Questions?

We've done discrete probability distributions.

Now let's look at continuous p.d.s

Suppose, as a NASA engineer, you've been given the task of developing the life support components on a manned mission to Mars. For safety, and to meet NASA requirements, the system you've designed has 5 fail-safe life-support components. In this way if one component fails there are backup systems to keep everyone alive.

Suppose the manufacturer guarantees the probability of a single component failing at 0.06.

As the engineer, you may be asked to answer questions such as these.

- What's the probability of all components failing during the 9 month trip to Mars?

- What's the probability of 3 failing?

- What's the probability of none failing?

- At a failure probability of 0.06 how many fail-safe components do you need to ensure a 99.99% chance of making it Mars alive?

To answer these and similar questions we use the **Binomial Distribution**.

Number of life insurance holders who will claim in a given period.

Number of loan holders who will default in a certain period.

Number of false starts of a car in n attempts.

Number of faulty items in n from a production line.

Number of n people randomly selected from a population who will have some characteristic.

Number of machines that last longer than T hours of operation without failure.

Blood samples that have zero, or >0 antibodies.

# 1. The Binomial Distribution

How to recognize and when to use:

- Only 2 possible outcomes.

e.g. {Success, Failure}, {Heads, Tails}, {Boy, Girl}

- Fixed or known number of trials.

- Probability of success remains the same for all trials.

## 1. The Binomial Distribution (cont.)

If the experiment has:

n trials

with p = probability of success

and q = probability of failure (remember: q = 1-p)

then the probability of x successes is:

$$P(x) = nCx * p^x * q^{n-x}$$

## 1. The Binomial Distribution (cont.)

The binomial distribution has:

measure of center: mean $\mu = n * p$

measure of spread:

standard deviation $\sigma = \sqrt{( n * p * q )}$

E.g. 1

A student takes a 10 question true-false test and guesses at all the answers. Find the probability of getting 6 out of the 10 correct.

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$n = 10 \quad P = 0.5 \quad q = 0.5 \quad x = 6$

$P(6) = 10C6 * 0.5^{6} * 0.5^{10-6}$

$P(6) = 210 * 0.0156 * 0.0625$

$P(6) = 0.205$

Which means 20.5% of the time you will get 6/10 correct if you guess at all the answers.

E.g. 2a

It has been reported that 5% of North Americans are afraid of being alone in their house at night. If a random sample of 20 people are selected find the probability that exactly 5 people in the sample are afraid of being alone.

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n=20 p=0.05 q=0.95 x=5

$P(5) = 20C5 * 0.05^{5} * 0.95^{15} = \underline{0.0022}$

E.g. 2b

It has been reported that 5% of North Americans are afraid of being alone in their house at night. If a random sample of 20 people are selected find the probability that at most 2 people in the sample are afraid of being alone.

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n=20 p=0.05 q=0.95 x=0, or 1, or 2

P(0) = 0.358, P(1) = 0.377, P(2) = 0.189

P(at most 2) = 0.358 + 0.377 + 0.189 = <u>0.924</u>

E.g. 2c

It has been reported that 5% of North Americans are afraid of being alone in their house at night. If a random sample of 20 people are selected find these probability that at least 3 people in the sample are afraid of being alone.

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n=20 p=0.05 q=0.95 x=3, 4, 5, ... 20

We can use 1 – P(at most 2)

P(at least three) = 1 – 0.924 = <u>0.076</u>

Depakote is a medication whose purpose is to reduce the pain associated with migraine headaches. In clinical trials of Depakote, 2% of the patients in the study experienced weight gain as a side effect.

Compute the mean and standard deviation of X, the number of patients experiencing weight gain in 600 trials of this experiment.

What does the mean tell us?

Would it be unusual to observe 19 patients experiencing weight gain in a random sample of 600 patients who take the medication? Justify!

Is the probability distribution from tossing a coin a binomial?

Which is the most likely outcome when tossing a coin 10 times?

{HTHTHTHTHT}

{HHHHHTTTTT}

{HHTHHTTTHT}

{HHTHTHHTHT}

Which is more likely when tossing a coin 10 times?

5 Heads and 5 Tails

6 Heads and 4 Tails

7 Tails and 3 Heads

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