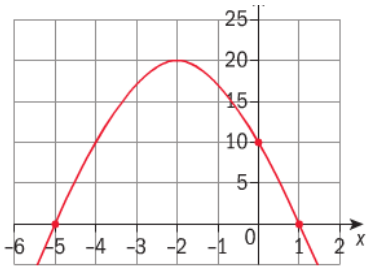
c For what length of time is the height of the stone greater than 20 m?

d How long does it take for the stone to hit the water below the bridge?

3 Let $f(x) = a(x-p)(x-q)$. Part of the graph of f is shown. The graph passes through the points $(-5, 0)$, $(1, 0)$ and $(0, 10)$.

a Write down the value of p and of q.

b Find the value of a.



Exponents and Logarithms

3 Solve $8(2^{x+1}) = 2\sqrt{2^x}$

1 Given that $p = \log_2 a$ and $q = \log_2 b$, find an expression in terms of p and/or q for:

a $\log_2 ab$ **b** $\log_2 a^3$ **c** $\log_2 \frac{b}{a}$

d $\log_2 \sqrt{b}$ **e** $\log_2 \frac{b^2}{\sqrt{a}}$

1 Solve these equations for x.

a $\log_2(x) = \log_2(6x-1)$ **b** $\ln(x+1) = \ln(3-x)$

c $\log_5(2-x) = \log_5(6x-1)$ **d** $\log_2(2x+3) + \log_2(x-1) = \log_2(x+1)$

e $\log_3 x - \log_3(x-1) = \log_3(x+1)$

3 Given that $\log_2 x + \log_2(2x+7) = \log_2 A$ find an expression for A in terms of x.

Hence or otherwise solve $\log_2 x + \log_2(2x + 7) = 2$

4 Solve $\log_4 x + \log_x 4 = 2$

5 Solve $\log_2 x^2 + \log_4 \sqrt{x} = 9$

5 Solve

a $\log_3(4x-1) = 3$ **b** $\log_{x+1}(x-1) = 2$

c $\log_3(2\log x) = 4$ **d** $\log_2(x-2) + \log_{\frac{1}{2}}(x-1) = 3$

5 The number, n, of insects in a colony, is given by $n = 4000e^{0.08t}$ where t is the number of days after observation commences.

a Find the population of the colony after 50 days.

b How long does it take the population to double from when the observations commenced?

Trigonometry

➔ The **angle of elevation** is the angle 'up' from horizontal.
The **angle of depression** is the angle 'down' from horizontal.

8 Buildings X and Y are across the street from each other, 95 m apart. From a point on the roof of Building X, the angle of depression to the base of Building Y is 55° and the angle of elevation to the top of Building Y is 35°. How tall are the two buildings?

The four cardinal **compass points** are north (N), south(S), east (E) and west (W).
Three-figure bearings give directions as angles measured clockwise from north.

3 A ship is sailing due west when the captain sees a lighthouse at a distance of 20 km on a bearing of 230°.
- **a** Draw a diagram to model this situation.
- **b** How far must the ship sail before the lighthouse is 16 km away?
- **c** How far must the ship sail beyond this point before the lighthouse is again at a distance of 16 km from the ship?
- **d** What is the bearing of the lighthouse from the ship the second time the two are 16 km apart?

2 A hiker leaves camp and walks 5 km on a bearing of 058°. He stops for a break, then continues walking for another 8 km on a bearing of 103°. He stops again before heading straight back. How far must he walk to get back to camp?

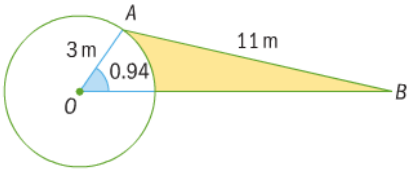
1 Convert these angles to radians.
Give exact values.

a 75° **b** 240° **c** 80° **d** 330°

3 Convert these angles to degrees.
Give exact values.

a $\frac{5\pi}{6}$ **b** $\frac{5\pi}{3}$ **c** $\frac{3\pi}{2}$ **d** $\frac{5\pi}{4}$

4 The diagram shows the circle, center O, with radius 3 m, $AB = 11$ and $A\hat{O}B = 0.94$ radians. Find the shaded area.



2 Given that $\sin\frac{\pi}{6} = \frac{1}{2}$ and $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, find each value.

a $\sin\frac{7\pi}{6}$ **b** $\cos\frac{5\pi}{6}$ **c** $\sin\left(-\frac{\pi}{6}\right)$ **d** $\cos\left(-\frac{11\pi}{6}\right)$

3 Given that $\sin A = 0.8$ and $\cos A = 0.6$, find each value.

a $\sin(180° - A)$ **b** $\cos(-A)$ **c** $\cos(360° - A)$
d $\sin(180° + A)$ **e** $\tan A$ **f** $\tan(-A)$
g $\sin(360° - A)$ **h** $\tan(180° + A)$

3 A Ferris wheel at an amusement park reaches a maximum height of 46 metres and a minimum height of 1 metre. It takes 20 minutes for the wheel to make one full rotation.

a If a child gets on the Ferris wheel when $t = 0$, how high will he be after riding for 10 minutes?
b Write a sine function to model the height of the child t minutes after boarding the Ferris wheel.
c How high is the child if he has been riding for 3 minutes?
d For what length of time will the child be higher than 40 metres?

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5 The graph of f, for $0 \le x \le 9$, is shown.

a Given that the function can be written in the form

$f(x) = a\sin(b(x-c)) + d$,

i find the values of a, c and d.

ii explain why $b = \frac{\pi}{4}$.

b Write down the interval for which $f(x) > 6$.

The graph has a maximum at (6, 11) and a minimum at (2, 1)

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6 Given that $\cos x = \frac{2}{5}$, and x is an acute angle, find

a $\sin x$ **b** $\tan x$ **c** $\sin 2x$

7 Sketch the graph of the function $f(x) = 3\cos\left(\frac{2\pi}{5}(x+1)\right) - 2$, for $-3 \le x \le 5$.

Statistics

2 A machine produces bolts with diameters distributed normally with a mean of 4 mm and a standard deviation of 0.25 mm. Bolts are measured accurately and any which are smaller than 3.5 mm or bigger than 4.5 mm are rejected. Out of a batch of 500 bolts how many would be acceptable?

1 The depths of snow at a ski resort are collected every year for 12 years on 31 January. All data is in centimetres.
30, 75, 125, 55, 60, 75, 65, 65, 45, 120, 70, 110.
Find **a** the range, **b** the median, **c** the lower quartile, **d** the upper quartile and **e** the interquartile range of the data set and show the data in a box and whisker plot.

7 The Onceonly family must drive an average of 250 km per day to complete their vacation on time. On the first five days, they travel 220 km, 300 km, 210 km, 275 km and 240 km. How many km must they travel on the sixth day in order to finish their vacation on time?

4 The number of children in the families in a class of 29 children is shown below. Find the mean and standard deviation.

Children	1	2	3	4	5	6	7
f	5	12	8	3	0	0	1

Other algebra problems

Solve using the algebraic method of your preference. Clearly state your answers as an ordered pair (x, y):

3) $\begin{aligned} \frac{1}{2}x - y &= 1 \\ y &= \sqrt{x-4}+1 \end{aligned}$

4) $\begin{aligned} 3x+7y &= 32 \\ x &= 5y-10 \end{aligned}$

5) $\begin{aligned} 6x+10y &= 32 \\ 4x-2y &= 4 \end{aligned}$

6) $\begin{aligned} \frac{1}{2}y &= x-1 \\ \frac{1}{3}x^2+1 &= y \end{aligned}$

7) $\begin{aligned} x &= y^2 \\ x-y &= 6 \end{aligned}$

8) $\begin{aligned} x^2+y^2 &= 25 \\ y &= x^2-13 \end{aligned}$

Solve for x. Eliminate extraneous solutions, if any.

a. $3x^2-4x+2=x^2+x-6$

b. $\sqrt{37-3x}=x-3$

c. $\frac{4x-1}{x+1}=x-1$

d. $\frac{4x-1}{x}=3x$

e. $5x^2-x+7=0$

f. $x-5=\sqrt{3x-11}$

g. $2|3x-1|+5=-2x+8$

h. $\frac{x+3}{x}-\frac{x-1}{x+3}=23$

i. $-3(2x+1)^3=-192$